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Properties of low-lying charmonia and bottomonia from lattice QCD + QED

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The precision of lattice QCD calculations has been steadily improving for some time and is now approaching, or has surpassed, the 1% level for multiple quantities. At this level QED effects, *i.e.* the fact that quarks carry electric as well as color charge, come into play. In this report we will summarise results from the first lattice QCD+QED computations of the properties of ground-state charmonium and bottomonium mesons by the HPQCD Collaboration.

Keywords: Charmonium; bottomonium; lattice QCD; lattice QCD+QED.

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1. Introduction

Lattice QCD has been the gold standard for calculating properties of hadrons in Standard Model for a long while [1]. For many quantities, such as masses and decay constants of ground-state pseudoscalar mesons, calculations have now reached, or surpassed, statistical precision of 1%. This precision of modern lattice QCD results means that sources of small systematic uncertainty that could appear at the percent level need to be understood and quantified. Here we focus on QED effects.

In the following section we briefly introduce the lattice QCD setup, as well as describe how we include QED in the calculation. In section 3. we summarise our results on charmonium and bottomonium hyperfine splittings and decay constants published in [2–4].

2. Lattice calculation

We use gluon field configurations generated by the MILC collaboration [5,6]. We use 17 different ensembles: six different lattice spacings from very coarse ($a \approx 0.15$ fm) to exafine ($a \approx 0.03$ fm), and a range of light quark masses (including close to physical masses) to control the chiral extrapolation. Most ensembles have $2+1+1$ flavours, *i.e.* light, strange and charm quarks in the sea (with degenerate u and d quarks whose mass is $m_l = (m_u + m_d)/2$). However, we use one ensemble with $n_f = 1+1+1+1$, where both u and d quarks have their respective physical masses.

The Highly Improved Staggered Quark (HISQ) action [7], which removes tree-level a^2 discretisation errors, is used for both sea and valence quarks. For heavy quarks the 'Naik' term is adjusted to remove $(am)^4$ errors at tree-level, which makes the action very well suited for calculations that involve c quarks. For the b quarks we use the so called heavy-HISQ method [8], *i.e.* do the calculation at several heavy valence quark masses $m_h > m_c$ to extract quantities at the physical b mass.

2.1. QED on the lattice

To study the systematic effects related to the fact that quarks carry both electric and color charge, we have to include QED in our QCD calculation. We use quenched QED, *i.e.* we include effects from the valence quarks having electric charge (the largest QED effect) but neglect effects from the electric charge of the sea quarks. In short, the calculation goes as follows (see [2] for details):

- Generate a random momentum space photon field $A_\mu(k)$ for each QCD gluon field configuration and set zero modes to zero using the QED$_L$ formulation (QED in finite box).
- Fourier transform A_μ into position space. The desired $U(1)$ QED field is then the exponential of A_μ, $\exp(ieQA_\mu)$, where Q is the quark electric charge in units of the proton charge e.
- c and b lattice quark masses have to be tuned separately in pure QCD and QCD+QED so that J/ψ and Υ masses match experiment.

2.2. Extraction of energies and decay constants

We calculate the quark-line connected correlation functions of pseudoscalar and vector mesons on each ensemble and use a multi-exponential fit to extract amplitudes and energies:

$$C_{\text{2-point}}(t) = \sum_i A_i \left(\mathrm{e}^{-E_i t} + \mathrm{e}^{-E_i (L_t - t)} \right). \qquad (1)$$

The decay constants are related to the ground state ($i = 0$) amplitude and meson mass:

$$f_P = 2 m_q \sqrt{\frac{2A_0^P}{(M_o^P)^3}}, \quad f_V = Z_V \sqrt{\frac{2A_0^V}{M_0^V}}. \qquad (2)$$

The renormalisation constant Z_V is needed to match the lattice vector current to that in continuum QCD, as we use a non-conserved lattice vector current [9]. The current used for the decay constant f_P is absolutely normalised, and no renormalisation factor is required.

