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Light front wave functions from AdS/QCD models

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In this talk, we present an extension to the matching procedure proposed by Brodsky and de Teramond to obtain the two-body wave functions in the light-front formalism for holographic models. We consider different static dilaton fields and AdS-like geometric deformations.

Keywords: AdS/QCD; light front holography; hadron wave function.

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1. Introduction

The hadronic wave function in terms of the quark and gluon degrees of freedom plays an essential role in making predictions for several QCD phenomena. However, in the process of doing a direct extraction of this object, some drawbacks arose. There are many non-perturbative approaches to obtain properties of the distribution amplitudes and hadronic wave functions from QCD. Some time ago, based on the AdS/CFT correspondence, Brodsky and de Teramond suggested a matching procedure to obtain mesonic Light-Front Wave Functions (LFWF) in terms of fields dual to hadrons and electromagnetic currents on the AdS side [1, 2].

The LFWF obtained from the matching procedure later was improved by adding different ingredients in the QCD side (*e.g.*, see [3–5]), But its direct effects on the AdS side just were considered in [6], which is the focus of this talk.

This work is organized as follows. In Sec. 2 we summarize the matching procedure, which allows us to relate the AdS modes with a two-body bound state LFWF. In Sec. 3, we study the large Q^2 limit in the Equation of Motion for the modes dual to photons. We further notice that we obtain the same equation as in the traditional quadratic dilaton in this limit, used for many AdS/QCD models. This similarity is the key ingredient that opens the door to relate the AdS modes with the two-body LFWF for several holographic models. In Sec. 4, we consider four AdS/QCD models and develop their associated LFWF.

2. Two-body wave function in holographic models

In Refs. [1, 2], the authors showed that based on the comparison of form factors, calculated in the light-front formalism and the AdS/QCD models, it is possible to relate bulk modes to light-front wave functions. Below we briefly discuss this matching procedure, including a generalization that will allow us to use this formalism with general AdS/QCD models.

In the light-front formalism, the electromagnetic form factor of the pion can be written as

$$F(Q^2) = 2\,\pi \int_0^1 dx\,\frac{1-x}{x}\int_0^\infty d\zeta\,\zeta \times J_0\left(\zeta Q\sqrt{\frac{1-x}{x}}\right)\widetilde{\rho}(x,\zeta), \qquad (1)$$

where Q^2 is the spacelike transferred momentum squared; J_0 is the Bessel function of zero-order, and ζ is a variable defined as

$$\zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j b_j\right|, \qquad (2)$$

representing the x-weighted transverse impact coordinate associated with the spectator system, b_j is the internal distance between constituents, and the sum is over the number of spectators. In two-body case $\zeta^2 = x(1-x)b^2$.

In (1), $\widetilde{\rho}(x,\zeta)$ is the effective transverse density of partons, which in the two-body case, is given by

$$\widetilde{\rho}_{n=2}(x,\zeta) = \frac{\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,\zeta)\right|^2}{A^2(1-x)^2}, \qquad (3)$$

where A is a normalization constant.

Now we pay attention to form factor calculations in the gravity side on AdS/QCD models. The matrix element for the spin-S current and the spin-J hadrons is given by [7]

$$\langle b|J^{\mu_1\mu_2\cdots\mu_s}|a\rangle = (\text{charge})(\text{kinematic factor})F_{ab}(Q^2), \qquad (4)$$

where $F_{ab}(Q^2)$ is the form factor.

Assuming a minimal coupling, the hadronic matrix element for the electromagnetic current in asymptotically AdS spaces used in AdS/QCD models has the form [1, 2]

$$ig_5\int d^4x\,dz\,\sqrt{g}\,e^{-\phi(z)}A^l(x,z)\Psi *_{p'}(x,z)\overleftrightarrow{\partial}_l\Psi_p(x,z), \qquad (5)$$

where g_5 is a five-dimensional effective coupling constant and $\Psi_p(x,z)$ is a normalizable mode representing a hadronic state, $\Psi_p(x,z) \sim e^{-ip\cdot x}\Psi(z)$, with hadronic invariant mass given by $p_\mu p^\mu = M^2$. Additionally, it considers an electromagnetic probe polarized along Minkowski coordinates, $A_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z)$, $A_z = 0$, where $J(Q,z)$ has the value 1 at zero momentum transfer. Since we are normalizing the bulk solutions to the total charge operator, and as a boundary limit, the external current is $A_\mu(x, z\to 0) = \epsilon_\mu e^{-iQ\cdot x}$. Thus $J(Q^2=0,z) = J(Q^2, z=0) = 1$.

