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New physics and the tau polarization vector in $b \to c\tau^- \bar{\nu}_\tau$ decays

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For a general $H_b \to H_c\tau\bar{\nu}_\tau$ decay, we analyze the role of the τ polarization vector $\mathcal{P}^\mu$ in the context of lepton flavor universality violation studies. We use a general phenomenological approach that includes several new physics terms and we make specific evaluations of $\mathcal{P}^\mu$ for different decays. We show that a $\mathcal{P}^\mu$ component orthogonal to the plane determined by the final hadron and τ three-momenta is only possible for complex Wilson coefficients and it would be associated to a violation of the CP symmetry.

Keywords: Beyond the Standard Model; CP violation; Lepton Flavour Universality.

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1. Introduction

The present discrepancies between experimental data for $b \to c$ semileptonic decays and standard model (SM) predictions are indications of lepton flavor universality (LFU) violation, and thus of the possible existence of new physics (NP) beyond the SM affecting only the third quark and lepton generations. The discrepancy between predictions and data have been especially seen in the ratios

$$\mathcal{R}_D = 0.340 \pm 0.027 \pm 0.013, \quad \mathcal{R}_D^{\rm SM} = 0.299 \pm 0.003,$$
$$\mathcal{R}_{D^*} = 0.295 \pm 0.011 \pm 0.008, \quad \mathcal{R}_{D^*}^{\rm SM} = 0.258 \pm 0.05,$$
$$\mathcal{R}_{J/\psi} = 0.71 \pm 0.17 \pm 0.18, \quad \mathcal{R}_{J/\psi}^{\rm SM} \approx 0.25, -0.28. \quad (1)$$

These results, compiled by the HFLAV group [1], combine data from BaBar, Belle and LHCb experiments [2-9]), and show a tension with the SM at the level of 3.1σ.

To study possible NP effects, a phenomenological approach is typically used, which is constructed out of the most general effective Hamiltonian comprising the full set of dimension-6 semileptonic $b \to c$ operators [10]

$$H_{\rm eff} = \frac{4G_F V_{cb}}{\sqrt{2}}[(1 + C_{V_L})\mathcal{O}_{V_L} + C_{V_R}\mathcal{O}_{V_R} + C_{S_L}\mathcal{O}_{S_L} + C_{S_R}\mathcal{O}_{S_R} + C_T\mathcal{O}_T], \quad (2)$$

where the different C_i are, complex in general, Wilson coefficients (Wc) that parameterize the deviations from the SM ($C_i^{\rm SM} = 0$). The numerical values of the different Wc depend on the NP model considered, but in the end they have to be fitted to the experimental data. Different NP scenarios can lead to the same, or very close, results for the $\mathcal{R}_{H_c}$ ratios. To differentiate among different NP models, one needs to simultaneously study other observables. Here, following the work of Ref. [11], we focus on the observables involving the τ-polarization vector, like its components and averages, and how they can be extracted.

2. $\mathcal{P}^\mu$ vector

For a given momentum configuration and when all particle's polarizations are summed up (except for the τ), the squared amplitude for a $H_b \to H_c\tau\bar{\nu}_\tau$ decay can be written as:

$$\overline{\sum_{rr'}} |\mathcal{M}|^2 = \bar{u}_h^S(k')\mathcal{O}u_h^S(k'), \quad (3)$$

with $u_h^S(k')$ the final τ spinor, corresponding to polarization $h = \pm 1$ along the four vector S, and r, r' polarization indexes of the initial and final hadrons, respectively. The operator $\mathcal{O}$ contains the physics of the decay and depends on the momenta of the particles. Using the spin-density matrix formalism, explained in detail in Ref. [11], this expression becomes:

$$\overline{\sum_{rr'}} |\mathcal{M}|^2 = \frac{1}{2}{\rm Tr}\left[(\not{k}' + m_\tau)\mathcal{O}\right](1 + h\,\mathcal{P}\cdot S), \quad (4)$$

where $\mathcal{P}^\mu$ is the τ polarization vector. It is defined by the relation

$$\mathcal{P}^\mu = \frac{{\rm Tr}[(\not{k}' + m_\tau)\mathcal{O}(\not{k}' + m_\tau)\gamma_5\gamma^\mu]}{{\rm Tr}[(\not{k}' + m_\tau)\mathcal{O}(\not{k}' + m_\tau)]}, \quad (5)$$

and it satisfies the constraints

$$\mathcal{P}^{\mu *} = \mathcal{P}^\mu, \qquad k' \cdot \mathcal{P} = 0. \quad (6)$$

