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Bound States in QED from a light-front approach

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We calculate the binding energy and structure of the positronium system in a light-front nonperturbative approach. The input interaction is from Quantum Electrodynamics first principles and one dynamical photon is explicitly included in the basis. We obtain the low-lying mass spectrum, which is found to be consistent with that from nonrelativistic quantum mechanics. Various excitation modes of the low-lying states can be identified from the associated wave functions. The photon distribution in the longitudinal direction is calculated for the low-lying positronium states.

Keywords: Bound states; QED; positronium; nonperturbative renormalization; BLFQ.

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1. Introduction

Although Quantum Chromodynamics (QCD) has been established to be the underlying theory for the strong interaction, solving hadron structure from QCD first principles remains a challenging task due to the nonperturbative nature in strongly-interacting many-body systems. Toward this goal, bound states in Quantum Electrodynamics (QED) are often used as models for hadrons, which is based on the observation that both QCD and QED are gauge theories and compared to QCD, QED lacks some complicated nonperturbative phenomena such as the confinement and spontaneous chiral symmetry breaking. Among the bound states in QED, the positronium, the bound state formed by one electron and one positron, has often been used as a model for studying the structure of mesons.

In this work we report the progress on solving the mass spectrum and structure of the positronium system in the recently constructed nonperturbative approach named Basis Light-front Quantization (BLFQ) [1]. BLFQ has so far been applied to many bound state systems [2–14] in QED and QCD. In the previous work [4] the positronium system has been studied in BLFQ using an effective one-photon-exchange interaction. In this work we attempt to solve the positronium based on the first-principles interaction in QED with one dynamical photon included in the basis.

This article is organized as follows: we first review the methodology of BLFQ in Sec. 2, then present the numerical results in Sec. 3 and finally conclude in Sec. 4.

2. Basis Light-front Quantization and renormalization scheme

BLFQ is a nonperturbative approach to bound state structure in the framework of quantum field theory. BLFQ is based on the light-front quantization, that is, the quantization plane is chosen to be the light front, a 3-dimensional surface in the 4-dimensional spacetime with fixed light-front time, $x^+ = x^0 + x^3$. One major advantage of light-front quantization lies in the fact that the resulting wave functions are frame-independent and thus naturally suitable for describing relativistic bound states. In BLFQ, the mass spectrum and amplitudes of the bound states are obtained by solving the following eigenvalue equation of the corresponding light-front Hamiltonian P^-:

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle. \tag{1}$$

The eigenvalues P^-_β correspond to the mass spectrum and the eigenvectors $|\beta\rangle$ correspond to the amplitudes of the bound states encoding their structural information. For the positronium system we employ the light-front QED Hamiltonian which can be obtained from the standard QED Lagrangian through the Legendre transformation in the light-cone gauge A^+_μ=0.

In order to cast the eigenvalue equation (Eq. (1)) into the matrix form, we write down the Hamiltonian P^- in a chosen basis. The basis is constructed in terms of Fock sector expansion. To generate the binding effect, we keep $|e^+e^-\rangle + |e^+e^-\gamma\rangle$ two Fock sectors in our basis. Within each Fock sector, the eigenstates of 2D harmonic oscillator and plane waves are used as the basis states for the transverse

and longitudinal directions, respectively, for each Fock particle. Further truncation on the transverse and longitudinal degrees of freedom of the basis states is needed. The truncation parameters for the transverse and longitudinal directions are denoted as $N_{\max}$ and K respectively. Larger $N_{\max}$ and K translate to simultaneous larger ultraviolet (UV) cutoff and smaller infrared (IR) cutoff of the basis, see Ref. [15] for the detailed procedure on basis construction in BLFQ.

In the truncated basis the light-front QED Hamiltonian takes the following form,

$$P^-_{\rm QED} = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_{e0}^2 + \left(i\partial^\perp\right)^2}{i\partial^+}\Psi + \frac{1}{2}A^j\left(i\partial^\perp\right)^2 A^j + ej^\mu A_\mu + \frac{e^2}{2}j^+\frac{1}{\left(i\partial^+\right)^2}j^+, \quad (2)$$

where Ψ and A_μ are the electron (positron) and photon field operator, respectively, and $j^\mu = \bar{\Psi}\gamma^\mu\Psi$. The first two terms are their corresponding kinetic terms and the remaining terms describe their interaction. m_{e0} is the bare fermion mass and e is the electric charge of the electron.

