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Proton gravitational form factors and mechanical properties in a light-front quark-diquark model

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We present our recent calculation of the gravitational form factors (GFFs) of proton using the light-front quark-diquark model constructed by the soft-wall AdS/QCD [1]. We extract the four quark GFFs $A(Q^2)$, $B(Q^2)$, $C(Q^2)$ and $\bar{C}(Q^2)$ by calculating the matrix elements of the symmetric energy momentum tensor. Using the D-term we calculate the pressure and shear distributions of quarks inside the proton in the impact parameter space. The GFFs, $A(Q^2)$ and $B(Q^2)$ are found to be consistent with the lattice QCD, while the qualitative behavior of the D-term form factor is in agreement with the extracted data from the deeply virtual Compton scattering (DVCS) experiments at JLab, the lattice QCD, and the predictions of different phenomenological models.

Keywords: Light-front; quark-diquark; generalized form factors; mechanical properties.

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1. Introduction

Mechanical properties like the spin and pressure distribution of quarks inside the nucleon has become a hot topic in recent times after the first experimental evidence of the pressure distribution of quarks inside the nucleon [2]. These mechanical properties are encoded in the matrix elements of the energy-momentum tensor in the form of gravitational form factors (GFFs). Experimentally, the GFFs can be assessed in hard exclusive processes like deeply virtual Compton scattering (DVCS) through generalized parton distributions (GPDs) [2]. For a spin-half particle, the symmetric form of the energy-momentum tensor can be parameterized in terms of four GFFs denoted as $A(Q^2)$, $B(Q^2)$, $4C(Q^2) = D(Q^2)$ (also called the D-term) and $\overline{C}(Q^2)$. The GFFs are functions of the square momentum transferred (Q^2). $A(Q^2)$ and $B(Q^2)$ are related to the mass and total angular momentum of the proton [3] and they are constrained by the conservation of momentum and total angular momentum. So the GFFs $A(Q^2)$ and $B(Q^2)$ satisfy the sum rule such that the total $A(0) + B(0) = 1$ and the angular momentum $J(0) = (1/2)\,(A(0) + B(0)) = 1/2$. These sum rules are a consequence of the Poincaré invariance of the theory. But the D-term is unconstrained at $Q^2 = 0$ and is connected to the internal properties of the nucleon like the pressure and stress distributions [4,7]. The D-term is found to be negative in stable interacting system. It is zero for a free fermion and -1 for a boson. The D-term for proton extracted from the recent JLab data [2] is negative and the corresponding pressure distribution is found to be repulsive near the center and attractive towards the periphery of the nucleus.

2. Light-front quark-diquark model

In this model for the nucleon, the light-front wave functions are modeled from the solution of soft-wall AdS/QCD [8]. We treat the three valence quarks inside the proton to be a quark (fermion) and composite state of a diquark (boson). This reduces a three body problem into a more tractable two body problem. We assume that the diquark has spin zero. So we can express the two-particle Fock-state expansion for the proton spin components, $J^z = \pm(1/2)$ in a frame where the proton transverse momentum vanishes, *i.e.* $P \equiv \big(P^+, \mathbf{0}_\perp, (M^2/P^+)\big)$, as follows:

$$|P;\uparrow(\downarrow)\rangle = \sum_q \int \frac{dx\; d^2\mathbf{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\bigg(\psi^{\uparrow(\downarrow)}_{+q}(x,\mathbf{k}_\perp)\Big| + \frac{1}{2},0;xP^+,\mathbf{k}_\perp\Big\rangle + \psi^{\uparrow(\downarrow)}_{-q}(x,\mathbf{k}_\perp)\Big| - \frac{1}{2},0;xP^+,\mathbf{k}_\perp\Big\rangle\bigg). \tag{1}$$