We then take the results at different lattice spacings and extrapolate to the continuum, taking into account $(am_q)^{2n}$ and $(a\Lambda)^{2n}$ discretisation effects. Terms that allow for mistuned sea quark masses are also included. For bottomonium, we map out the dependence in quark mass to extract the result at the physical m_b.

3. Charmonium and bottomonium

Let us now summarise our results on charmonium and bottomonium hyperfine splittings and decay constants.

3.1. Hyperfine splitting

In Fig. 1 we plot the hyperfine splitting as a function of lattice spacing, the blue hexagons and violet triangles showing our results on different ensembles in pure QCD and in QCD+QED, respectively. Our extrapolation to the continuum and to physical quark masses is shown by the turquoise error band. The red error band gives our physical result, and the black cross and the black error band show the average experimental result from Particle Data Group [10]. Our final

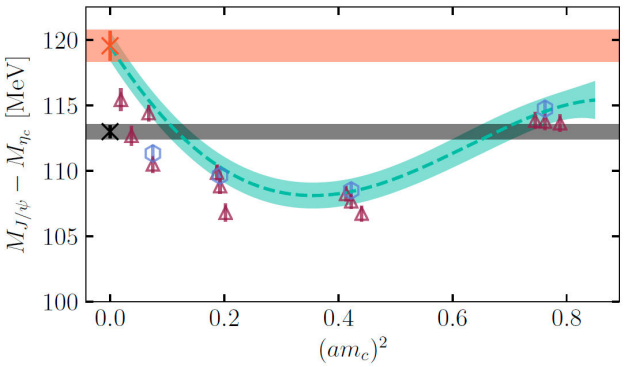

FIGURE 1. Charmonium hyperfine splitting as a function of lattice spacing. This figure is from [2].

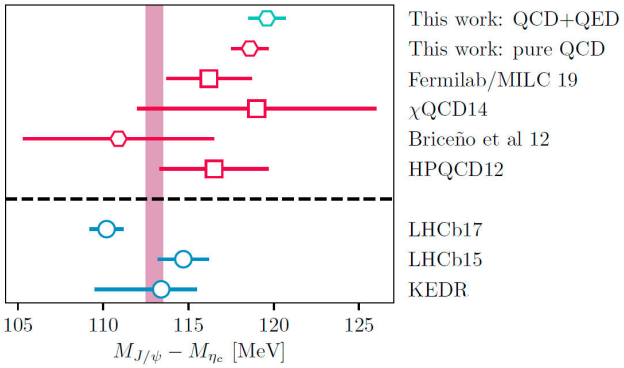

FIGURE 2. Charmonium hyperfine splitting. This figure is from [2].

QCD+QED result for the charmonium hyperfine splitting is $M_{J/\psi} - M_{\eta_c} = 120.3(1.1)$ MeV.

For the first time we see a significant, 6σ difference between the experimental average and a lattice calculation. Note that quark-line disconnected correlation functions are not included in the lattice calculation. The difference between our result and the experimental result is then taken to be the effect of the η_c decay to two gluons (prohibited in the lattice calculation): $\Delta M_{\eta_c}^{\text{annihln}} = +7.3(1.2)$ MeV.

In Fig. 2 we compare our result for $M_{J/\psi} - M_{\eta_c}$ with other lattice QCD results as well as with experimental results that measure this difference. The results are from the following publications: Fermilab/MILC [11], χQCD [12], Briceno [13], HPQCD [14], LHCb [15, 16] and KEDR [17]. The PDG average, shown as the purple error band, is obtained from taking the differences of the PDG J/ψ and η_c masses rather than only from experiments that directly measure the splitting.

To study the bottomonium hyperfine splitting, we map out the dependence in m_h to extract the result at physical m_b. This is illustrated in Fig. 3, where we plot our results on different lattice ensembles as a function of the heavy vector meson mass M_{ϕ_h} (which is a proxy for the heavy quark mass). The error band shows the extrapolation to the continuum, and the black cross shows the experimental average from Particle Data Group [18].

