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Gluon transversity and TMDs for spin-1 hadrons

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We explain a gluon transversity, transverse-momentum-dependent parton distribution functions (TMDs), and parton distribution functions (PDFs) for spin-1 hadrons. The gluon transversity exists in hadrons with spin more than or equal to one, and it does not exist in the spin-1/2 nucleons. Since there is no direct contribution from the nucleons, it is an appropriate quantity to probe an exotic component in the spin-1 deuteron beyond a simple bound system of the nucleons. We show how the gluon transversity can be measured at hadron accelerator facilities by the Drell-Yan process in addition to lepton-accelerator experiments. Next, possible TMDs are explained for the spin-1 hadrons at the twists 3 and 4 in addition to twist-2 ones by considering tensor polarizations. We found that 30 TMDs exist in the tensor-polarized spin-1 hadron at the twists 3 and 4 in addition to 10 TMDs at the twist 2. There are 3 collinear PDFs at the twists 3 and 4. We also indicate that the corresponding TMD fragmentation functions exist at the twists 3 and 4. Due to the time-reversal invariance in the collinear PDFs, there are new sum rules on the time-reversal odd TMDs. In addition, we obtained a useful twist-2 relation, a sum rule, and relations with multiparton distribution functions by using the operator product expansion and the equation of motion for quarks. These findings are valuable for experimental investigations on polarized deuteron structure functions in 2020's and 2030's at world accelerator facilities.

Keywords: QCD; quark; gluon; spin-1 hadron; structure function; transversity.

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1. Introduction

In spin-1 hadrons, there are structure functions in addition to the ones of the spin-1/2 nucleons, and they are related to their tensor polarizations. These new structure functions were proposed in 1980's; however, experimental progress was rather slow except for the HERMES b_1 measurement in 2005 [1]. In spite of this situation, we have a bright future prospect because there are experimental projects in 2020's and 2030's to investigate polarized deuteron structure functions at various accelerator facilities, such as Thomas Jefferson National Accelerator Facility (JLab), Fermilab (Fermi National Accelerator Laboratory), Nuclotron-based Ion Collider fAcility (NICA), LHC (Large Hadron Collider)-spin, and electron-ion colliders (EIC, EicC) [2]. Therefore, time has come to investigate them theoretically before experimental measurements.

In this report, we discuss a gluon transversity, especially how to find it in a proton-deuteron Drell-Yan process. There are already significant works on a quark transversity both theoretically and experimentally. However, there is no experimental measurement on the gluon transversity because it does not exist in the spin-1/2 nucleons. The gluon transversity is defined by an amplitude with a gluon-spin flip, namely the difference of two units of spin ($\Delta s = 2$), so that the hadron spin needs to be larger than or equal to one. The most stable spin-1 target is the deuteron, which can be used for experimental measurements. Since the proton and neutron cannot contribute directly, the gluon transversity is an appropriate observable to find any exotic signature in the deuteron beyond the simple bound system of the nucleons. If a finite distribution is found experimentally, it could lead a new field of hadron physics. On this topic, the purpose of our study is to provide a theoretical formalism for investigating the gluon transversity at hadron accelerators, for example, by the Drell-Yan process [3,4] as discussed in Sec. 3, whereas the lepton scattering measurement was already considered at JLab [2,5].

The second topic is on transverse-momentum-dependent parton distribution functions (TMDs) and parton distribution functions (PDFs) of tensor-polarized spin-1 hadrons up to twist 4 [6] as explained in Secs. 4 and 5. Polarized PDFs of the nucleons have been investigated up to twist 4 [7]; however, they were investigated only at the twist-2 level [8] until recently for spin-1 hadrons. The purpose of our study is to provide full TMDs, PDFs, and fragmentation functions up to twist 4 for the spin-1 hadrons [6]. Due to the time-reversal (T) invariance in the collinear PDFs, there are sum rules for T-odd TMD distributions. For the collinear PDFs, a useful twist-2 relation and a sum rule were found [9] in the similar way to the Wandzura-Wilczek relation and the Burkhardt-Cottingham sum rule. Furthermore, the equation of motion for quarks was used for obtaining relations among the collinear parton- and multiparton-distribution functions for spin-1 hadrons [10]. We explain these results in this paper.