The light-front wave functions $\psi^{\lambda_N}_{\lambda_q q}(x,\mathbf{k}_\perp)$ for different spin configurations of nucleon helicities $\lambda_N = \pm(1/2)$ and for quark helicities $\lambda_q = \pm(1/2)$ can be written as follows:

$$\psi^{\uparrow}_{+q}(x,\mathbf{k}_\perp) = \varphi^{(1)}_q(x,\mathbf{k}_\perp)\,,$$
$$\psi^{\uparrow}_{-q}(x,\mathbf{k}_\perp) = -\frac{k^1 + ik^2}{xM}\,\varphi^{(2)}_q(x,\mathbf{k}_\perp)\,,$$

$$\psi^{\downarrow}_{+q}(x,\mathbf{k}_\perp) = \frac{k^1 - ik^2}{xM}\,\varphi^{(2)}_q(x,\mathbf{k}_\perp)\,,$$
$$\psi^{\downarrow}_{-q}(x,\mathbf{k}_\perp) = \varphi^{(1)}_q(x,\mathbf{k}_\perp), \qquad (2)$$

where, the wave functions $\varphi_q^{(i=1,2)}(x,\mathbf{k}_\perp)$ are the modified form of the soft-wall AdS/QCD prediction constructed by introducing the parameters $a_q^{(i)}$ and $b_q^{(i)}$ for quark q [8, 9],

$$\varphi^{(i)}_q(x,\mathbf{k}_\perp) = N^{(i)}_q \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a^{(i)}_q}(1-x)^{b^{(i)}_q} \times \exp\left(-\frac{\mathbf{k}^2_\perp}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right). \qquad (3)$$

We consider massless quarks and the parameters are tuned to reproduce the electromagnetic properties of the nucleons. The full set of parameters used here can be found in Ref. [10].

3. Gravitational Form Factors

The symmetric energy-momentum tensor $T^{\mu\nu}$ for a spin half system can be parameterized in terms of four GFFs [11, 12]

$$\langle P', S'|T^{\mu\nu}_i(0)|P,S\rangle = \bar{U}(P',S')\bigg(-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\bigg)U(P,S), \qquad (4)$$

where, $\bar{P}^\mu = (1/2)(P'+P)^\mu$, $q^\mu = (P'-P)^\mu$, $U(P,S)$ is the spinor, $Q^2 = -q^2$ and M is the system mass. In the Drell-Yan frame with $q^+ = 0$, the light-front four momenta are defined as :

$$P = (P^+, P_\perp, P^-) = \left(P^+, 0, \frac{M^2}{P^+}\right),$$
$$P' = (P'^+, P'_\perp, P'^-) = \left(P^+, q_\perp, \frac{q^2_\perp + M^2}{P^+}\right),$$
$$q = P' - P = \left(0, q_\perp, \frac{q^2_\perp}{P^+}\right). \qquad (5)$$

The GFFs are calculated by considering the energy momentum tensor of a free quark inside the proton:

$$T^{\mu\nu} = \frac{i}{2}[\bar{\psi}\gamma^\mu(\overrightarrow{\partial}^\nu\psi) - \bar{\psi}\gamma^\mu\overleftarrow{\partial}^\nu\psi], \qquad (6)$$

where ψ is the quark field. The matrix element needed to calculate the GFFs can be written in the compact form as follows:

$$\mathcal{M}^{\mu\nu}_{SS'} = \frac{1}{2}\left(\langle P+q, S|T^{\mu\nu}_i(0)|P,S'\rangle\right), \qquad (7)$$

where the Lorentz indices $(\mu,\nu) \equiv \{+,-,1,2\}$, $(S,S') \equiv \{\uparrow,\downarrow\}$ is the helicity of the initial and final state. $\uparrow$ ($\downarrow$) represents positive (negative) spin projection along $z-$ axis.

The GFFs $A_i(Q^2)$ and $B_i(Q^2)$ are calculated using the T_i^{++} component.

$$\mathcal{M}^{++}_{\uparrow\uparrow} = 2(P^+)^2 A_i(Q^2),$$
$$\mathcal{M}^{++}_{\uparrow\downarrow} = -2(P^+)^2\frac{(q^1_\perp - iq^2_\perp)}{2M}B_i(Q^2). \qquad (8)$$

The GFFs $C_i(Q^2)$ are $\bar{C}_i(Q^2)$ are calculated using the transverse components $T_i^{\perp\perp}$.