The dynamical photon in the $|e^+e^-\gamma\rangle$ Fock sector not only generates the binding through the exchange between the electron and the positron, but also induces the self-energy interaction for the electron and positron inside the positronium, thus their bare mass, m_{e0}, is not equal to the physical mass and the mass renormalization procedure is needed. We follow the recently developed basis-state-dependent renormalization procedure [16], which is a natural extension of the sector-dependent renormalization [17,18]: in the truncation scheme adopted in BLFQ, basis states in the positronium systems may have different available quanta for self-energy interaction leading to different values of the mass counterterm. In order to determine these counterterms, we perform a series of calculations of the physical electrons (or positrons) in the basis containing $|e\rangle + |e\gamma\rangle$ Fock sectors. By matching the resulting physical electron mass, m_e, with its experimental value, we obtain the electron mass counterterm, $\Delta m_e = m_{e0} - m_e$, as a function of the truncation parameters, $N_{\max}$ and K. This obtained set of the mass counterterms is subsequently substituted into the kinetic energy term in $|e^+e^-\rangle$ Fock sector of the positronium system and the resulting Hamiltonian is ready to be diagonalized for the evaluation of the positronium's mass spectrum and structure.

3. Numerical results

Results from nonrelativistic quantum mechanics suggest that the binding energy ($E_B \equiv M_{\rm Ps} - 2m_e$) of the positronium is proportional to the square of the electromagnetic coupling constant $\alpha = e^2/4\pi$, in order to obtain the binding energy with sufficient numerical precision, we choose to work with an artificially enlarged value for the electromagnetic coupling constant, $\alpha = 0.3$, instead of the physical value of $1/137$. We take the physical electron mass $m_e = 1\,$MeV. Following the renormalization procedure we outlined in Sec. 2, we first obtain the fermion mass counterterm, Δm_e, as the function of the truncation parameters $N_{\max}$ and K, as shown in Fig. 1.

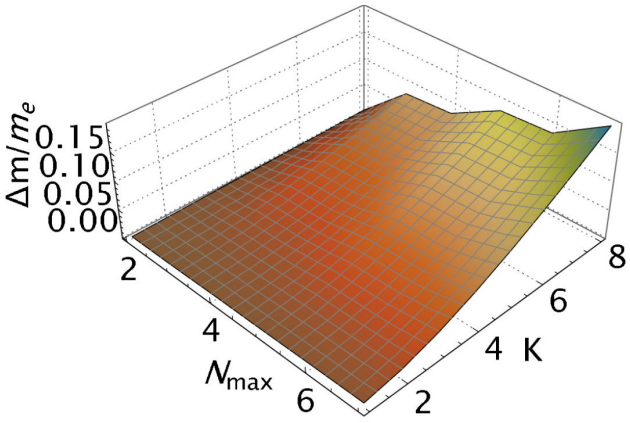

FIGURE 1. Mass counterterm (in units of the physical electron mass m_e) as a function of basis truncation parameters $N_{\max}$ and K.

The magnitude of the counterterm increases with the truncation parameters since larger truncation parameters translate to larger UV cutoff and smaller IR cutoff, which leads to larger phase space for self-energy correction to the electron mass.

Substituting the mass counterterms into the positronium Hamiltonian, we solve for the mass spectrum and wave functions of the low-lying positronium states. In Fig. 2, we present the obtained binding energies for the lowest 8 eigenstates. For the $M_J = 0$ states, from bottom to up, the lowest eight states are identified as 1^1S_0, 1^3S_1, 2^1S_0, 2^3S_1, 2^3P_0, 2^3P_1, 2^1P_1 and 2^3P_2. The binding energy for the ground state 1^1S_0 is 0.0195 MeV, close to the prediction from the nonrelativistic quantum mechanics $E_0 = \alpha^2 m_e/4$=0.0225 MeV. In our current truncated basis space, the excited states above 1^3S_1 are not bound. We expect that as the basis size increases the invariant mass of these excited states will drop and they eventually become bound. At the same time, the rotational symmetry is expected to gradually

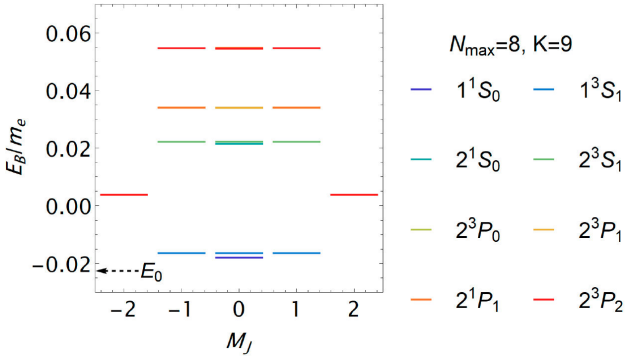

FIGURE 2. The binding energy (E_B) spectrum of the positronium system at $N_{\max} = 8$, $K = 9$ and $\alpha = 0.3$. The binding energy is represented in units of the physical electron mass. The arrow (E_0) indicates the ground state (1^1S_0) binding energy from nonrelativistic quantum mechanics.

