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BLFQ calculations of the proton leading twist quark TMDs

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For hadron collisions, transverse-momentum-dependent parton distribution functions (TMD PDFs) are important quantities to connect theoretic calculations and experimental cross sections. Recently, basis light-front quantization (BLFQ) has become a powerful tool to investigate the structures of the bound state. We present the first calculations within the BLFQ framework for the leading twist quark TMDs under a trivial assumption of the gauge link. We compare our calculations with previous calculations via the PDF limit and evolve our results of the unpolarized TMDs.

Keywords: Protons; Transverse-momentum dependent distributions; SIDIS.

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1. Introduction

Via TMD factorization and evolution [1–3], transverse-momentum-dependent parton distribution functions (TMD PDFs) enter the cross-sections of the semi-inclusive deep-inelastic-scattering (SIDIS) and Drell-Yan process. Being an important subject, TMDs have been studied in many models [5, 6, 17] and also lattice simulations [8, 9]. Much progress has also been made on the experimental side to extract TMD PDFs from the experiment data, mainly focusing on f_1 [10, 11], $f_{1T}^{\perp}$ [12, 13], g_{1T} [14], h_1 [13, 15] and $h_{1T}^{\perp}$ [16]. Our theoretical understandings of the behaviour of TMDs have also reached a certain level [31, 32].

Recently, as a promising non-perturbative framework to calculate the internal structures of the bound state, basis light-front quantization (BLFQ) has been employed to investigate the physical electron [18], positronium [7], meson [19, 20] and most importantly, hadron [21, 22, 33]. The basic idea of BLFQ is to simultaneously get the mass spectrum and the light-front wavefunction (LFWF) of the bound states within a feasible computation time by diagonalizing the light-front stationary Schrödinger equation.

We follow the study from [21, 22] to present the first analysis of the leading twist quark TMD PDFs of the proton system within the BLFQ framework. We also present some discussions and comparisons concerning our results.

2. BLFQ framework

In this section, we provide a brief introduction to the BLFQ calculations of the hadron system within the leading, three-quark, Fock sector truncation. In general, we start from a certain Hamiltonian, which enters the light-front stationary Schrödinger equation. Via basis function and truncation, we then transform this equation into a standard matrix problem. After getting the eigenvector we then reconstruct the light-front wavefunction (LFWF), based on which we further calculate observables like TMDs.

In this work, we diagonalize the following effective Hamiltonian [22]

$$H'_{\text{eff.}} = H_{\text{eff.}} + H' \,, \tag{1}$$

$$H_{\text{eff.}} = \sum_i \frac{m_i^2 + \left(p_i^{\perp}\right)^2}{x_i} - \left(\sum_i p_i^{\perp}\right)^2 + \frac{1}{2}\sum_{i,j} V_{i,j}^{\text{conf.}} + \frac{1}{2}\sum_{i,j} V_{i,j}^{\text{OGE}} \,. \tag{2}$$

Here $V_{i,j}^{\text{conf.}}$ is the confinement potential and $V_{i,j}^{\text{OGE}}$ is the one-gluon-exchange. The H' term is added here to facilitate the factorization between the center of mass and intrinsic motion

$$H' = \lambda_L \left\{ \left(\sum_i p_i^{\perp}\right)^2 + b^4 \left(\sum_i r_i^{\perp}\right)^2 - 2b^2 \mathbb{1} \right\} . \tag{3}$$

Here we use two-dimensional harmonic oscillators (2D HO) as basis in the transverse direction and plane waves in the longitudinal direction. As for the truncation, we introduce $N_{\max}$ and K in the transverse and longitudinal directions respectively [22]. Thus we translate the eigenproblem $H'_{\text{eff.}} \left|P, \Lambda\right\rangle = M^2 \left|P, \Lambda\right\rangle$ into a standard matrix problem and finally get the light-front wavefunction (LFWF) $\psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}(\{x_i, p_i^{\perp}\})$. Here M is the proton mass.