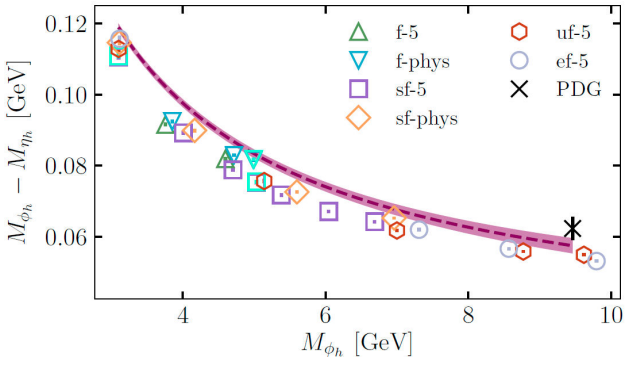

FIGURE 3. Bottomonium hyperfine splitting. This figure is from [4].

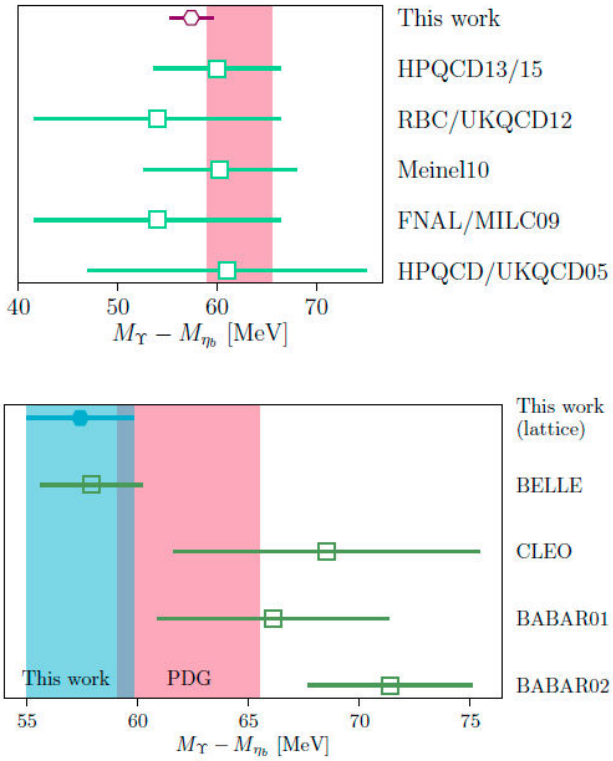

FIGURE 4. Bottomonium hyperfine splitting. These figures are from [4].

Our QCD+QED result for bottomonium hyperfine splitting is $M_\Upsilon - M_{\eta_b} = 57.5(2.3)(1.0)$ MeV. The missing quark-line disconnected contributions (allowed for by the second uncertainty) are expected to be smaller for bottomonium than charmonium, and here we find good agreement with experiment.

We compare our results to other lattice QCD results and experimental results in Fig. 4. These results are from the following publications: lattice calculations by HPQCD/UKQCD [19], Fermilab/MILC [20], Meinel [21], RBC/UKQCD [22] and HPQCD [23], and experimental results from Belle [24], CLEO [25] and BaBar [26, 27] as well as the experimental average from Particle Data Group [18]. All lattice calculations show good agreement, but there is some tension between the different experimental results with our value favouring (but not significantly) the most recent lower result from Belle.

