Progress on $d^*(2380)$ in a chiral SU(3) quark model

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The experimental information and theoretical predictions of $d^*(2380)$ are briefly introduced. The salient features of the chiral SU(3) quark model are presented, and the results of $d^*(2380)$ from traditional calculation in this model are shown and discussed. The problems in such quark model calculations are pointed out, and a revised quark model investigation of $d^*(2380)$ is given. It is shown that the $d^*(2380)$ has not yet been fully understood in quark model.

Keywords: Dibaryon; chiral symmetry; quark model.

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1. Introduction

In 2011, the COSY Collaboration reported a resonance structure in $pn \rightarrow dπ^0π^0$ reaction [1]. Supposing this structure is caused via an s-channel resonance, the isospin $I$ and spin-parity $J^P$ of this resonance will be $I(J^P) = 0(3^+)$, and the mass and width of this resonance will be $M \approx 2380$ MeV, $Γ \approx 70$ MeV. This resonance was called $d^*(2380)$. Later, this resonance was also observed in $pn \rightarrow pnπ^0π^0$ and $pn \rightarrow ppm^{-}π^0$ reactions [2,3]. It was also reported to be observed in proton-deuteron and deuteron-deuteron fusion reactions to helium isotopes [4-7].

In 2014, the WASA-at-COSY Collaboration and the SAID data analysis center have reanalyzed the proton-neutron scattering data on both cross sections and polarizations [8]. It was found that with the inclusion of the newly observed analyzing power data $A_y$ from the COSY Collaboration, a pole in the coupled $^3D_3 - ^3G_3$ partial waves can be found in the revised fit of the data. The pole position is $(2380 \pm 10) - i(40 \pm 5)$ MeV, in accordance with the mass and width of $d^*(2380)$.

In 2017, the experiment at ELPH has reported the total cross-section data for $γd \rightarrow dπ^0π^0$ [9]. It was shown that the total cross section data can be better described with the inclusion of the $d^*(2380)$ resonance. However, as only one data point at the $d^*(2380)$ position is available, more high precision data are needed to further confirm the resonance information in this reaction.

In 2020, the experiments at MAMI have reported the polarization data $P_y$ for the reaction $γd \rightarrow pπ^0$ [10]. It was found that the new $P_y$ data for polarized neutron are consistent with their old $P_y$ data for polarized proton in $γp \rightarrow p\bar{n}$ [11], and both sets of data are in accordance with the inclusion of a $d^*(2380)$ resonance.

In 2020, the lattice results for $d^*(2380)$ became available [12]. It was reported that a quasi-bound state corresponding to $d^*(2380)$ is formed with the binding energy $25 - 40$ MeV below the $ΔΔ$ threshold for heavy pion masses ($m_π = 679$, $841$, and $1018$ MeV).

Theoretically, we are interested in $d^*(2380)$ mainly because of its unusual narrow decay width. It is true that the $d^*(2380)$ is 84 MeV below the threshold of $ΔΔ$, but it is still above the thresholds of $ΔNπ$, $NNππ$, and $NN$. In principle, it can decay to these channels via strong interactions. Thus, it is expected to have a large decay width. But the experimental decay width is only 70 MeV. It is even below 1/3 of the decay widths of two $Δs$. This may indicate that the $d^*(2380)$ has an unconventional structure.

In literature, there are many predictions for $ΔΔ$ binding energies, and the predicted $ΔΔ$ binding energies are in a rather large range from few MeV to more than 300 MeV in various models. But before the experimental information being available in 2009 and 2011, only in a few theoretical works the predicted binding energies are roughly consistent with the mass of $d^*(2380)$. In the rest parts of the present paper, we present and discuss the results of $d^*(2380)$ from the chiral SU(3) quark model.