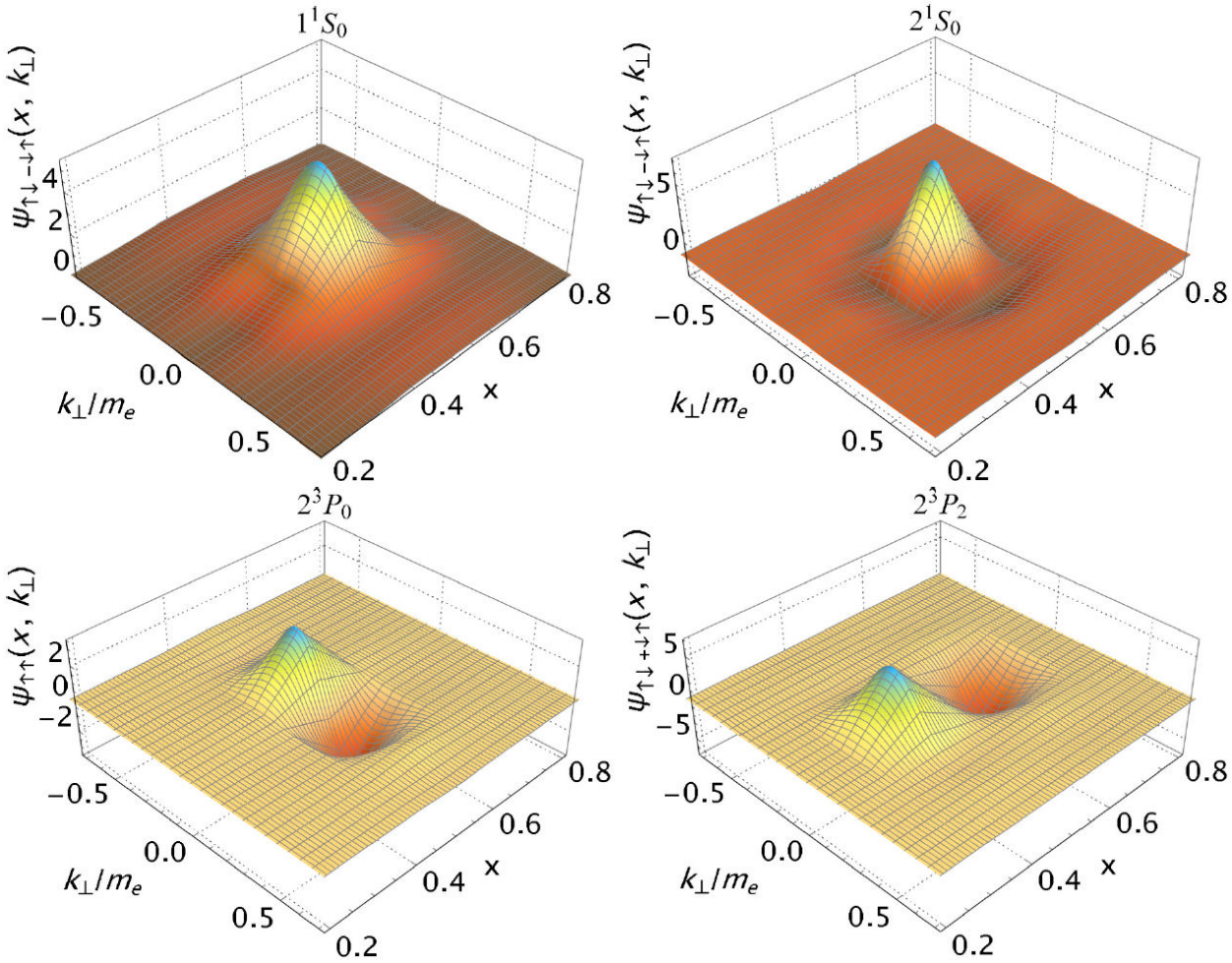

FIGURE 3. The light-front wave functions (in the $|e^+e^-\rangle$ sector) of the positronium system at $N_{\max} = 8$ and $K = 9$, $\alpha = 0.3$. x is the longitudinal momentum fraction of the electron and $k_\perp$ represents the relative transverse momentum between e^+ and e^-. For 2^3P_2, the M_J=0 component is shown. $\uparrow$ ($\downarrow$) denotes the helicity of the electron or positron and $\psi_{\downarrow\uparrow\pm\uparrow\downarrow} = (\psi_{\downarrow\uparrow} \pm \psi_{\uparrow\downarrow})/\sqrt{2}$.

restore so that the mass difference between the M_J=2 component and the M_J=1,0 component will decrease. The convergence of the results with respect to the basis size will be studied in a future work [19].