3.2. Decay constants

The decay constant of a pseudoscalar meson P (*e.g.* η_c or η_b) is defined in terms of the axial current as

$$\langle 0|A_\alpha|P\rangle = p_\alpha f_P. \tag{3}$$

Using the PCAC relation this can be written as

$$\langle 0|\bar{\Psi}_q\gamma_5\Psi_q|P\rangle = \frac{(M_0^P)^2}{2m_q} f_P. \tag{4}$$

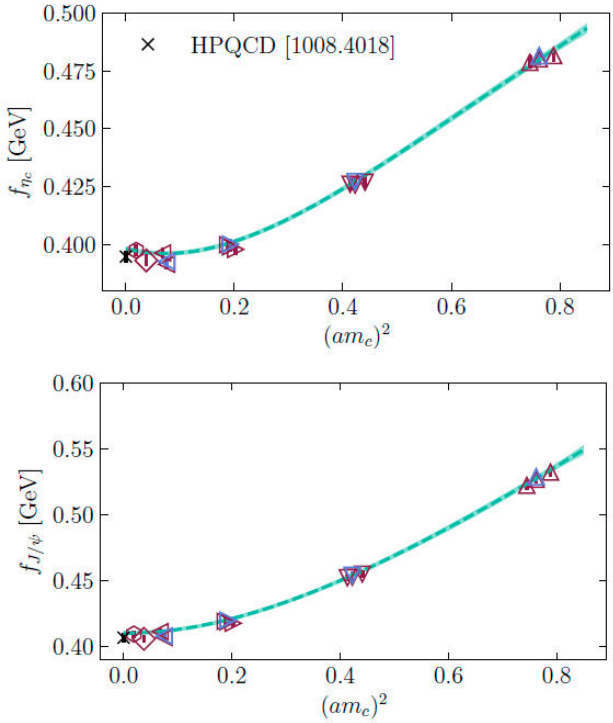

FIGURE 5. Charmonium decay constants. These figures are from [2].

For a vector meson (*e.g.* J/ψ or Υ) the vector decay constant is defined through the vector current

$$\langle 0|\bar{\Psi}_q\gamma_\alpha\Psi_q|V\rangle = f_V M_V \epsilon_\alpha, \tag{5}$$

where ϵ is the polarisation vector of the meson.

The tensor decay constant of the vector meson is

$$\langle 0|\bar{\Psi}_q\sigma_{\alpha\beta}\Psi_q|V\rangle = i f_V^T(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha). \tag{6}$$

Note that the tensor decay constant is scale- and scheme-dependent, unlike the vector decay constant f_V.

The decay constants can be written in terms of meson masses and amplitudes — see Eq. (2) along with

$$f_T = Z_T\sqrt{\frac{2A_0^T}{M_0^V}}, \tag{7}$$

using amplitudes from a tensor-tensor correlation function.

Our results for the charmonium pseudoscalar and vector decay constants f_{η_c} and $f_{J/\psi}$ on different lattice ensembles are plotted as a function of the lattice spacing in Fig. 5. The error band shows our extrapolation to the physical point. For f_{η_c}, the black cross shows the result from an earlier lattice calculation by the HPQCD collaboration [28], whereas for $f_{J/\psi}$ the black cross shows the result determined from the experimental average for $\Gamma(J/\psi \to e^+e^-)$. Our QCD+QED results at the physical point are [2] $f_{J/\psi} = 410.4(1.7)$ MeV, $f_{\eta_c} = 398.1(1.0)$ MeV and $f_{J/\psi}/f_{\eta_c} = 1.0284(19)$.

The decay constants from the QCD+QED calculation are compared with the pure QCD results in Fig. 6. The QED effects are very small, but at this precision they have to be

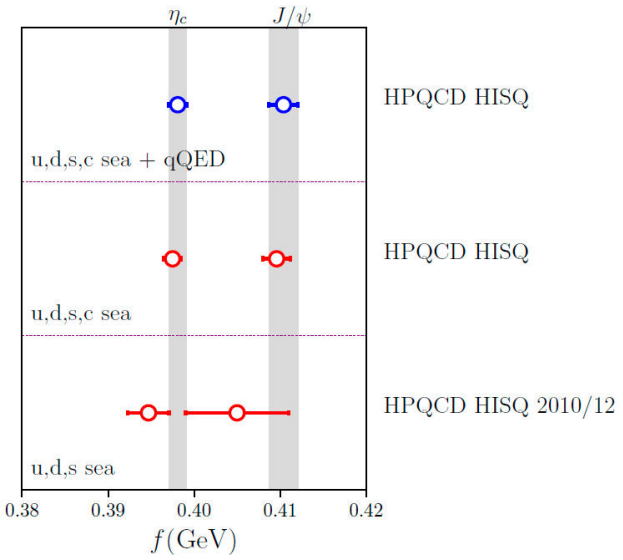

FIGURE 6. Charmonium decay constants.