2. The chiral SU(3) quark model

2.1. The model Hamiltonian

The chiral SU(3) quark model is an extension of the SU(2) linear σ model which consists of $σ$ and $π$ as the chiral fields and works well for SU(2) non-strange systems. In the chiral SU(3) quark model, the $σ$ field in SU(2) linear $σ$ model is extended to the scalar nonet fields, and the $π$ field in SU(2) linear $σ$ model is extended to the pseudo-scalar nonet fields. The SU(3) chiral field reads

$$\Sigma = \sum_{a=0}^{8} λ_a σ_a + i \sum_{a=0}^{8} λ_a π_a,$$

with $λ_a$ being a unitary matrix and $λ_a$ ($a = 1, 2, \cdots , 8$) being the Gell-Mann matrix of the flavor SU(3) group. The quark and chiral fields interacting Lagrangian reads...
\[ \mathcal{L}_I = -g_{ch}(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^\dagger \psi_L) \]
\[ = -g_{ch}\psi \left( \sum_{a=0}^8 \sigma_a \lambda_a + i\gamma_5 \sum_{a=0}^8 \pi_a \lambda_a \right) \psi, \quad (2) \]

with \( \psi \) being the quark spinors and \( g_{ch} \) the quark and chiral-field coupling constant. By introducing the scalar and pseudoscalar fields of Eq. (1), the chiral symmetry of the Lagrangian in Eq. (2) is restored, and the constituent quarks obtain their constituent masses via the spontaneous chiral symmetry breaking. The Goldstone bosons get their physical masses via the explicit chiral symmetry breaking caused by the tiny current quark masses.

In practical calculations, one also needs to consider the one-gluon-exchange (OGE) potential to describe the short-range perturbative effects and the phenomenological confinement potential to describe the long-range non-perturbative effects. The OGE potential reads

\[ V_{ij}^{OGE} = V_{cen}^{OGE}(r_{ij}) + V_{ls}^{OGE}(r_{ij}) + V_{ten}^{OGE}(r_{ij}), \quad (3) \]

with

\[ V_{cen}^{OGE}(r_{ij}) = \frac{g_1 g_2}{4} \left( \lambda^c_i \cdot \lambda^c_j \right) \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \right\} \times \left[ \frac{1}{m^2_i} + \frac{1}{m^2_j} + \frac{4}{3m_i m_j} (\sigma_i \cdot \sigma_j) \right], \quad (4) \]

\[ V_{ls}^{OGE}(r_{ij}) = -\frac{g_1 g_2}{4} \left( \lambda^c_i \cdot \lambda^c_j \right) \frac{m^2_i + m^2_j + 4m_i m_j}{8m^2_i m^2_j} \times \frac{1}{r_{ij}} \left[ L \cdot (\sigma_i + \sigma_j) \right], \quad (5) \]

\[ V_{ten}^{OGE}(r_{ij}) = -\frac{g_1 g_2}{4} \left( \lambda^c_i \cdot \lambda^c_j \right) \frac{1}{4m_i m_j} \frac{1}{r^3_{ij}} \times (3\sigma_i \cdot \bar{r}_{ij}\sigma_j \cdot \bar{r}_{ij} - \sigma_i \cdot \sigma_j). \quad (6) \]

The confinement potential is usually chosen to be of quadratic or linear type. In the former case, it reads

\[ V_{ij}^{conf} = -\left( \lambda^c_i \cdot \lambda^c_j \right) \left( a^2_i r^2_{ij} + a^0_i \right). \quad (7) \]

Here, in above equations, \( m_{ij} \) is the mass of the \( i(j) \)-th constituent quark, and \( \lambda^c \) is the Gell-Mann matrix of the color SU(3) group.