The structural information of the positronium states is encoded in the corresponding wave functions. In Fig. 3, we show the wave functions for 1^1S_0, 2^1S_0, 2^3P_0 and 2^3P_2 states in the momentum space, where we plot the wave functions in the $|e^+e^-\rangle$ Fock sector with dominant helicity configuration. These plotted wave functions are normalized to one. We can see the radial excitation in the s-wave state 2^1S_0 and azimuthal excitation in the p-wave states 2^3P_0 and 2^3P_2.

Since the interaction between the electron and the positron is mediated by the photon, there is finite probability to find the photon in the positronium. The probability of finding a photon in our basis is equivalent to the probability of finding the positronium to stay in the $|e^+e^-\gamma\rangle$ Fock sector. In Table I, we list the probability taken by the $|e^+e^-\gamma\rangle$ Fock sector for the lowest 8 eigenstates. We can identify the general trend that the probability of finding a photon increases as we go higher up in the excited states. This is as expected since the higher-lying states are more energetically favorable to stay in the $|e^+e^-\gamma\rangle$ Fock sector compared to the lower-lying states. In order to gain more differential knowledge about the photon distribution in the positronium, in Fig. 4, we present the photon distribution function in the longitudinal direction as a function of the fraction of the longitudinal momentum carried by the photon out of the total longitudinal

TABLE I. The probability of finding a photon in positronium states.

State	1^1S_0	1^3S_1	2^1S_0	2^3S_1	2^3P_0	2^3P_1	2^1P_1	2^3P_2
Prob. in $\|e^+e^-\gamma\rangle$	0.10035	0.10079	0.11111	0.10754	0.11281	0.11237	0.15651	0.15736

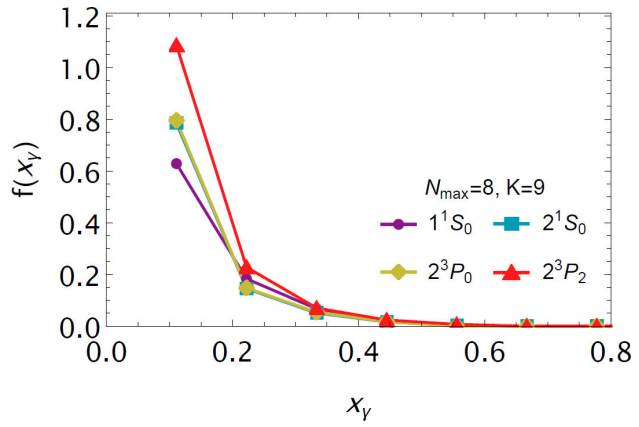

FIGURE 4. Photon distribution in the longitudinal direction for 1^1S_0, 2^1S_0, 2^3P_0 and 2^3P_2 states.

momentum carried by the positronium, $x_\gamma = p^+_\gamma / P^+_{\rm Ps}$. Similar to the physical electron system [2], the photon is mainly distributed in the small-x_γ region. Compared to the ground state, the excited states' photon distribution is enhanced in the small-x_γ region. The 3-dimensional distribution of the photon in the positronium will be studied in a future work [19].

4. Conclusion and outlook

We report preliminary results for the mass spectrum and structure of the positronium system based on the fundamental QED interaction in Basis Light-front Quantization. One dynamical photon is kept in the basis. Nonperturbative mass renormalization is performed with the mass counterterm determined from solving a series of physical electron systems in different basis sizes. The obtained low-lying energy levels are approximately consistent with nonrelativistic quantum mechanics. The longitudinal distribution of the dynamical photon is obtained for the low-lying states. The photon is found to be predominantly distributed in the small-x_γ region, which is qualitatively similar to the photon distribution in the physical electron. As a next step, we plan to study the convergence of the results with respect to the basis truncation parameters. The application of similar approaches to the meson systems in QCD is already ongoing [20].

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