The full expression for the polarization vector is obtained in Ref. [11] to be

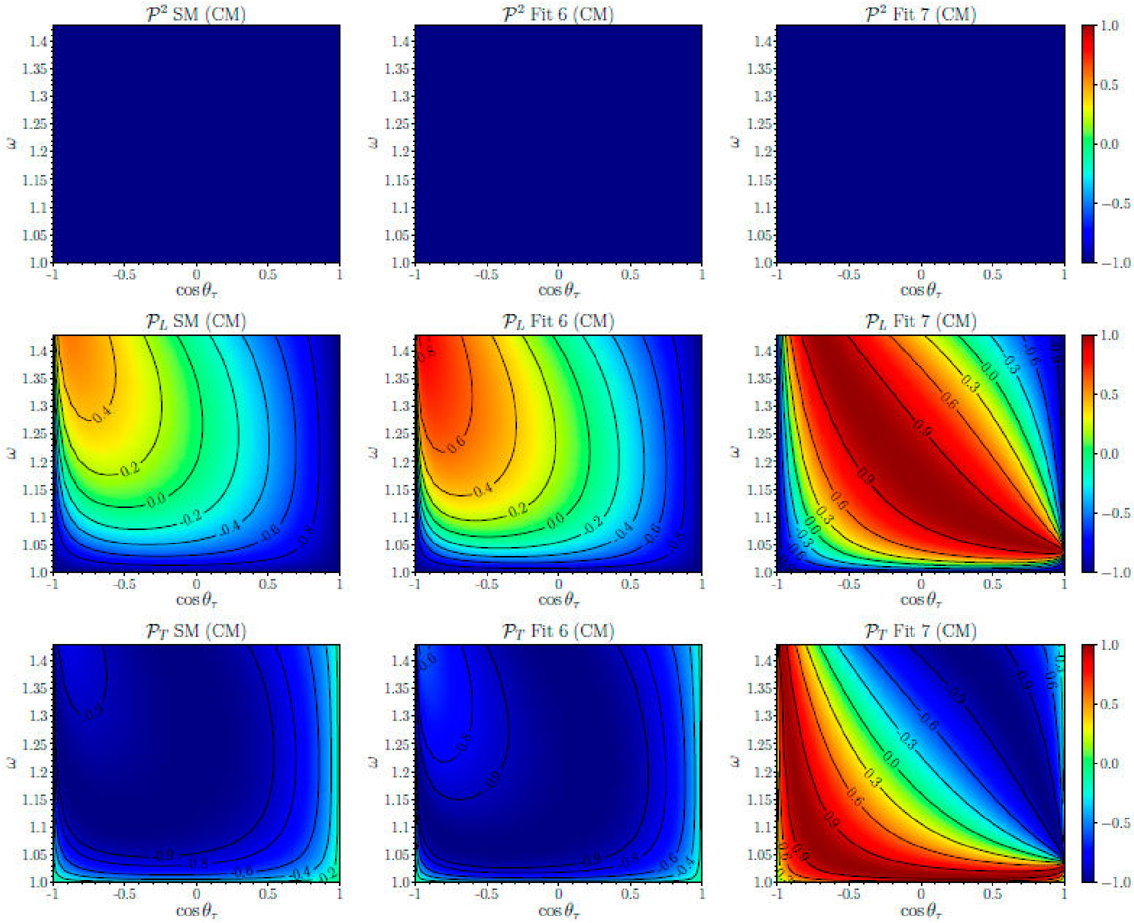

FIGURE 1. $\mathcal{P}_T$, $\mathcal{P}_L$ and $\mathcal{P}^2$ (CM system) for the $\bar{B} \to D\tau\bar{\nu}_\tau$ decay. The NP scenarios are Fits 6 and 7 of Ref. [10].