The total Hamiltonian of the chiral SU(3) quark model reads

\[ H = \sum_i \left[ m_i + \frac{\bar{p}^2_i}{2m_i} \right] - \frac{\bar{p}^2_{cm}}{2M} + \sum_{i<j} \left[ V_{ij}^{OGE} + V_{ij}^{conf} \right] \]
\[ + \sum_{a=0}^8 \left( V_{ij}^{\sigma_a} + V_{ij}^{\pi_a} \right). \quad (8) \]

Here \( V_{ij}^{\sigma_a} \) and \( V_{ij}^{\pi_a} \) are potential originated from the Lagrangian of Eq. (2), i.e. they are quark-quark potential induced by the \( \sigma_a \) field and \( \pi_a \) field, respectively. Their explicit expressions are

\[ V_{ij}^{\sigma_a} = V_{cen}^{\sigma_a}(r_{ij}) + V_{ls}^{\sigma_a}(r_{ij}), \quad (9) \]
\[ V_{ij}^{\pi_a} = V_{cen}^{\pi_a}(r_{ij}) + V_{ten}^{\pi_a}(r_{ij}), \quad (10) \]

with

\[ V_{cen}^{\sigma_a}(r_{ij}) = -\frac{g_2}{4\pi} \frac{\Lambda^2 m_a}{\Lambda^2 - m_a^2} Y_1(m_a, r_{ij}) (\lambda^a_i \lambda^a_j), \quad (11) \]
\[ V_{ls}^{\sigma_a}(r_{ij}) = -\frac{g_2}{4\pi} \frac{\Lambda^2 m_a}{\Lambda^2 - m_a^2} \frac{m_a^2}{4m_i m_j} Z_3(m_a, r_{ij}) \times [L \cdot (\sigma_i + \sigma_j)] (\lambda^a_i \lambda^a_j), \quad (12) \]
\[ V_{cen}^{\pi_a}(r_{ij}) = \frac{g_2}{4\pi} \frac{\Lambda^2 m_a}{\Lambda^2 - m_a^2} \frac{m_a^2}{12m_i m_j} Y_3(m_a, r_{ij}) \times (\sigma_i \cdot \sigma_j) (\lambda^a_i \lambda^a_j), \quad (13) \]
\[ V_{ten}^{\pi_a}(r_{ij}) = \frac{g_2}{4\pi} \frac{\Lambda^2 m_a}{\Lambda^2 - m_a^2} \frac{m_a^2}{12m_i m_j} H_3(m_a, r_{ij}) \times (3\sigma_i \cdot \bar{r}_{ij}\sigma_j - \sigma_i \cdot \sigma_j) (\lambda^a_i \lambda^a_j). \quad (14) \]

Here

\[ Y_1(m_a, r) = Y(m_a r) - \left( \frac{\Lambda}{m_a} \right) Y(\Lambda r), \quad (15) \]
\[ Y_3(m_a, r) = Y(m_a r) - \left( \frac{\Lambda}{m_a} \right)^3 Y(\Lambda r), \quad (16) \]
\[ Z_3(m_a, r) = Z(m_a r) - \left( \frac{\Lambda}{m_a} \right)^3 Z(\Lambda r), \quad (17) \]
\[ H_3(m_a, r) = H(m_a r) - \left( \frac{\Lambda}{m_a} \right)^3 H(\Lambda r), \quad (18) \]

with

\[ Y(x) = \frac{1}{x} e^{-x}, \quad (19) \]
\[ Z(x) = \left( \frac{1}{x^2} + \frac{1}{x^3} \right) e^{-x}, \quad (20) \]
\[ H(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) Y(x). \quad (21) \]

\( \Lambda \) in above equations is the cutoff parameter in the form factor introduced in each quark and chiral-field vertex,

\[ F(q^2) = \left( \frac{\Lambda^2}{\Lambda^2 + q^2} \right)^{1/2}. \quad (22) \]

2.2. The wave functions of two-baryon system

In resonating group method (RGM), the wave functions of \( \Delta \Delta \rightarrow CC \) system in six-quark center-of-mass frame is constructed as

\[ \Psi_{6q} = \sum_{B=\Delta,C} A \left[ \phi_B(\xi_1, \xi_2) \phi_B(\xi_4, \xi_5) \eta_{BB}(r) \right]. \quad (23) \]
where $A$ is the antisymmetrizing operator,

$$A = 1 - 9P_{36}.$$  \hspace{1cm} (24)