3. Transverse-momentum dependent distributions

Transverse-momentum dependent distributions emerge from the parameterization [23, 24] of the following TMD correlators

$$\Phi^{[\Gamma]}(P,S;x=\frac{p^+}{P^+},p^\perp)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3N_0}e^{ip\cdot z}$$
$$\times\langle P,S|\,\bar{\Psi}(0)\mathcal{W}(0,z)\Gamma\Psi(z)\,|P,S\rangle\,|_{z^+=0}\,, \tag{4}$$

where color and flavor indexes and summation, if needed, are implicit. For the leading twist ($\Gamma=\gamma^+,\gamma^+\gamma^5,i\sigma^{j+}\gamma^5$), the parameterizations are

$$\Phi^{[\gamma^+]}(x,p^\perp;P,S)=f_1^e-\frac{\epsilon^{ij}p^iS^j}{M_e}f_{1T}^{\perp e}\,, \tag{5}$$

$$\Phi^{[\gamma^+\gamma^5]}(x,p^\perp;P,S)=S^3g_{1L}^e+\frac{p^\perp\cdot S^\perp}{M_e}g_{1T}^e\,, \tag{6}$$

$$\begin{aligned}\Phi^{[i\sigma^{j+}\gamma^5]}(x,p^\perp;P,S)=&\,S^jh_1^e+S^3\frac{p^j}{M_e}h_{1L}^{\perp e}\\&+S^i\frac{2k^ip^j-(p^\perp)^2\delta^{ij}}{2M_e^2}h_{1T}^{\perp e}\\&+\frac{\epsilon^{ij}p^i}{M_e}h_1^{\perp e}\,.\end{aligned} \tag{7}$$

The kinematic conventions we use are the same as those in [18, 25, 26]:

$$P=\left(P^+,\frac{M}{P^+},0^\perp\right)\,, \tag{8}$$

$$(S^+,S^-,S^\perp)=\left(\frac{S^3P^+}{M},\frac{-S^3M}{P^+},S^1,S^2\right)\,, \tag{9}$$

$$(S^1,S^2,S^3)=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)\,, \tag{10}$$

where M is the proton mass.

The gauge link, $\mathcal{W}(0,z)$, theoretically, is needed to maintain the gauge invariance of the TMD correlations, Eq. (4) [27–29]. In this work, for simplicity, we use the following approximation

$$\mathcal{W}(0,z)\approx\mathbb{1}. \tag{11}$$

This is an assumption commonly used in the theoretical calculations of TMD [30, 35]. Under this approximation, one would find that all the T-odd TMDs reduce to zero.

With the above assumptions and conventions, we substitute the LFWF from the BLFQ framework into the TMD correlator and get the six non-vanishing TMDs under the trivial assumption of the gauge link:

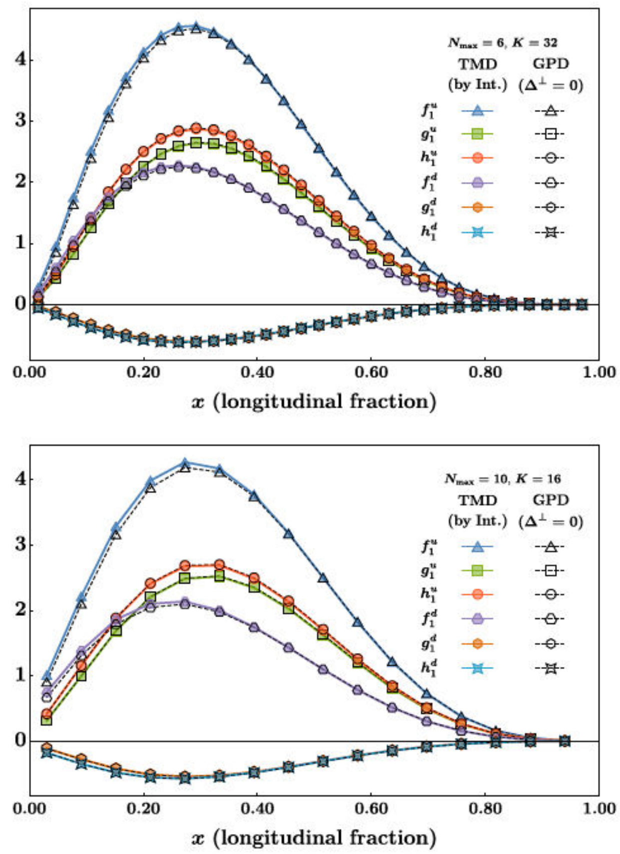

FIGURE 1. (Color online) Comparison of the PDF limit of our TMD and the PDF calculations from [22] at initial scale. Different panels are comparisons for different truncation parameters $N_{\max}$ (2D HO truncation) and K (longitudinal planwave truncation). See § 2. for detailed discussions of $N_{\max}$ and K. Solid lines are the PDF limit of the TMDs we got here and dashed results are PDFs gotten from [30]. Lines with different markers (colors) are results for different TMDs.

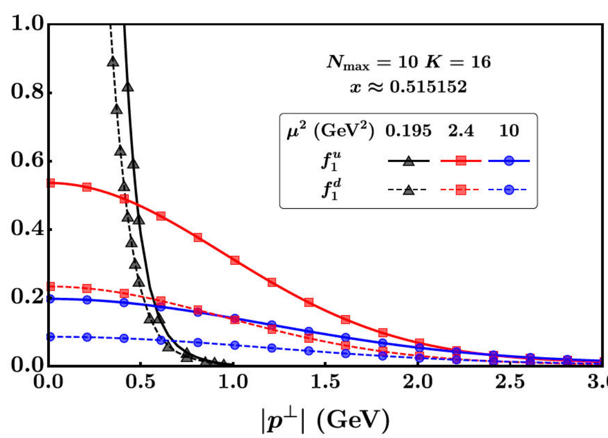

FIGURE 2. (Color online) f_1 at the initial scale, $0.195\,\mathrm{GeV}^2$ [22] and two higher scales $2.4\,\mathrm{GeV}^2$ and $10\,\mathrm{GeV}^2$ following evolution scheme described in § 4.. Lines with different markers (colors) represent f_1 at different scales. Solid lines represent u quark distributions and dashed lines represent d quark distributions. This plot shows the distributions at $x=(9.5/16.5)\approx0.515$ in the transverse direction.