$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} - \mathcal{M}^{22}_{\uparrow\downarrow} - \mathcal{M}^{22}_{\downarrow\uparrow} = i\left(\frac{B_i(Q^2)}{4M} - \frac{C_i(Q^2)}{M}\right) \times \left((q^1_\perp)^2 q^2_\perp - (q^2_\perp)^3\right),$$
$$\mathcal{M}^{11}_{\uparrow\downarrow} + \mathcal{M}^{11}_{\downarrow\uparrow} + \mathcal{M}^{22}_{\uparrow\downarrow} + \mathcal{M}^{22}_{\downarrow\uparrow} = i\bigg(\frac{B_i(Q^2)}{4M} - C_i(Q^2)\frac{3Q^2}{M} + 2M\overline{C}_i(Q^2)\bigg)\left(q^2_\perp\right). \qquad (9)$$

The significance of the D-term is the fact that they are related to the pressure and shear distribution functions of the quarks inside the nucleon. We calculate these mechanical properties in the impact parameter space (IPS) by performing a Fourier transformation from the momentum space to the IPS. The distributions of pressure and shear forces inside the nucleon are given by

$$p(b) = \frac{1}{6M}\frac{1}{b^2}\frac{d}{db}b^2\frac{d}{db}\tilde{D}(b),$$
$$s(b) = -\frac{1}{4M}b\frac{d}{db}\frac{1}{b}\frac{d}{db}\tilde{D}(b), \qquad (10)$$

where $\tilde{D}(b)$ represents the Fourier transform of GFF $D(Q^2) = 4C(Q^2)$ and $b = |\vec{b}_\perp|$ is the impact parameter.

Our results for the four GFFs are shown in Fig. 1. In Fig. 1a) and b), we show our result for the GFFs $A^{u+d}(Q^2)$ and $B^{u+d}(Q^2)$. We compare our results with the lattice QCD prediction [13] available at the scale of $\mu^2 = 4$ GeV^2. In order to compare with the lattice results we perform the QCD evolution of our results using the higher order perturbative parton evolution toolkit (HOPPET) [14] following Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations of QCD with next-to-next-to-leading order. Both the initial ($\mu_0^2 = 0.32$ GeV^2) and final scale ($\mu^2 = 4$ GeV^2) results are shown. The initial scale results are in accordance with the constraints set by the conservation laws such that $A^{u+d}(0) = 1$ and the vanishing of the anomalous gravitomagnetic moment $B^{u+d}(0) \approx 0$ [15]. The evolved results for $A^{u+d}(Q^2)$ and $B^{u+d}(Q^2)$ are consistent with the lattice QCD results. The error bands reflect a 10% uncertainty in the model parameters.

We show our result for the D-term ($4C^{u+d}(Q^2) = D^{u+d}(Q^2)$) in Fig. 1c). The D-term is found to be negative as expected for a stable bound system. We compare our