The $\Delta$ has isospin $3/2$ and three quarks are in color singlet state, and the color state $C$ has isospin $1/2$ and three quarks are in color octet state. $\phi_B(\xi_1, \xi_2)$ and $\phi_B(\xi_4, \xi_5)$ are internal wave functions of two baryons taken as gaussians, with $\xi_1$ and $\xi_2$ being the internal coordinates of one baryon, and $\xi_4$ and $\xi_5$ being the internal coordinates of another baryon,

$$\phi_B(\xi_1, \xi_2) = \left(\frac{2}{3\pi b_u^2}\right)^{3/2} \exp\left[-\frac{1}{b_u^2} \left(\frac{\xi_1^2}{4} + \frac{\xi_2^2}{3}\right)\right],$$ \hspace{1cm} (25)

$$\phi_B(\xi_4, \xi_5) = \left(\frac{2}{3\pi b_u^2}\right)^{3/2} \exp\left[-\frac{1}{b_u^2} \left(\frac{\xi_4^2}{4} + \frac{\xi_5^2}{3}\right)\right].$$ \hspace{1cm} (26)

The spin, flavor, and color quantum numbers are suppressed in Eq. (23) for the sake of simplicity. $\eta_{BB}(r)$ is the trial wave function of the relative motion between two baryons $BB$, which is known and will be determined by the dynamics of the two-baryon system:

$$\langle \delta \Psi_{6q} | (H - E) | \Psi_{6q} \rangle = 0.$$ \hspace{1cm} (27)

By solving this equation, one gets the wave function and the binding energy or scattering phase shifts of the two-baryon system.

2.3. The model parameters

In previous quark model calculations [13–15], the model parameters are fixed as following. The $u(d)$ quark mass is chosen to be $313$ MeV, and the size parameter $b_u$ of $u(d)$ quark gaussian wave function is set to be $0.5$ fm. The coupling constant for quark and chiral fields coupling is fixed by the relation:

$$g_{ch}^2 = \left(\frac{3}{5}\right) \frac{2 g_{NN} m_u^2}{4\pi M_N^2},$$ \hspace{1cm} (28)

with $M_N$ being the nucleon mass and $g_{NN}^2/4\pi = 13.67$ taken as the empirical value. The masses of mesons are chosen to be their experimental values except for the $\pi$ meson, whose mass is treated as a parameter to be fixed by the binding energy of deuteron. The coupling constant of one-gluon exchange potential is fixed by the mass split of $N - \Delta$. The parameters in confinement potential are fixed by the stability condition of $N$,

$$\frac{\partial M_N}{\partial b_u} = 0,$$ \hspace{1cm} (29)

and the mass of $N$, $M_N = 939$ MeV.

We mention that there is no free parameter when the chiral SU(3) quark model is applied to study the $\Delta\Delta$ interaction.

3. Results for $d^*(2380)$

In 1999, the $\Delta\Delta$ interaction was investigated in chiral SU(3) quark model by Yuan et al., and the effects of the hidden-color channel were also studied [13]. Here, the hidden color channel $CC$ is composed of two color-octet states, and it has the same quantum numbers as $\Delta\Delta$. It was found that the binding energy of $\Delta\Delta$ single channel is about $30 - 63$ MeV, and for $\Delta\Delta - CC$ coupled channels, the binding energy is about $42 - 80$ MeV, which is very close to the experimental value for $d^*(2380)$, $84$ MeV. These results showed that the hidden-color channel $CC$ is rather important for the $\Delta\Delta$ interaction, as it leads to $12 - 17$ MeV increment of the binding energy for the $\Delta\Delta$ system.