$$\mathcal{P}^\mu = \frac{1}{\mathcal{N}(\omega, k\cdot p)}\left(\frac{p_\perp^\mu}{M}\mathcal{N}_{\mathcal{H}_1}(\omega, k\cdot p) + \frac{q_\perp^\mu}{M}\mathcal{N}_{\mathcal{H}_2}(\omega, k\cdot p) + \frac{\epsilon^{\mu k' qp}}{M^3}\mathcal{N}_{\mathcal{H}_3}(\omega, k\cdot p)\right), \quad (7)$$

with $l_\perp = [l - (l\cdot k'/m_\tau^2)k']$ $(l = p, q)$ and where the different $\mathcal{N}$ functions depend on the product of the initial hadron (p) and the neutrino (k) momenta and on ω (the product of the initial and final hadron four-velocities). Looking closer to these functions, we see that they depend on 10 independent functions of ω [11-13]

$$\begin{aligned}
\mathcal{N} &= \frac{1}{2}\Big[\mathcal{A}(\omega) + \mathcal{B}(\omega)\frac{(k\cdot p)}{M^2} + \mathcal{C}(\omega)\frac{(k\cdot p)^2}{M^4}\Big],\\
\mathcal{N}_{\mathcal{H}_1} &= \mathcal{A}_\mathcal{H}(\omega) + \mathcal{C}_\mathcal{H}(\omega)\frac{(k\cdot p)}{M^2},\\
\mathcal{N}_{\mathcal{H}_2} &= \mathcal{B}_\mathcal{H}(\omega) + \mathcal{D}_\mathcal{H}(\omega)\frac{(k\cdot p)}{M^2} + \mathcal{E}_\mathcal{H}(\omega)\frac{(k\cdot p)^2}{M^4},\\
\mathcal{N}_{\mathcal{H}_3} &= \mathcal{F}_\mathcal{H}(\omega) + \mathcal{G}_\mathcal{H}(\omega)\frac{(k\cdot p)}{M^2}.
\end{aligned} \quad (8)$$

Note that that $\mathcal{F}_\mathcal{H}(\omega)$ and $\mathcal{G}_\mathcal{H}(\omega)$ depend on the imaginary part of structure functions (SFs) and they can be nonzero only in the presence of complex Wc's [11].

Choosing the orthogonal basis of the four-vector Minkowski space[i]

$$\begin{aligned}
N_0^\mu &= \frac{k'^\mu}{m_\tau}, \quad N_T^\mu = \left(0, \frac{(\vec{k}\,' \times \vec{p}\,') \times \vec{k}\,'}{|(\vec{k}\,' \times \vec{p}\,') \times \vec{k}\,'|}\right),\\
N_L^\mu &= \tilde{s}^\mu = \left(\frac{|\vec{k}\,'|}{m_\tau}, \frac{k'^0\vec{k}\,'}{m_\tau|\vec{k}\,'|}\right),\\
N_{TT}^\mu &= \left(0, \frac{\vec{k}\,' \times \vec{p}\,'}{|\vec{k}\,' \times \vec{p}\,'|}\right),
\end{aligned} \quad (9)$$

and since $\mathcal{P}\cdot k' = 0$, we have that

$$\mathcal{P}^\mu = \mathcal{P}_L\, N_L^\mu + \mathcal{P}_T\, N_T^\mu + \mathcal{P}_{TT}\, N_{TT}^\mu\,. \quad (10)$$

We can also compute the Lorentz scalar

$$\mathcal{P}^2 = -(\mathcal{P}_T^2 + \mathcal{P}_{TT}^2 + \mathcal{P}_L^2). \quad (11)$$

This quantity takes values between -1 and 0, corresponding -1 to a fully polarized τ and 0 to an unpolarized τ.