2. Polarizations of spin-1 hadrons

Polarizations of spin-1 hadrons are described by the spin vector $\vec{S}$ and tensor T_{ij} defined by the polarization vector $\vec{E}$ as

$$\begin{aligned}
\vec{S} &= \mathrm{Im}\,(\,\vec{E}^{\,*} \times \vec{E}\,) = (S_T^x,\, S_T^y,\, S_L),\\
T_{ij} &= \frac{1}{3}\delta_{ij} - \mathrm{Re}\,(\,E_i^* E_j\,)\\
&= \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}, \quad (1)
\end{aligned}$$

where S_T^x, S_T^y, S_L, S_{LL}, S_{TT}^{xx}, S_{TT}^{xy}, S_{LT}^x, and S_{LT}^y are parameters to express the vector and tensor polarizations. The polarizations of the spin-1 hadrons, for example the deuteron, are listed in Table I by showing the polarization $\vec{E}$ and the polarization parameters for the longitudinal, transverse, and linear polarizations of a spin-1 hadron. The longitudinal polarizations contain both S_L and S_{LL}, and the transverse ones do S_T^i, S_{LL}, and S_{TT}^{xx}. It is interesting to see that these polarizations partially have the tensor polarization parameter S_{LL} and that the linear polarization parameter S_{TT}^{xx} is contained in the transverse polarization. The linear polarizations are defined by the polarization vector $\vec{E}_x = (\,1,\,0,\,0\,)$ and $\vec{E}_y = (\,0,\,1,\,0\,)$. They also have the parameter S_{LL} in addition to S_{TT}^{xx} as shown in Table I, so that the S_{LL} terms should be cancelled in order to extract the gluon transversity defined in association with S_{TT}^{xx}.

TABLE I. Longitudinal, transverse, and linear polarizations of a spin-1 hadron, polarization vectors, and parameters of the spin vector and tensor [2,3].

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1,\,-i,\,0)$	0	0	$+1$	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1,\,-i,\,0)$	0	0	-1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0,\,-1,\,-i)$	$+1$	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0,\,+1,\,-i)$	-1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i,\,0,\,-1)$	0	$+1$	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i,\,0,\,+1)$	0	-1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	$(1,\,0,\,0)$	0	0	0	$+\frac{1}{2}$	-1
Linear y	$(0,\,1,\,0)$	0	0	0	$+\frac{1}{2}$	$+1$

3. Gluon transversity in Drell-Yan process

The gluon transversity has not been measured yet, although there are global analysis results on the quark transversity. In principle, it exists in the spin-1 deuteron although it does not for the spin-1/2 nucleons, so that it is a unique quantity to probe a new hadronic physics within the deuteron.

The gluon transversity $\Delta_T g$ is defined by the matrix element between the linearly polarized (E_x) deuteron as [3]

$$\begin{aligned}
\Delta_T g(x) &= \varepsilon_{TT,\alpha\beta} \int \frac{d\xi^-}{2\pi}\, xp^+\, e^{ixp^+\xi^-} \\
&\quad \times \langle\, p\, E_x \,|\, A^\alpha(0)\, A^\beta(\xi) \,|\, p\, E_x \,\rangle_{\xi^+ = \vec{\xi}_\perp = 0}, \qquad (2)
\end{aligned}$$

where x is the momentum fraction for a gluon, $\varepsilon_{TT}^{\alpha\beta}$ is the transverse parameter given by $\varepsilon_{TT}^{11} = +1$ and $\varepsilon_{TT}^{22} = -1$, ξ is the space-time coordinate expressed by the lightcone coordinates $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$ and $\vec{\xi}_\perp$, p is the deuteron momentum, and A^μ is the gluon field. It is expressed by the gluon distribution difference as

$$\Delta_T g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x), \qquad (3)$$

where $\hat{y}/\hat{x}$ is the gluon linear polarization ε_y in the deuteron with the polarization E_x. Because this distribution is defined by the linear polarizations for the gluon and the spin-1 hadron, it could be called gluon linearity. However, the gluon transversely is commonly used in literature, so that this expression is used throughout this paper. In terms of parton-hadron forward scattering amplitudes, $A_{\Lambda_i\lambda_i,\,\Lambda_f\lambda_f}$, with the initial and final hadron helicities, Λ_i and Λ_f, and parton ones, λ_i and λ_f, the gluon transversity is given by

$$\Delta_T g(x) \sim \mathrm{Im}\, A_{++,\,--}\,. \qquad (4)$$

Namely, it is defined by the amplitude with the gluon helicity flip, so that the change of two spin units ($\Delta s = 2$) is needed between the initial and final states. It is the reason why the spin-1/2 nucleons cannot accommodate this distribution.