Considering an asymptotically AdS space with a metric defined as

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \tag{6}$$

where $\eta_{\mu\nu}$ is the Minkowski 4D spacetime metric, z is the holographic coordinate, $A(z)$ defines the warp factor for an asymptotically AdS space, *i.e.*, $e^{2A(z\to 0))} = R^2/z^2$. Additionally, the model considers a dilaton field $\phi(z)$, which breaks the conformal invariance. With these ingredients, the corresponding expression for the form factors related to scalar hadrons in AdS is

$$F(Q^2) = \int_0^\infty dz\, e^{3A(z)-\phi(z)} \Psi(z) J(Q^2,z) \Psi(z), \tag{7}$$

where $\Psi(z)$ and $J(Q^2,z)$ are the AdS modes dual to scalar hadrons and photons. These modes are the solutions of the bulk equation of motion (EOM) in the Sturm Liouville form associated with each bulk field. The latter form factor can be transformed into the following expression

$$F(Q^2) = \int_0^\infty dz\, \Phi(z) J(Q^2,z) \Phi(z), \tag{8}$$

where $\Phi(z)$ corresponds to the solutions of the EOM transformed into an Schrödinger-like form, while $J(Q^2,z)$ remains as the same solution used before.

In order to generalize the ideas exposed in Refs. [1,2], *i.e.*, to put (8) in the same mathematical form as (1), we have to find a matching between AdS hadronic modes and the LFWF. The crucial step (not considered in general by any AdS/QCD model before) is to write the electromagnetic current as [4]

$$J(Q^2,z) = \int_0^1 dx\, g(Q^2,x)\, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right). \tag{9}$$

This rule allows us, after putting $z=\zeta$ and considering x variable with the same physical interpretation as in Eq. (1), to compare both form factors, allowing us to establish a matching that gives a relationship between the AdS modes and the LFWF:

$$\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,\zeta)\right|^2 = A^2\, x(1-x)\, g(Q^2,x)\, \frac{|\Phi(\zeta)|^2}{2\pi\zeta}. \tag{10}$$

Here the factor A is constrained by the probability condition $P_{q_1\bar{q}_2} = \int_0^1 dx \int d^2b\, |\widetilde{\psi}_{q_1\bar{q}_2}(x,b)|^2 \leq 1$ with $P_{q_1\bar{q}_2}$ being the probability of finding the Fock valence state $|q_1\bar{q}_2\rangle$ inside the meson M.

Recalling that in two-body case we have $\zeta^2 = x(1-x)b^2$, the relationship between AdS modes and the LFWF is written as follows

$$\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,b)\right|^2 = A^2\, \frac{\sqrt{x(1-x)}}{2\,\pi\, b} \times g\left(Q^2,x\right) \left|\Phi\left(\sqrt{x(1-x)}\, b\right)\right|^2. \tag{11}$$

In general, is not so difficult to obtain $\Phi(\zeta)$ numerically in most of the known AdS/QCD models. However, obtaining $g(Q^2,x)$ is a problem still not addressed. The expression (10) has been used in AdS/QCD models where $g(Q^2,x)=1$ as in the hard wall [1], and in the soft wall with quadratic dilaton [2]. In the latter, there is an extra condition: $g(Q^2,x)=1$, which is only achieved in the large Q^2 case.

3. General soft-wall model at high Q^2

As we mention in the last section, this function equals one in the hard wall [1]. It can be approximated to one in the large Q^2 limit for the traditional soft wall model with quadratic dilaton in the AdS geometry [2]. This section will analyze what happens if we consider an arbitrary dilaton and with a generic asymptotically AdS geometry.

To do this extension, we will consider a general AdS-like warp factor given by

$$A(z) = \ln\left(\frac{R}{z}\right) + h(z), \tag{12}$$

TABLE I. Summary of the AdS/QCD models used to construct the LF wave function.