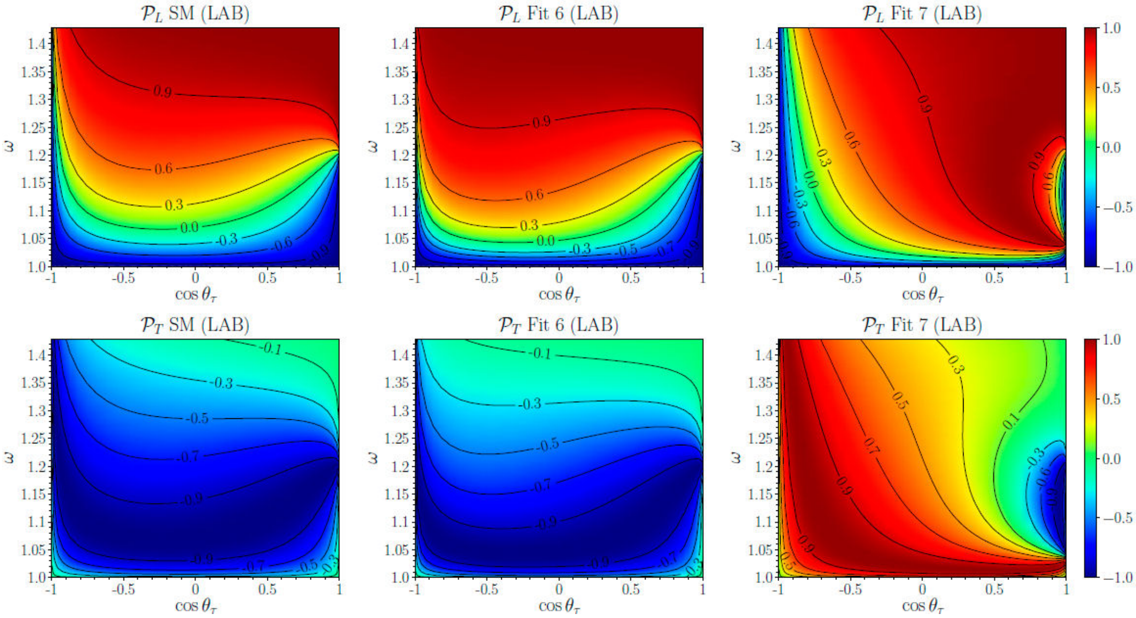

FIGURE 2. $\mathcal{P}_T$, $\mathcal{P}_L$ and $\mathcal{P}^2$ (LAB system) for the $\bar{B} \to D\tau\bar{\nu}_\tau$ decay. The NP scenarios are Fits 6 and 7 of Ref. [10].

In Fig. 1, we present results for these observables for the $B \to D$ decay and evaluated in the CM frame within the SM and the NP models corresponding to Fits 6 and 7 of Ref. [10]. One sees that the SM and Fit 6 give very similar results, while the predictions obtained from Fit 7 are very different and can be easily distinguished from the other two sets of results. The exception is P^2, for which one can prove that it is exactly minus one for $0^- \to 0^-$ decays [11]. In Fig. 2 we show the same observables, but evaluated this time in the frame where the initial hadron is at rest (LAB). Again, Fit 7 results are very different from SM and Fit 6 ones.

3. $\mathcal{P}_a$ averages

In the literature, see for instance Ref. [14], it is however more common to use the name polarization vector for the averages that we define below. We will denote those averages as $\langle P_a\rangle(\omega), a = L, T, TT$, and in the CM and LAB frames they are defined as

$$\langle\mathcal{P}_a^{\rm CM}\rangle(\omega) = \frac{\int_{-1}^{+1} d\cos\theta_\tau\, \mathcal{N}(\omega, k\cdot p)\, \mathcal{P}_a^{\rm CM}(\omega, k\cdot p)}{\int_{-1}^{+1} d\cos\theta_\tau\, \mathcal{N}(\omega, k\cdot p)},$$

$$\langle\mathcal{P}_a^{\rm LAB}\rangle(\omega) = \frac{\int_{E_\tau^-(\omega)}^{E_\tau^+(\omega)} dE_\tau \mathcal{N}(\omega, k\cdot p)\, \mathcal{P}_a^{\rm LAB}(\omega, k\cdot p)}{\int_{E_\tau^-(\omega)}^{E_\tau^+(\omega)} dE_\tau \mathcal{N}(\omega, k\cdot p)}. \quad (12)$$