The gluon transversity will be measured at charged-lepton scattering by looking by the angle dependence of the deuteron spin in the cross section [2]. It is the angle between the lepton-scattering plan and the target-spin orientation. The intension of our studies is to make the measurement possible at hadron accelerator facilities by supplying a theoretical formalism for the Drell-Yan process [3, 4]. As an example, the proton-deuteron Drell-Yan process was investigated because it is possible at Fermilab. The formalism details are explained in the paper [3], where the cross section of $p(A) + d(B) \to \mu^+\mu^- + X$ is given by the difference $d\sigma(E_x) - d\sigma(E_y)$ as

$$\frac{d\sigma_{pd\to\mu^+\mu^- X}}{d\tau\, d\vec{q}_T^{\,2}\, d\phi\, dy}(E_x - E_y) = -\frac{\alpha^2\,\alpha_s\, C_F\, q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1} dx_a \frac{\sum_q e_q^2\, x_a[\,q_A(x_a)+\bar{q}_A(x_a)\,]\, x_b \Delta_T g_B(x_b)}{(x_a x_b)^2\,(x_a - x_1)(\tau - x_a x_2)^2}, \tag{5}$$

by considering the deuteron linear polarizations (E_x, E_y). Here, τ is defined by the dimuon mass or momentum squared as $\tau = M_{\mu\mu}^2/s = Q^2/s$ with the center-of-mass energy squared s, $\vec{q}_T^{\,2}$ is the dimuon transverse momentum squared, ϕ is its azimuthal angle, y is the rapidity in the center-of-mass frame, α is the fine structure constant, α_s is the QCD running coupling constant, C_F is the color factor $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, and e_q is the quark charge. The momentum fraction x_b is given by $x_b = (x_a x_2 - \tau)/(x_a - x_1)$, and the minimum of x_a is $\min(x_a) = (x_1 - \tau)/(1 - x_2)$ with $x_1 = e^y\sqrt{(Q^2 + \vec{q}_T^{\,2})/s}$ and $x_2 = e^{-y}\sqrt{(Q^2 + \vec{q}_T^{\,2})/s}$. The $q_A(x_a)$ and $\bar{q}_A(x_a)$ are quark and antiquark distribution functions in the proton, and $\Delta_T g(x_b)$ is the gluon transversity in the deuteron.

In estimating the cross section numerically, we used the CTEQ14 PDFs for the unpolarized PDFs of the proton and also the deuteron by ignoring nuclear corrections. Since there is no available gluon transversity at this stage, we assumed it is equal to the longitudinally-polarized gluon distribution given by the NNPDF1.1; however, it is likely an overestimation of the cross section. In Fig. 1, the polarization asymmetry $A_{E_{xy}} \equiv d\sigma(E_x - E_y)/d\sigma(E_x + E_y)$ is shown for the Fermilab kinematics with $p_p = 120$ GeV by taking $\phi = 0$, $y = 0.5$, and $q_T = 0.5$ or 1.0 GeV as the function $M_{\mu\mu}^2$. The asymmetry is typically a few percent. However, if a finite gluon transversity is found in an experiment, it could lead to an interesting new hadron physics. Fortunately, this experiment will be proposed at Fermilab within the E-1039 collaboration [2].

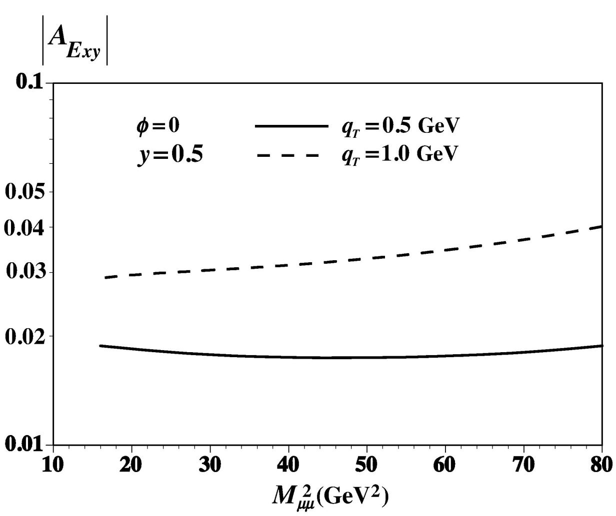

FIGURE 1. Polarization asymmetry $|A_{E_{xy}}|$.