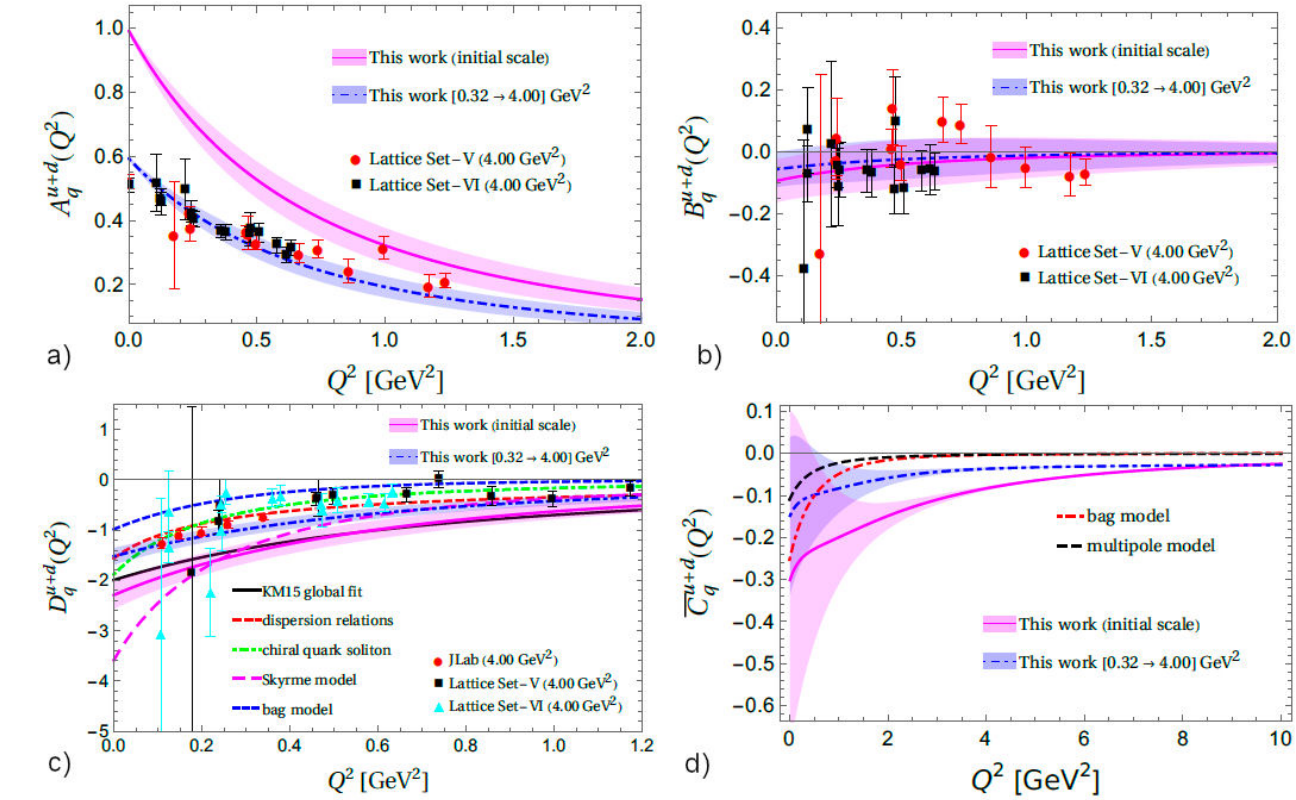

FIGURE 1. The plots of the four GFFs as functions of Q^2. The solid magenta lines with magenta bands are the results at the initial scale, while the dash-dotted blue lines with blue bands represent the results at $\mu^2 = 4$ GeV^2 evolved from the initial scale $\mu_0^2 = 0.32$ GeV^2. In plots (a) and (b) our results are compared with the lattice results (red circle and black square) at scale $\mu^2 = 4$ GeV^2 [13]. In plot (c) the red circles are the experimental data from the Jefferson Lab [2] and the cyan triangles and black squares correspond to the lattice results [13]. In plot (c) our results are compared with KM15 global fit [16] (solid black), dispersion relations [17] (dashed red), chiral quark soliton [18] (dash-dotted green), Skyrme model [19] (big dashed magenta), and bag model [20] (dashed blue). In plot (d) our results are compared with the bag model [20] (dash-dotted red) and the multipole model [7] (dashed black).

D-term result with various theoretical models, lattice results and the JLab data. We find that the qualitative behavior of our result is compatible with lattice QCD [13] and the experimental data from JLab [2] as well as other theoretical predictions from the KM15 global fit [16], dispersion relation [17], χQSM [18], Skyrme model [19], and bag model [20]. The evolved value at zero momentum transfer is $D^{u+d}(0) = -1.521$. The fitted function $-1.521/(1+0.531\mathrm{Q}^2)^{3.026}$ reproduces the evolved result for $D^{u+d}(Q^2)$. In Fig. 1d) our result for the GFF $\bar{C}^{u+d}(Q^2)$ is shown. We observe that the central line for $\bar{C}^{u+d}(Q^2)$ is negative. This is consistent with the behavior of $\bar{C}(Q^2)$ seen in the bag model [20] and the multipole model [7]. After evolution the results for $\bar{C}^{u+d}(Q^2)$ are in better agreement with the bag model [20] and the multipole model [7]. We also observe that our parameters become more sensitive as $Q^2 \to 0$ as seen from the error bands on $\bar{C}^{u+d}(Q^2)$.