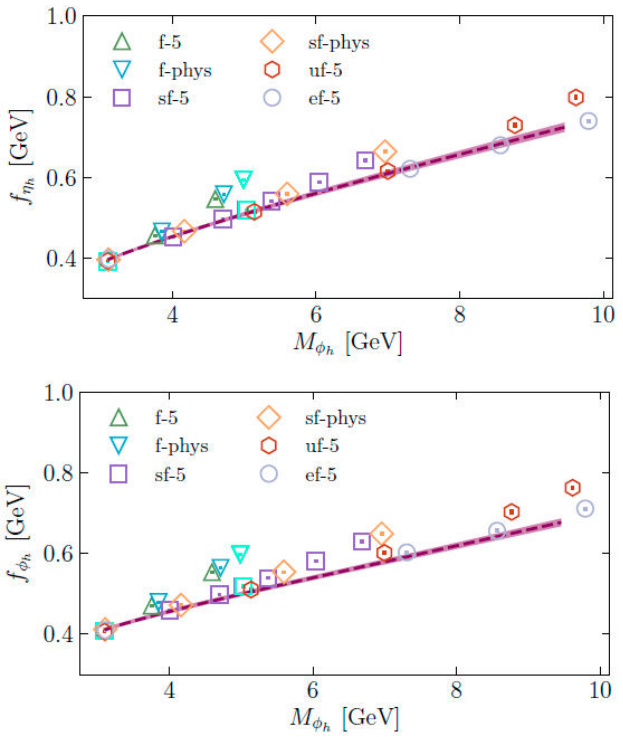

FIGURE 7. Bottomonium decay constants. These figures are from [4].

taken into account. Figure 6 also compares these new results to an earlier lattice calculation by the HPQCD collaboration that had only u, d and s quarks in the sea [14, 28]. The improvement in the precision highlights how far lattice calculations have come.

For bottomonium, we map the dependence of the pseudoscalar decay constant f_{η_h} and the vector decay constant f_{ϕ_h} on the heavy quark mass, and extrapolate to the continuum and physical masses in the same way as for the hyperfine splitting. This is illustrated in Fig. 7, that shows lattice results from individual ensembles as well as the extrapolation for both decay constants as a function of the vector meson mass M_{ϕ_h}. The results at the physical point are [4] $f_\Upsilon = 677.2(9.7)$ MeV, $f_{\eta_b} = 724(12)$ MeV, and $f_\Upsilon/f_{\eta_b} = 0.9454(99)$. For charm the ratio $f_{J/\psi}/f_{\eta_c}$ is greater than 1, but for b quarks this is now shown to be < 1.

As we briefly mentioned earlier, the partial decay width of a vector meson to a lepton pair is directly related to the decay constant:

$$\Gamma(\phi_h \to l^+l^-) = \frac{4\pi}{3}\alpha_{\rm QED}^2 Q^2 \frac{f_{\phi_h}^2}{M_{\phi_h}}, \qquad (8)$$

where Q is the electric charge of the quark. We can thus use our results for the vector decay constants to calculate leptonic widths and compare with experiments, or vice versa.