4. TMDs and PDFs for spin-1 hadrons

Next, we discuss the TMDs and PDFs at the twists 3 and 4 for tensor-polarized spin-1 hadrons. Recently, fully consistent investigations have been done for finding possible twist 3 and twist 4 TMDs and PDFs, whereas the higher-twist PDFs were found many years ago for the nucleons. In general, the TMDs and PDFs are defined from the correlation function $\Phi_{ij}^{[c]}$, which is the amplitude to extract a parton from a hadron and then to insert it into the hadron at a different space-time point:

$$\Phi_{ij}^{[c]}(k,P,T\,|\,n) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \times \langle\, P,T \left|\, \bar{\psi}_j(0)\, W^{[c]}(0,\xi)\psi_i(\xi) \,\right| P,\, T\,\rangle. \tag{6}$$

Here, k and P are quark and hadron momenta, T indicates the tensor polarization of a spin-1 hadron, n is the lightcone vector $n^\mu = (1,0,0,-1)/\sqrt{2}$, ξ is a space-time coordinate, ψ is the quark field, and $W^{[c]}(0,\xi)$ is the gauge link with the integral path c.

This correlation function is expanded in a Lorentz invariant way with the constraints of the Hermiticity and parity invariance. The time-reversal invariance does not have to be satisfied in the TMD level due to the existence of the color flow given by the gauge link; however, it is imposed in the collinear PDFs. Then, we obtain [6]

$$\Phi(k,P,T\,|n) = \frac{A_{13}}{M}T_{kk} + \cdots + \frac{A_{20}}{M^2}\varepsilon^{\mu\nu Pk}\gamma_\mu\gamma_5 T_{\nu k} + \frac{B_{21}M}{P\cdot n}T_{kn} + \cdots + \frac{B_{52}M}{P\cdot n}\sigma_{\mu k}T^{\mu n}, \tag{7}$$

for the tensor polarization part. The twist-2 expression was given in Ref. [8] for spin-1 hadrons. Higher-twist expressions were investigated for the spin-1/2 nucleons in Ref. [7] by including the lightcone vector n to accommodate twist-3 and 4 effects. In the same way, we included n terms for the spin-1 hadrons for defining the higher-twist TMDs and PDFs. Here, A_i and B_i are expansion coefficients, the tensor polarization is expressed by $T^{\mu\nu}$, and the contraction $X_{\mu k} \equiv X_{\mu\nu}k^\nu$ is used. The TMDs are given by integrating the function over the quark momenta as

$$\Phi^{[c]}(x,k_T,P,T) = \int dk^+ dk^-\, \Phi^{[c]} \times (k,P,T\,|n)\,\delta(k^+ - xP^+). \tag{8}$$

The TMDs and collinear PDFs are defined by traces of the correlation functions with γ matrices (Γ) as $\Phi^{[\Gamma]} \equiv$

$\mathrm{Tr}\,[\,\Phi\Gamma\,]/2$. The twist-2 TMDs were defined by the traces $\Phi^{[\gamma^+]}$, $\Phi^{[\gamma^+\gamma_5]}$, and $\Phi^{[i\sigma^{i+}\gamma_5]}$ (or $\Phi^{[\sigma^{i+}]}$) [8]. The twist-3 TMDs were obtained by $\Phi^{[\gamma^i]}$, $\Phi^{[\mathbf{1}]}$, $\Phi^{[i\gamma_5]}$ $\Phi^{[\gamma^i\gamma_5]}$ $\Phi^{[\sigma^{ij}]}$, and $\Phi^{[\sigma^{-+}]}$, and the twist-4 TMDs were obtained by $\Phi^{[\gamma^-]}$, $\Phi^{[\gamma^-\gamma_5]}$, and $\Phi^{[\sigma^{i-}]}$ [6]. For example, we have