These averages correspond to some experimental asymmetries and they are easier to measure. Note that, while the $\mathcal{P}_a^{\rm CM}$ and $\mathcal{P}_a^{\rm LAB}$ components carry the same information, the equivalence of the two frames is lost for the averages [11]. Therefore, if only the averages are measured, CM and LAB give complementary information. Note, however, that the $\langle\mathcal{P}^2\rangle(\omega)$ average is a scalar and thus the same in both frames.

In Fig. 3, we show the results for $\langle\mathcal{P}_T\rangle$,$\langle\mathcal{P}_L\rangle$ and $\langle\mathcal{P}^2\rangle$ corresponding to the five $\bar{B} \to D^{(*)}$, $\bar{B}_c \to \eta_c, J/\Psi$ and $\Lambda_b \to \Lambda_c$ semileptonic decays. As before, Fit 7 results are very different from those of Fit 6 and SM. Also, it can be seen that some decays are better for discriminating NP effects. $0^- \to 0^-$ meson decays are the best, followed by the $\Lambda_b \to \Lambda_c$ decay, while $0^- \to 1^-$ meson decays are not that good, even if one could reduce the hadronic uncertainties.

4. $\mathcal{P}_{TT}$ and complex Wilson coefficients

The $\mathcal{P}_{TT}$ component of the τ polarization vector only depends on the $\epsilon^{\mu k' qp}\mathcal{N}_{\mathcal{H}_3}$ term. $\mathcal{N}_{\mathcal{H}_3}$ is proportional to the imaginary part of some of the structure functions. This requires complex Wilson coefficients, thus incorporating violation of the CP symmetry in the NP effective Hamiltonian. Moreover, it only depends on the SFs that are generated from the interference of vector-axial with scalar-pseudoscalar and tensor terms and the interference of scalar-pseudoscalar with tensor terms. Therefore, at least one of the C_S, C_P, C_T Wilson coefficients must be different from zero.

In the previous sections, all the fits used had real Wilson coefficients and, thus, the $\mathcal{P}_{TT}$ component was exactly zero. That is why we now focus on the R_2 leptoquark model fit of Ref. [15] since it has complex Wc's. The result of this fit has a degeneracy in the sign of the imaginary part of the Wilson coefficients that cannot be broken using other observables [11].