After the experimental information of $d^*(2380)$ became available, we have restudied the $\Delta\Delta - CC$ interaction within the chiral SU(3) quark model with more refined quark-quark interactions [14, 15]. It was found that the binding energy of the $\Delta\Delta$ system is about $29 - 62$ MeV, and when the hidden color channel was further considered, the binding energy of the $\Delta\Delta - CC$ coupled channels is found to be $47 - 84$ MeV. Again, these results showed that the hidden-color channel $CC$ is very important for the $\Delta\Delta$ interaction, as it causes $18 - 22$ MeV increment of the binding energy for the $\Delta\Delta$ system.

By extracting the components of $CC$ in $\Delta\Delta - CC$ system, it was found that the $d^*(2380)$ has a fraction of hidden-color channel of about $2/3$. We know that a pure hexaquark state can be expanded as

$$[6]_{\text{orb}} [33]_{03} = \sqrt{\frac{1}{5}} |\Delta\Delta\rangle_{03} + \sqrt{\frac{4}{5}} |CC\rangle_{03}.$$ \hspace{1cm} (30)

This means that a pure hexaquark state has a fraction of hidden-color channel of $4/5$. Now the $d^*(2380)$ has a fraction of hidden-color channel of about $2/3$, thus it is fair to say that the $d^*(2380)$ is a hexaquark dominated exotic state.

Once the wave functions are ready, the partial decay widths of $d^*(2380)$ can be calculated straightforwardly. At the lowest level, the hidden-color components do not decay, and the $d^*(2380)$ decays to $d\pi\pi$ and $NN\pi\pi$ via $\Delta\Delta$ components. Under such assumptions, the partial decay widths of $d^* \to d\pi^+\pi^-$, $d^* \to d\pi^0\pi^0$, $d^* \to N\pi^+\pi^-$, $d^* \to N\pi^0\pi^0$, $d^* \to pp\pi^0\pi^0$, and $d^* \to nn\pi^+\pi^-$ were calculated, and the results for all these decay channels were found to be consistent with the experimental values [16, 17]. For the $d^* \to NN\pi\pi$ decay, the calculated branching ratio is $0.9\%$, consistent with the experimental upper limit, $9\%$ [18].

So it seems that the $d^*(2380)$ can be well understood in the chiral SU(3) quark model. The calculated binding energy of the $\Delta\Delta - CC$ system is consistent with the $d^*(2380)$ mass. The large hidden-color channel components suppress the decay width of $d^*(2380)$, and consequently, the calculated partial decay widths of $d^*(2380)$ are in good agreement with the data. Is the situation really so?

4. Problems in previous quark model calculations

In previous quark model calculations in literature, the wave function of single quark is chosen to be a gaussian wave
function, and the size parameters of these gaussians are set to be a constant for all considered baryons, e.g. $b_u = 0.5$ fm. The problem is, why the size parameter is the same for all baryons? We know that different baryons have different quantum numbers. Then it is difficult to understand that different baryons have the same size although their Hamiltonians are different due to their different quantum numbers. The consequence of setting the size parameter for all baryons to be the same is the following. The wave functions of single baryons with specified size parameter might not be the solutions of the given Hamiltonian. In this situation, when one studies the baryon-baryon interactions with the same quark-quark interaction, non-physical channels might be needed to change the internal wave functions of single baryons. Thus, one needs to be very careful to explain the structures of the bound baryon-baryon states.

On the other hand, in constituent quark model study of nucleon-nucleon interaction, the OGE is found to be one of the most important sources of the short-range repulsion. Therefore, one needs a credible determination of the coupling constants of OGE to get a proper understanding of the nucleon-nucleon short-range interaction mechanisms. In previous quark model calculations, the coupling constants in OGE potential were claimed to be determined by the mass splittings of $N - \Delta$ and $\Lambda - \Sigma$. But the masses of all baryons were calculated as averaged values of the given Hamiltonian with the spatial wave functions of constituent quarks setting as gaussians with the same size parameter $b_u$. In this case the obtained baryons’ masses may not be the minimums of the given Hamiltonian. As a consequence, the coupling strengths of OGE potential were not well determined in previous quark model calculations, and the short-range nucleon-nucleon interaction mechanisms were not properly understood at the quark level.