Our results are: $\Gamma(J/\psi \to e^+e^-) = 5.637(47)(13)$ keV and $\Gamma(\Upsilon \to e^+e^-) = 1.292(37)(3)$ keV, and we show the

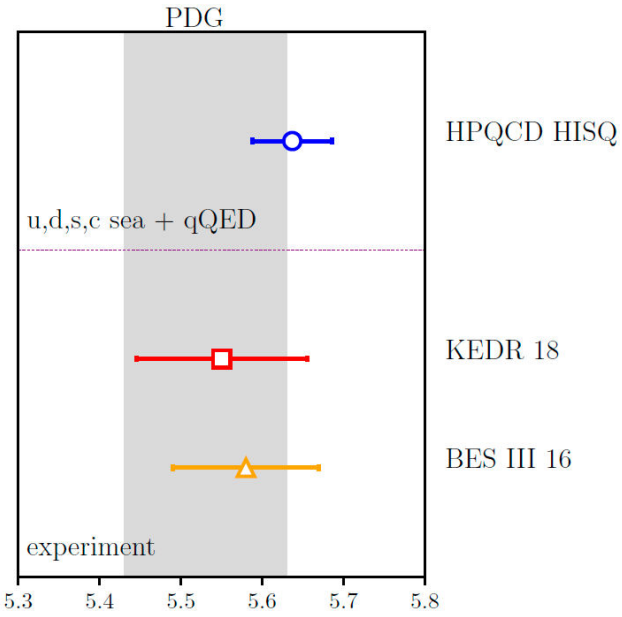

FIGURE 8. Leptonic width $\Gamma(J/\psi \to e^+e^-)$ [keV] (from [2]).

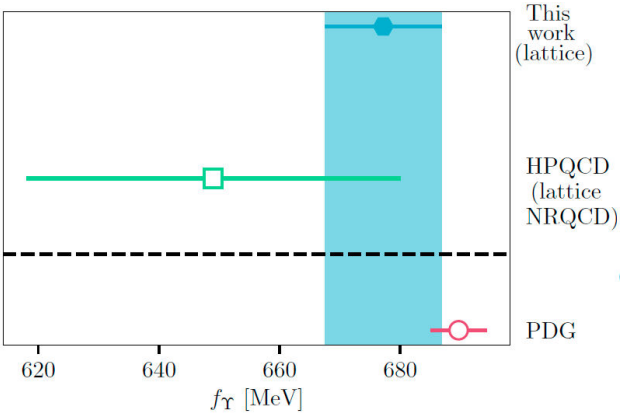

FIGURE 9. Bottomonium decay constant — comparing lattice QCD (top result) with that inferred from experiment for $\Gamma(\Upsilon \to e^+e^-)$ (bottom result). This figure is from [4].

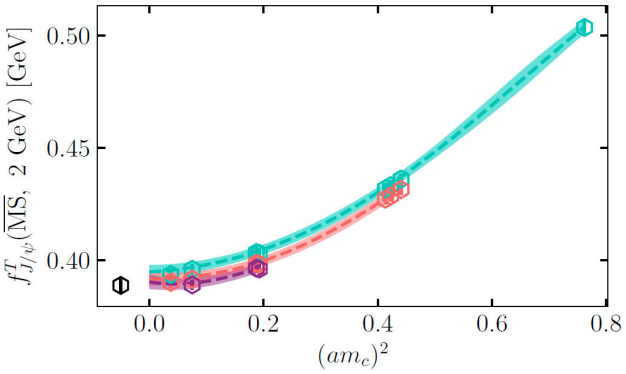

FIGURE 10. Tensor decay constant $f^T_{J/\psi}$. This figure is from [3].

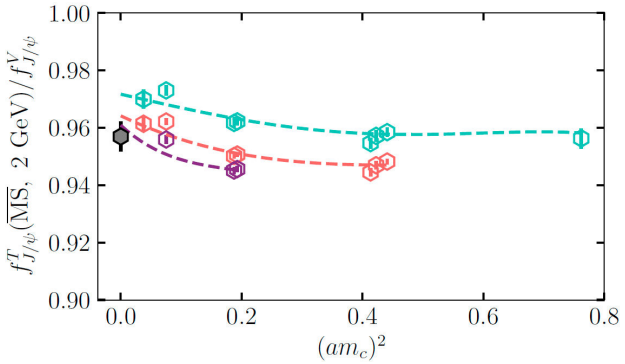

FIGURE 11. The ratio of tensor and vector decay constants. This figure is from [3].