Model	Dilaton	Deformation	Ref.
1	$\phi_1(z) = \kappa^2 z^2$	$h_1(z) = 0$	[10]
2	$\phi_2(z) = \mu_G^2 z^2 \tanh\left(\frac{\mu_{G^2}^4}{\mu_G^2} z^2\right)$	$h_2(z) = 0$	[11]
3	$\phi_3(z) = \kappa^2 z^2 + M z + \tanh\left(\frac{1}{M z} - G\right)$	$h_3(z) = 0$	[12]
4	$\phi_4(z) = 0$	$h_4(z) = \frac{1}{2} k z^2$	[13]

where R is the AdS radius and $h(z \to 0)$ is a deformation function that vanishes in the limit $z \to 0$. The EOM for the current $J(Q^2, z)$ reads as [8]

$$\partial_z^2 J(Q^2,z) - \left[\frac{1}{z} - \partial_z\left(h(z) - \phi(z)\right)\right] \times \partial_z J(Q^2,z) + Q^2 J(Q^2,z) = 0. \quad (13)$$

Notice that to get the LFWF, we consider a matching involving the expression (8), where the modes dual to hadrons must be normalizable. Since we are interested in the current $J(Q^2, z)$ written in the large Q^2 limit, holographically, we can fulfill this condition with the low z limit. In other words, large Q^2 is equivalent to the limit $z \to 0$ in this context. In this limit, the Eq. (13) reduces to

$$\partial_z^2 J(Q^2,z) - \left(\frac{1}{z}\right)\partial_z J(Q^2,z) + Q^2 J(Q^2,z) = 0\,, \quad (14)$$

which is the same equation to the hard-wall case, i.e. $g(Q^2, x) = 1$. Recall that the asymptotically AdS condition is translated into the vanishing of the deformation function at the conformal boundary. This condition ensures the field/operator matching condition via the conformal dimension of the bulk fields [9].

Thus, for a general asymptotically AdS space, we have the expression for the two-body LF wave function:

$$\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,\zeta)\right|^2 = A^2\, x(1-x)\, \frac{\left|\Phi(\zeta)\right|^2}{2\pi\zeta}, \quad (15)$$

that in terms of x and b is written as

$$\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,b)\right|^2 = A^2\,\frac{\sqrt{x(1-x)}}{2\pi b}\,\left|\Phi\left(\sqrt{x(1-x)}\,b\right)\right|^2. \quad (16)$$

Therefore, the expression (11) with $g(Q^2, x) = 1$ is general and valid to obtain a two-parton holographic LFWF for other models different from the usual soft-wall or hard-wall.

4. Examples

In this section, we compare the LFWF obtained for different dilatons. The recipe is as follows once we have defined the dilaton and the AdS-like warp factor, we construct the holographic potential for scalar modes defined by

$$V(z) = \frac{15}{4\,z^2} + \frac{1}{4}\left[\phi'(z)^2 + 9\,h(z)^2\right] - \frac{3}{2}\phi'(z)\,h'(z) - \frac{1}{2}\left[\phi''(z) - 3\,h''(z)\right] - \frac{3}{2\,z}\left[\phi'(z) - 3\,h'(z)\right] + \frac{e^{2\,h(z)} M_5^2\, R^2}{z^2}, \quad (17)$$

where $M_5^2\,R^2$ is the bulk mass associated with the scalar modes. We will fix $M_5^2\,R^2 = -3$ in our analysis. This particular choice implies that we have dual scalar meson states.