In Fig. 2a) and b) we show our results for the pressure and shear distributions respectively of the quark inside the nucleon in the impact parameter space. The pressure distribution $p(b)$ must satisfy the stability condition, also known as the von Laue condition [22],

$$\int_0^\infty db\, b^2\, p(b) = 0. \tag{11}$$

The stability condition is a consequence of the conservation of the energy momentum tensor and it helps in understanding how the internal forces balance inside a composite system [4, 5]. We compare our results with the distribution evaluated in leading order light-cone sum rule [21] and the distribution obtained from the fitting functions of the experimental data for $D(Q^2)$ at JLab [2]. In order to maintain stability, the pressure, in accordance with the von Laue condition, must have at least one node. We observe that our pressure distribution satisfies the von Laue stability condition [22] showing a positive core and a negative tail. The presence of a positive core indicates a repulsive central core which is essential to prevent the inner collapse and the negative tail indicates an attractive outer region essential for maintaining the bound system. The node in our pressure distribution is near 0.5 fm (central line), whereas the node appears around 0.7 fm in the result based on light-cone sum rule [21] and around 0.6 fm for the JLab [2] results.

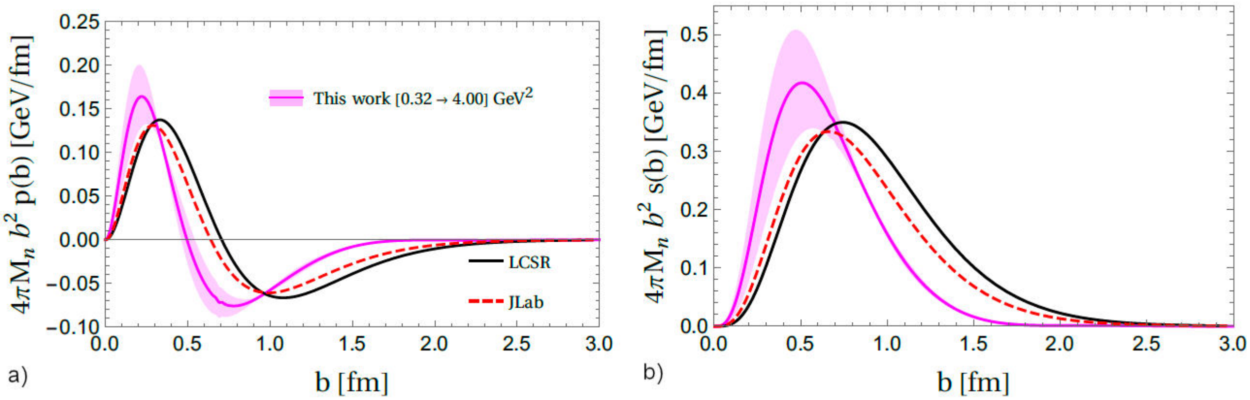

FIGURE 2. Plots of a) the pressure distribution $4\pi M_n b^2 p(b)$, and b) the shear distribution $4\pi M_n b^2 s(b)$ as a function of b. Our results are compared with results based on LCSR evaluated in Ref. [21] (black line) and using the fitting function based on the JLab experimental data [2] (red dashed line).

The shear distribution for stable hydrostatic systems is positive and has connection to surface tension and surface energy [5]. Our shear distribution is also positive and the qualitative behavior is in accord with other approaches [18,19,21]. The peak value of our result is higher than the results based on light-cone sum rule [21] and JLab data [2].

4. Conclusion

We extracted the four proton GFFs by calculating the matrix elements of the symmetric energy-momentum tensor using a light-front scalar quark-diquark model. The GFFs $A(Q^2)$ and $B(Q^2)$ were extracted from the diagonal plus component whereas the D-term and $\bar{C}(Q^2)$ were extracted from the transverse component of the energy-momentum tensor. We evolved our GFFs to compare them with the lattice predictions. The results for $A(Q^2)$ and $B(Q^2)$ after evolution were found to be consistent with lattice QCD predictions. Our D-term results after evolution were consistent with the lattice prediction and the experimental data extracted from DVCS process at JLab. Our value for $\bar{C}(Q^2)$ is found to be negative at zero momentum transfer. The behavior of the pressure $p(b)$ and shear force $s(b)$ distributions were also in accordance with the experimental observation and other theoretical predictions.

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