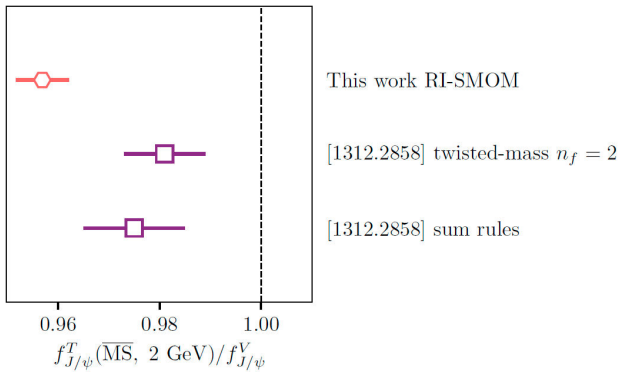

FIGURE 12. Comparison of the ratio of tensor and vector decay constants.. This figure is from [3].

comparison with experiment in Figs. 8 (charmonium) and 9 (bottomonium). The agreement is seen to be good, and the result from lattice for $\Gamma(J/\psi \to e^+e^-)$ is now more precise than the experimental average from Particle Data Group. There is no experimental decay rate that can be directly compared with the pseudoscalar decay constant.

We now turn to determining the J/ψ tensor decay constant $f^T_{J/\psi}$. Recall that the tensor decay constant is scale and scheme dependent, unlike the pseudoscalar and vector decay constants. The calculation (published in [3]) can be summarised as follows:

1. Extract $\sqrt{2A_0^T/M_0^T}$ from tensor-tensor correlators.
2. Calculate the renormalisation factor $Z_T^{\rm SMOM}$. Convert f^T to the $\overline{MS}$ scheme at multiple scales μ using the RI-SMOM scheme as an intermediate scheme on each ensemble.
3. Run all the $\overline{MS}$ tensor decay constants at a range of scales μ to a reference scale of 2 GeV using a three-loop calculation of the tensor current anomalous dimension. Here $\mu = 2, 3, 4$ GeV.
4. Fit all of the results for the $\overline{MS}$ decay constant at 2 GeV to a function that allows for discretisation effects and non-perturbative condensate contamination coming from $Z_T^{\rm SMOM}$.

The continuum extrapolation is illustrated in Fig. 10. We plot the tensor decay constant in the $\overline{MS}$ scheme at a scale of 2 GeV using lattice tensor current renormalisation in the RI-SMOM scheme at multiple μ values. These three values are shown as different coloured lines. The blue line is 2 GeV, the orange, 3 GeV and the purple, 4 GeV. The black hexagon is the physical result for $f^T_{J/\psi}(2\ {\rm GeV})$ obtained from the fit (with the condensate contamination removed).

In addition to the tensor decay constant $f^T_{J/\psi}(2\ {\rm GeV})$, we also determine the ratio of the tensor and vector decay constants, $f^T_{J/\psi}/f^V_{J/\psi}$. The extrapolation of the ratio to continuum is illustrated in Fig. 11. The colour coding for the lines and data points is the same as in Fig. 10.

Our (pure QCD) results for the J/ψ tensor decay constant and its ratio with the vector decay constant are [3] $f^T_{J/\psi}(\overline{MS}, 2\,{\rm GeV}) = 0.3927(27)\ {\rm GeV}$ and $f^T_{J/\psi}(\overline{MS}, 2\,{\rm GeV})/f^V_{J/\psi} = 0.9569(52)$. The ratio is compared to other lattice QCD and QCD sum rule calculations [29] in Fig. 12. Our result for the ratio is slightly (but not significantly) lower than other results. The new determination of $f^T_{J/\psi}$ is much more precise than the previous determinations. This is potentially useful for tests of BSM physics.

HPQCD's results show the high precision achievable now for the properties of ground-state heavyonium mesons. In future this precision will be extended up the spectrum to excited states.

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