$$\Phi^{[\gamma^i]}(x,k_T,T)=\frac{M}{P^+}\Bigg[f_{LL}^{\perp}(x,k_T^2)S_{LL}\frac{k_T^i}{M}+f'_{LT}(x,k_T^2)S_{LT}^i - f_{LT}^{\perp}(x,k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2}-f'_{TT}(x,k_T^2)\frac{S_{TT}^{ij}k_{T\,j}}{M} + f_{TT}^{\perp}(x,k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\frac{k_T^i}{M}\Bigg], \quad (9)$$

as the trace for defining some of the twist-3 TMDs. Instead of the primed TMDs we define other TMDs by $F(x,k_T^2)\equiv F'(x,k_T^2)-(k_T^2/(2M^2))\,F^{\perp}(x,k_T^2)$ where $k_T^2=-\vec{k}_T^2$, so that the primed TMDs $'$ may not be used in actual TMD lists. From these traces, we find that the following tensor-polarized TMDs exist [6]:

$$\begin{aligned}
&\text{Twist-2 TMD: } f_{1LL},\ f_{1LT},\ f_{1TT},\ g_{1LT},\ g_{1TT},\\
&\qquad h_{1LL}^{\perp},\ h_{1LT},\ h_{1LT}^{\perp},\ h_{1TT},\ h_{1TT}^{\perp},\\
&\text{Twist-3 TMD: } f_{LL}^{\perp},\ e_{LL},\ f_{LT},\ f_{LT}^{\perp},\ e_{1T},\ e_{1T}^{\perp},\ f_{TT},\ f_{TT}^{\perp},\\
&\qquad e_{TT},\ e_{TT}^{\perp},\ g_{LL}^{\perp},\ g_{LT},\ g_{LT}^{\perp},\ g_{TT},\ g_{TT}^{\perp},\\
&\qquad h_{1L},\ h_{LT},\ h_{LT}^{\perp},\ h_{TT},\ h_{TT}^{\perp},\\
&\text{Twist-4 TMD: } f_{3LL},\ f_{3LT},\ f_{3TT},\ g_{3LT},\ g_{3TT},\\
&\qquad h_{3LL}^{\perp},\ h_{3LT},\ h_{3LT}^{\perp},\ h_{3TT},\ h_{3TT}^{\perp}.
\end{aligned} \quad (10)$$

Namely, there are 10, 20, and 10 tensor-polarized TMDs at twists 2, 3, and 4, respectively. These are classified by chiral even/odd and time-reversal even/odd. Since the time-reversal invariance should be satisfied in collinear PDFs by the integral over the transverse momentum $\vec{k}_T$, there are sum rules for the T-odd TMDs as

$$\int d^2k_T\,h_{1LT}(x,k_T^2)=\int d^2k_T\,g_{LT}(x,k_T^2) =\int d^2k_T\,h_{LL}(x,k_T^2)=\int d^2k_T\,h_{3LT}(x,k_T^2)=0. \quad (11)$$

The TMD fragmentation functions are also found up to twist 4 [6] simply by changing kinematical variables and function names as [8]:

$$\begin{aligned}
&\text{Kinematical variables: } x,k_T,S,T,M,n,\gamma^+,\sigma^{i+}\\
&\qquad \Rightarrow\ z,k_T,S_h,T_h,M_h,\bar{n},\gamma^-,\sigma^{i-},\\
&\text{Distribution functions: } f,g,h,e\\
&\qquad \Rightarrow\ \text{Fragmentation functions: } D,G,H,E.
\end{aligned} \quad (12)$$

In addition, if the TMDs are integrated over $\vec{k}_T$, we obtain the tensor-polarized PDFs up to twist 4 as:

$$\begin{aligned}
&\text{Twist-2 PDF: } f_{1LL},\ \text{Twist-3: } e_{LL},\ f_{LT},\\
&\text{Twist-4: } f_{3LL}.
\end{aligned} \quad (13)$$

The collinear fragmentation functions were investigated in Ref. [11].