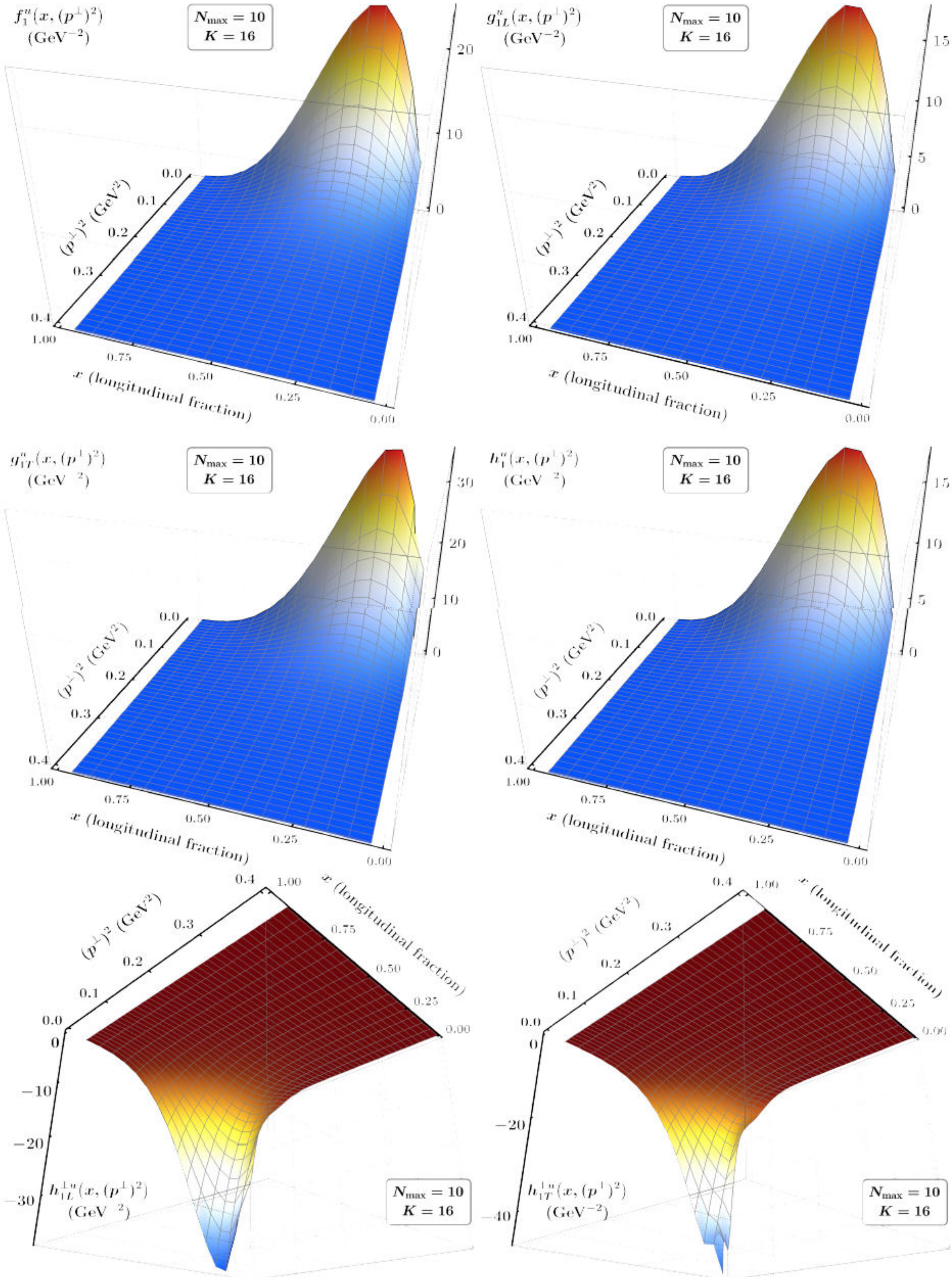

FIGURE 3. 3D plots of BLFQ results for six non-vanishing TMDs of the u quark.

$$f_1 = \int [\mathrm{d}123] \sum_{\lambda_2\lambda_3} \left[|\psi^{+}_{+,\lambda_2\lambda_3}|^2 + |\psi^{+}_{-,\lambda_2\lambda_3}|^2 \right] , \quad (12)$$

$$g_{1L} = \int [\mathrm{d}123] \sum_{\lambda_2\lambda_3} \left[|\psi^{+}_{+,\lambda_2\lambda_3}|^2 - |\psi^{+}_{-,\lambda_2\lambda_3}|^2 \right] , \quad (13)$$

$$g_{1T} = \frac{M}{(p^\perp)^2} \int [\mathrm{d}123] \times \sum_{\lambda_2\lambda_3} 2\,\mathrm{Re} \left[k^\perp_R \, \psi^{+\,*}_{+,\lambda_2\lambda_3} \psi^{-}_{+,\lambda_2\lambda_3} \right] , \quad (14)$$

$$h_1 = \int [\mathrm{d}123] \sum_{\lambda_2\lambda_3} \left[\psi^{+\,*}_{+,\lambda_2\lambda_3} \psi^{-}_{-,\lambda_2\lambda_3} \right] , \quad (15)$$

$$h^\perp_{1L} = \frac{M}{(p^\perp)^2} \int [\mathrm{d}123] \times \sum_{\lambda_2\lambda_3} 2\,\mathrm{Re} \left[k^\perp_R \, \psi^{+\,*}_{-,\lambda_2\lambda_3} \psi^{+}_{+,\lambda_2\lambda_3} \right] , \quad (16)$$

$$h^\perp_{1T} = \frac{2M^2}{|p^\perp|^2} [\mathrm{d}123] \sum_{\lambda_2\lambda_3} \left[\psi^{+\,*}_{-,\lambda_2\lambda_3} \psi^{-}_{+,\lambda_2\lambda_3} \right] , \quad (17)$$

where $k^\perp_R = k^1 + ik^2$, and we use the abbreviation

$$[\mathrm{d}123] = \frac{\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \mathrm{d}^2 p^\perp_2 \mathrm{d}^2 p^\perp_2 \mathrm{d}^2 p^\perp_3}{16\pi^3} \times \delta(x_1 + x_2 + x_3 - 1) \times \delta^2(k^\perp_1 + k^\perp_2 + k^\perp_3) \delta(x - x_1) \delta^2(p^\perp - p^\perp_1) . \quad (18)$$