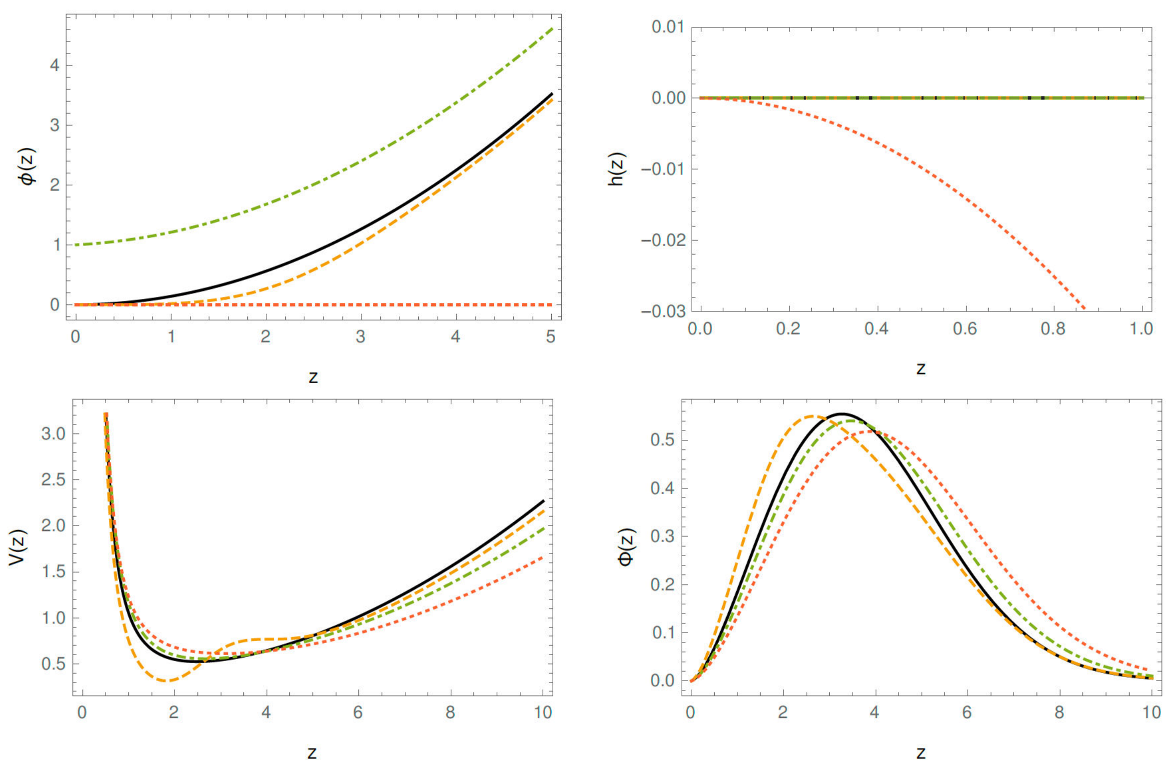

FIGURE 1. From left to right, the set of dilaton fields, deformation functions, holographic potentials, and AdS modes considered in this work with the following conventions: for model 1, we used black lines; for model 2, we use dashed lines; for model 3, we use dot-dashed lines and model 4 is depicted with dotted lines.