5. Revised quark model calculations

Recently, we have solved the above mentioned inconsistency problems [19]. Instead of setting the size parameters to be the same for all baryons, we let them be determined by variation principle, i.e. for each baryon, the size parameter is chosen to make sure that the baryon mass is in the minimum of the Hamiltonian. In Fig. 1 of Ref. [19], we have shown the energies of octet and decuplet baryon ground states as a function of the size parameter of Gaussian wave functions. The minimum of each curve should be regarded as the mass of each baryon. The model parameters are then adjusted to make the theoretical baryon masses consistent with the experimental values. One sees from this figure that each baryon has different size parameter as one expects. In particular, the size parameters for octet baryons are quite different from those for decuplet baryons, e.g. $b_u = 0.47$ fm for $N$ and $0.59$ fm for $\Delta$. If one uses the size parameter for $N$ to study the $\Delta \Delta - CC$ system, one needs to be very careful in explaining the $CC$ components of the bound state obtained. The $CC$ channel might not be physical one. Instead, it might be partially needed to change the internal wave function of $\Delta$ which is not the solution of the Hamiltonian as the chosen size parameter does not guarantee that the calculated $\Delta$ energy is the minimum of the matrix element of the Hamiltonian.

Apart from the energies of the octet and decuplet baryon ground states, our new calculation describes quite well the $NN$ interactions simultaneously. Figures 2-9 of Ref. [19] presents the partial-wave phase shifts and the mixing parameters of $NN$ scattering up to total angular momentum $J = 6$. Table III of Ref. [19] shows the binding energy of deuteron obtained in our new quark model calculation. One sees that all those observables obtained in our new theoretical calculation are consistent with the corresponding data.

We emphasize that our work of Ref. [19] is the first quark model calculation that describes the masses of octet and decuplet baryon ground states, the binding energy of deuteron, and the partial-wave phase shifts and mixing parameters of $NN$ scattering in a rather consistent manner.

Using such a model, we have re-investigated the $\Delta \Delta - CC$ system in a parameter-free way. Our preliminary results show that the binding energy of $\Delta \Delta$ system is 18 MeV. When the channel coupling of $\Delta \Delta$ and $CC$ is further considered, the binding energy of the system is found to be 21 MeV. This means that: a) when single baryons and two-baryon systems are treated consistently, the binding energy of the $\Delta \Delta - CC$ system will be largely reduced, and b) the effects of hidden-color channel are much less important in the new calculation as there is only 3 MeV increment of the binding energy when the hidden-color channel is included.

In previous quark model calculations, the narrow $d^*(2380)$ decay width can be explained because of the very large $CC$ components. However, in our new quark model calculation, the contribution of the $CC$ channel is much less important. In this case, it might be difficult to explain the narrow decay width of $d^*(2380)$ in this revised model.

We mention that in Ref. [20], the possibility of explaining the $d^*(2380)$ as a three-diquark state was investigated. In Ref. [21], a triangle singularity mechanism was proposed to explain the peak of $d^*(2380)$.

6. Summary

The WASA-at-COSY Collaboration has reported the $d^*(2380)$ with an unusual narrow decay width [1]. In previous chiral quark model calculation, the binding energy of $\Delta \Delta - CC$ system are qualitatively consistent with the mass of $d^*(2380)$ [13–15], and the narrow $d^*(2380)$ decay width can be explained by the large hidden-color channel components [16–18].

Recently, we have updated the chiral quark model calculation by treating the single baryons and two-baryon systems in a rather consistent way, and we found that in the new calculation, the binding energy of $\Delta \Delta - CC$ system will be largely reduced, and the effects of hidden-color channel are much less important, which makes the explanation of narrow $d^*(2380)$ width difficult.
We conclude that the $d^*(2380)$ has not yet been fully understood in quark model.

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