We compute the $\mathrm{d}^2 p^\perp$ integration of the three TMDs which have a proper PDF limit, f_1, g_{1L}, h_1, and compare our calculations with the initial-scale results from [22]. PDFs in Ref. [22] are calculated by setting the momentum transfer $\Delta = 0$ for corresponding generalized parton distributions (GPDs). This comparison is presented in Fig. 1 and we are pleased to find good consistency for different truncation parameters. The 3D plots of all the six non-vanishing TMDs are presented in Fig. 3. It is easy to find that all those results are in reasonable behaviour. We will present more results in Ref. [33].

4. TMDs after evolution

The comparison with the previous calculations of the PDFs also enables us to assign the same initial scale $\mu_0^2 = 0.195\,\mathrm{GeV}^2$ to our TMD results. Thus we can implement TMD evolutions to see how it affects our results. For the unpolarized TMD, f_1, the following evolution scheme from Refs. [30, 34, 36] is quite common in the literature

$$\tilde{f}_1(x, |b^\perp|; \mu) = \tilde{f}_1(x, |b^\perp|; \mu_0) \tilde{R}(\mu, \mu_0, |b^\perp|) \times e^{-g_K(|b^\perp|) \ln\left(\frac{\mu}{\mu_0}\right)} . \quad (19)$$

Here, $\tilde{f}_1(x, |b^\perp|; \mu)$ is the unpolarized TMD in the impact parameter space, ***i.e.***, the two-dimensional Fourier transformation of $f_1(x, |p^\perp|; \mu)$.

The evolution kernel, $\tilde{R}$, takes the following form

$$\tilde{R}(\mu, \mu_0, |b^\perp|) = \exp\left(\ln\left[\frac{\mu}{\mu_0}\right] \int_{\mu_b}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{\mu}^{\mu_0} \frac{\mathrm{d}\mu'}{\mu'} \gamma_F(\mu, \mu') \right) , \quad (20)$$

where $\mu_b = C_1/b_*$, $b_* = |b^\perp| / \sqrt{1 + ((b^\perp)^2/b^2_{\text{max}})}$ are in charge of the transition between perturbative and non-perturbative regions [37]. The anomalous dimensions are

$$\gamma_K(\mu') = \alpha_s(\mu') \frac{2C_F}{\pi} , \qquad \gamma_F(\mu, \mu') = \alpha_s(\mu') \frac{C_F}{\pi} \left(\frac{3}{2} - \ln\left[\frac{\mu^2}{\mu'^2}\right] \right) . \quad (21)$$

We also use the same non-perturbative function, $g_K(|b^\perp|) = (1/2) g_2 \left(b^\perp\right)^2$ and parameters as in [30].

Evolved BLFQ results of f_1 are shown in the transverse direction as Fig. 2. It is easy to observe that in the transverse direction, with increasing scale, the magnitudes of both f_1^u and f_1^d decrease and the widths of them increase, which is quite as expected.

5. Conclusions

In this paper, we present the first investigation of the proton quark TMDs in the leading twist within the BLFQ framework. We deduce the corresponding formulas for TMDs and compare the PDF limit of our TMDs with PDFs got in [22]. We find good consistency in this comparison. First, this strongly signifies that our calculation is self-consistent. Second, after DGLAP evolution, PDF calculations in the previous papers [21, 22] agree well with the experiment data and thus we choose the same initial scale $\mu_0^2 = 0.195\,\mathrm{GeV}^2$ for our TMDs. Following evolution scheme from Refs. [30, 34, 36], we also evolve our TMDs to higher scales to investigate the effect of scale evolution. We find that with higher scale we would generally get TMDs with lower magnitude and larger extension in the transverse direction.

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