5. Useful relations among PDFs and multiparton distribution functions

For the new twist-3 PDFs, we derived useful relations. First, we obtained a twist-2 relation and a sum rule [9], analogous to the Wandzura-Wilczek (WW) relation and the Burkhardt-Cottingham (BC) sum rule, for the twist-2 and twist-3 tensor-polarized parton distribution functions f_{1LL} and f_{LT}, respectively. Using the formalism of the operator product expansion and defining multiparton distribution functions for twist-3 terms, we obtained the relation

$$f_{LT}(x)=\frac{3}{2}\int_{\epsilon(x)}^{x}dy\frac{f_{1LL}(y)}{y}+\int_{\epsilon(x)}^{x}dy\frac{f_{LT}^{(HT)}(y)}{y}. \quad (14)$$

Here, $\epsilon(x)$ is defined by $\epsilon(x)=1$ (-1) at $x>0$ $(x<0)$, and the last term is the twist-3 effect given by the multiparton distribution functions. We define $+$ PDFs by $f^+(x)=f(x)+\bar{f}(x)$ in the range $0\le x\le 1$. The f_{1LL}^+ is the same as b_1 with the relation $b_1^{q+\bar{q}}=-(3/2)f_{1LL}^+$. Then, neglecting the higher-twist term, we obtain

$$f_{LT}^+(x)=\frac{3}{2}\int_1^x\frac{dy}{y}\,f_{1LL}^+(y). \quad (15)$$

Namely, the twist-2 part of f_{LT} is expressed by an integral of f_{1LL} (or b_1). If the function f_{2LL} is define by $f_{2LL}=(2/3)f_{LT}-f_{1LL}$, it leads to the twist-2 relation similar to the WW relation as

$$f_{2LT}^+(x)=-f_{1LL}^+(x)+\int_1^x\frac{dy}{y}f_{1LL}^+(y).$$

Integrating this equation over x, we obtain the BC-like sum rule as

$$\int_0^1 dx\,f_{2LT}^+(x)=0.$$

If the parton-model sum rule for f_{1LL} (b_1), $\int dx f_{1LL}^+(x)=0$ $(\int dx b_1^{q+\bar{q}}(x)=0)$ [12], is applied by assuming vanishing tensor-polarized antiquark distributions, another sum rule exists for f_{LT} itself, $\int_0^1 dx\,f_{LT}^+(x)=0$. In deriving these relations, we showed that the following tensor-polarized multiparton distribution functions exist: $F_{LT}(x,y)$, $G_{LT}(x,y)$, $H_{LL}^{\perp}(x,y)$, $H_{TT}(x,y)$.

Next, from the equation of motion for quarks, useful relations were also obtained (1) for the twist-3 PDF f_{LT}, the trasverse-momentum moment PDF $f_{1LT}^{(1)}$, and the multiparton distribution functions $F_{G,LT}$ and $G_{G,LT}$; (2) for the twist-3 PDF e_{LL}, the twist-2 PDF f_{1LL}, and the multiparton distribution function $H^{\perp}_{G,LL}$ as [10]

$$x f_{LT}(x) - f^{(1)}_{1LT}(x) - \mathcal{P}\int_{-1}^{1} dy\, \frac{F_{G,LT}(x,y)+G_{G,LT}(x,y)}{x-y} = 0,$$

$$x\, e_{LL}(x) - 2\mathcal{P}\int_{-1}^{1} dy\, \frac{H^{\perp}_{G,LL}(x,y)}{x-y} - \frac{m}{M} f_{1LL}(x) = 0.$$

The transverse-momentum moments of the TMDs are defined by $f^{(1)}(x) = \int d^2k_T(\vec{k}_T^{\,2}/(2M^2))\, f(x,k_T^2)$, $\mathcal{P}$ is the principle integral, and m is the quark mass. In addition, the Lorentz-invariance relation was obtained as [10]

$$\frac{df^{(1)}_{1LT}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2\mathcal{P}\int_{-1}^{1} dy\, \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0.$$

In these derivations, we also obtained relations in $F_{D/G,LT}(x,y)$, $G_{D/G,LT}(x,y)$, $H^{\perp}_{D/G,LL}(x,y)$, and $H_{D/G,TT}(x,y)$.

6. Summary

We explained a possible gluon transversity measurement by the proton-deuteron Drell-Yan process. Then, possible twist-3 and twist-4 TMDs and PDFs were shown for tensor-polarized spin-1 hadrons. In addition, the corresponding TMD fragmentation functions exist at twists 3 and 4. A useful twist-2 WW-like relation and a BC-like sum rule were derived by defining multiparton distribution functions at twist 3. Furthermore, from the equation of motion for quarks, the twist-3 PDFs are related to other PDFs and multiparton distribution functions, and so called the Lorentz-invariance relation was also obtained. Since there are various experimental projects to investigate spin-1 hadrons, these studies should be useful.

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