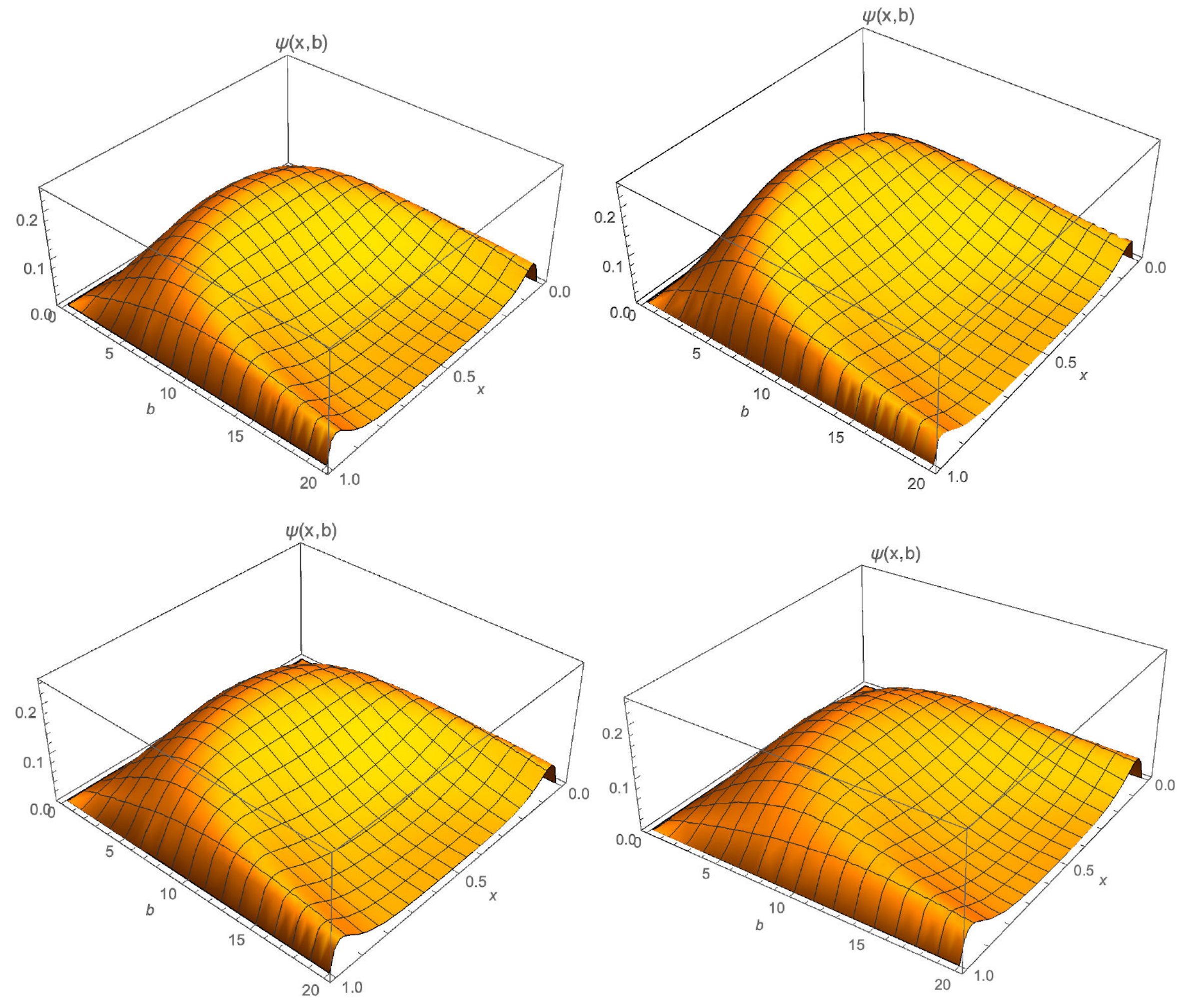

FIGURE 2. The LFWF $\widetilde{\psi}_{q_1\bar{q}_2}(x, b)$ for each model discussed in this work. From left to right: model 1, model 2, model 3 and model 4.

The bulk Schrödinger-like modes are obtained by solving the Schrödinger-like EOM on the AdS side with the general holographic potential (17). With these modes, we will construct the two-body LF wave function (16).

We will consider four different models from the AdS/QCD literature, characterized by their specific form of the dilaton field and the deformation function used. A summary of these models can be found in Table I.

In Fig. 1, it is depicted a comparison between the four dilatons and deformation functions, with their corresponding holographic potentials, given by the general expression (17), and its AdS Schrodinger-like modes calculated from such potentials.

In Fig. 2 we plot the LFWF calculated for each one of the four models discussed in this paper. Notice that although they are qualitatively similar, they have different behavior in b and x, which can impact the shape of hadron properties calculated, for example in pion form factor calculations, these wave functions must produce a difference in how $Q^2F(Q^2)$ goes to constant value according to counting rules at high Q^2.

5. Conclusions

The relation between AdS modes and the LFWF is an interesting topic that has been restricted the hard-wall [1] and soft-wall models with quadratic dilaton [2] or their phenomenological modifications in the QCD side (*e.g.*, see [3–5]).

There are plenty of AdS/QCD models considering different dilatons or asymptotically AdS geometries, which try to catch many aspects of hadronic phenomenology that the standard hard-wall or soft-wall with quadratic dilaton do not address. For these sorts of AdS/QCD models, it was not studied their LFWF extension. Mainly because the matching procedure, allowing us to compare form factors at both sides and extract the LFWF in terms of AdS modes, was not discussed before in the specialized literature up to [6], which was used as the basis for this presentation. Therefore, the approach considered here could be interesting because it allows computing the LFWF associated with these AdS/QCD approaches.

The key point to calculate the LFWF related to AdS modes is to know appropriately the distribution $g(x, Q^2)$ defined in the expression (10). As it was discussed in Sec. 3, for the large Q^2 limit case, this distribution is equal to one. Thus, we can obtain a two-body holographic LFWF for a wide variety of AdS/QCD models.

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