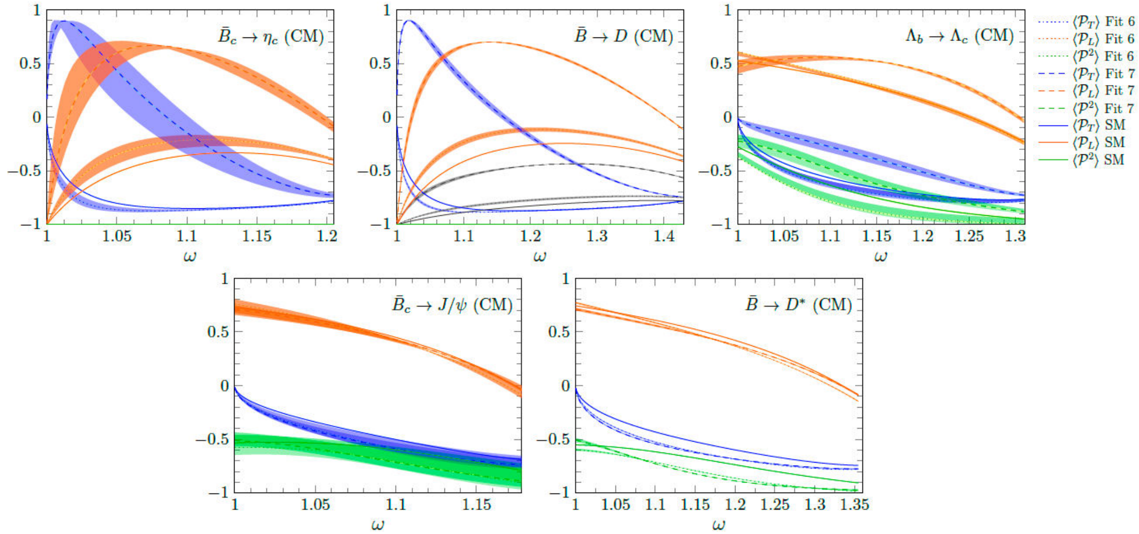

FIGURE 3. $\langle \mathcal{P}_T \rangle$, $\langle \mathcal{P}_L \rangle$ and $\langle \mathcal{P}^2 \rangle$ (CM system) for the five decays considered. The NP scenarios are Fits 6 and 7 of Ref. [10].

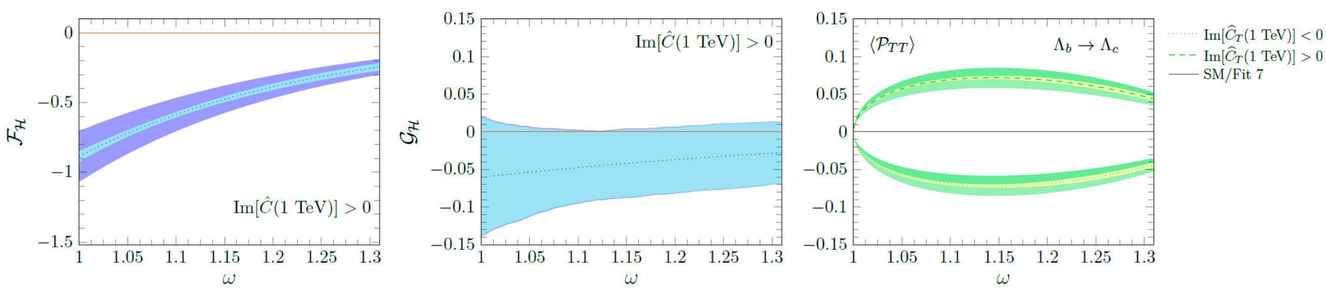

FIGURE 4. Independent functions $\mathcal{F}_\mathcal{H}$ and $\mathcal{G}_\mathcal{H}$ and the $\langle \mathcal{P}_{TT} \rangle$ average using the R2 leptoquark model fit of Ref. [15].

In Fig. 4, we show the results of the functions $\mathcal{F}_\mathcal{H}$ and $\mathcal{G}_\mathcal{H}$ with a positive $\Im[C_T(1\ \mathrm{TeV})] > 0$, see [16]. In the third panel, the average $\langle \mathcal{P}_{TT}^{\rm CM} \rangle(\omega)$, is shown for both signs of the imaginary part of C_T. This polarization vector component breaks the degeneracy present in other observables.

5. $H_b \to H_c\tau(\to d\nu_\tau)\bar{\nu}_\tau$ 4-body decays

The τ is very short-lived and thus it is very difficult to measure. However, most of the information that could be obtained from actually observing the final τ is contained in the 4-body differential decay width [16,17]

$$\frac{d^3\Gamma_d}{d\omega d\xi_d d\cos\theta_d} = \mathcal{B}_d \frac{d\Gamma_{\rm SL}}{d\omega} \Big\{ F_0^d(\omega,\xi_d) + F_1^d(\omega,\xi_d)\cos\theta_d + F_2^d(\omega,\xi_d) P_2(\cos\theta_d) \Big\}, \tag{13}$$

corresponding to the sequential decay $H_b \to H_c\tau(\to d\nu_\tau)\bar{\nu}_\tau$ with $d = \pi, \rho, \mu\bar{\nu}_\mu$. The variable ξ_d is related to the CM energy of the charged particle in which the tau decays ($\pi,\ \rho$ or μ) whereas θ_d is the angle made by the three-momenta of this particle with that of the H_c hadron, also measured in the CM frame. A scheme of the latter can be seen in the diagram of Fig. 5. The $F_{0,1,2}(\omega,\xi_d)$ functions are given by

$$\begin{aligned} F_0(\omega,\xi_d) =& C_n(\omega,\xi_d) + C_{P_L}(\omega,\xi_d)\langle P_L^{\rm CM}\rangle, \\ F_1(\omega,\xi_d) =& C_{A_{FB}}(\omega,\xi_d) A_{FB} + C_{Z_L}(\omega,\xi_d) Z_L \\ &+ C_{P_T}(\omega,\xi_d)\langle P_T^{\rm CM}\rangle, \\ F_2(\omega,\xi_d) =& C_{A_Q}(\omega,\xi_d) A_Q + C_{Z_Q}(\omega,\xi_d) Z_Q \\ &+ C_{Z_\perp}(\omega,\xi_d) Z_\perp. \end{aligned} \tag{14}$$

The C_i are kinematical factors that depend on the tau decay mode ($\pi,\ \rho$ or $\mu\bar{\nu}_\mu$) and they can be computed analytically [16]. The seven $\omega-$functions $A_{FB}, Z_L, A_Q, Z_Q, Z_\perp$ and $\langle P_{L,T}^{\rm CM}\rangle$ are a combination of the 7 independent $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{A}_\mathcal{H}$, $\mathcal{B}_\mathcal{H}$ $\mathcal{C}_\mathcal{H}$, $\mathcal{D}_\mathcal{H}$ and $\mathcal{E}_\mathcal{H}$ ones. Information on $\mathcal{F}_\mathcal{H}$, $\mathcal{G}_\mathcal{H}$ is lost after integration over the azimuthal angle ϕ_d. Note that the latter can not be measured since the τ momentum can not be established.

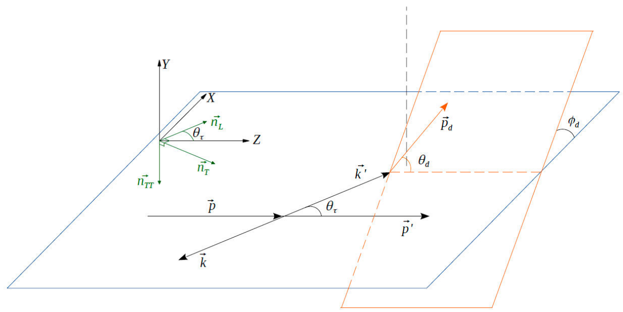

FIGURE 5. Kinematics in the $\tau\bar{\nu}_\tau$ CM reference system and the unit vectors ($\vec{n}_L, \vec{n}_T$ and $\vec{n}_{TT}$) which define the Minkowski base of Eq. (9).

6. Conclusions

We have seen, especially through Fig. 3, that the meson $0^- \to 0^-$ and the baryon $\Lambda_b \to \Lambda_c$ decays are the best for distinguishing among NP models. However, one has 5 unknown, complex in general, Wilson coefficients and, in order to fix their values, it is necessary to combine the study of different decays and observables. While measuring the momentum and polarization state of the final τ would be ideal to obtain the maximum information, this is precluded by the fact that the τ is very short-lived. One has then to rely in sequential processes where the final τ subsequently decays. These processes retain most of the information which could be extracted from $H_b \to H_c\tau\bar{\nu}_\tau$ decays if the τ were detected. In particular, the values of the $\langle P_{L,T}^{\rm CM}\rangle$ averages can be obtained from such sequential decays. As we have shown, those averages can help to distinguish between different NP models.

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i. The definition of the correspondent three-vectors can be seen, for the center of mass frame of the two final leptons (CM), in the diagram of Fig. 5.

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