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In medium dynamics of heavy particles

P. Vander Griend

Physik-Department, Technische Universität München,
James-Franck-Str. 1, 85748 Garching, Germany.

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The hierarchy of scales of heavy quarks and their bound states in a quark gluon plasma makes the system ideally suited for the use of effective field theories and the formalism of open quantum systems. We utilize these tools to perform a first principles treatment of these heavy particles in medium and analyze the regimes in which the dynamics takes the form of a Langevin equation in which the medium and the heavy particle interact via random "kicks" altering the particle's momentum.

Keywords: Quark gluon plasma; heavy quarks; effective field theories; Langevin dynamics.

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1. Introduction

Heavy quarks are excellent probes of the medium formed in heavy ion collision experiments at the Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) and the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL). Their presence from the earliest stages of the collision and persistence through the expansion and evolution of the medium to the end stages of the collision make them ideal probes of the medium formed. It is thus incumbent upon theorists to quantify the motion of heavy probes of the medium with the aim of extracting observables for comparison against experiment.

Due to the large mass M of a heavy quark relative to the temperature T of the medium, Brownian motion described by a Langevin equation has been postulated to describe the diffusion of a heavy quark in medium. In general, the Langevin equation describing the Brownian motion of a heavy particle interacting with a medium is

$$\frac{dp_i}{dt} = -\eta p_i + \xi_i(t), \tag{1}$$

where p_i is the momentum of the heavy particle, η is the drag coefficient, and $\xi_i(t)$ encodes the random interactions of the heavy particle with the medium. The force-force correlator defines the momentum diffusion coefficient κ

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t'). \tag{2}$$

The drag and diffusion coefficients are related by the Einstein relation

$$\eta = \frac{\kappa}{2MT}\,. \tag{3}$$

In Ref. [1], for a heavy quark in a non-Abelian plasma, Casalderrey-Solana and Teaney integrated the force-force correlator along the Schwinger-Keldysh contour and thereby showed the heavy quark momentum diffusion coefficient κ takes the form of a dressed chromoelectric-electric correlator

$$\kappa = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}t \left\langle \left\{ \tilde{E}^{i,a}(t,\mathbf{0}), \tilde{E}^{i,a}(0,\mathbf{0}) \right\} \right\rangle, \tag{4}$$

where

$$\tilde{E}^i(t,\mathbf{0}) = \Omega^\dagger(t)\, E^i(t,\mathbf{0})\, \Omega(t), \tag{5}$$

and

$$\Omega(t) = \exp\left[-ig \int_{-\infty}^{t} \mathrm{d}t' A_0(t',\mathbf{0}) \right]. \tag{6}$$

In Ref. [2], Blaizot and Escobedo used the formalism of open quantum systems (OQS) and the effective field theory (EFT) nonrelativistic QCD (NRQCD) [3,4] to treat a heavy quark in a non-Abelian plasma. They derived the equation of motion of the density matrix of the heavy quark and the time evolution of the heavy quark momentum showing it takes the form of a Fokker-Planck equation with corresponding Langevin equation. In this proceeding, we present an analogous analysis starting with a modified version of NRQCD with the aim of deriving a Langevin equation for the time evolution of the momentum of the heavy quark where the force-force correlator is related to κ in its dressed chromoelectric-electric correlator form given in Eq. (4).

The remainder of this proceeding is structured as follows: in Sec. 2, we introduce the modified NRQCD Lagrangian used to treat the interaction of the heavy quark with the medium; in Sec. 3, we derive the master equation describing the in-medium evolution of the density matrix of the heavy quark; in Sec. 4, we derive a Langevin equation from the master equation; we conclude in Sec. 5.

2. Medium interactions

We aim to describe the interaction of a heavy quark of mass M interacting with thermal medium particles of temperature T in the regime $M \gg T$. Our starting point is the NRQCD Lagrangian describing a nonrelativistic heavy quark

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left(i\partial_0 - gA_0 + \frac{\nabla^2}{2M} \right)\psi. \tag{7}$$

We isolate the gauge structure of the heavy quark field using the field redefinition

$$\psi(t,\mathbf{x}) \to \exp\left[ig\int\limits_{\mathbf{0}}^{\mathbf{x}} \mathrm{d}\mathbf{x}'\cdot\mathbf{A}(t,\mathbf{x}')\right]\psi(t,\mathbf{x}), \qquad (8)$$

and, furthermore, give it the color structure $\psi \sim \psi_i/\sqrt{N_c}$, where N_c is the number of colors. As we seek to isolate contributions from thermal gluons of temperature T, we multipole expand the gauge fields to first order. Under this redefinition and expansion, the NRQCD Lagrangian takes the form

$$\mathcal{L}_{\rm NRQCD} \to \psi^\dagger\left\{i\partial_0 - gA_0(t,\mathbf{0}) + \mathbf{x}\cdot g\mathbf{E}(t,\mathbf{0}) + \frac{\nabla^2}{2M}\right\}\psi, \quad (9)$$

where we have discarded terms containing thermal gluons and factors of $1/M$. Further redefining the heavy quark field by

$$\psi(t,\mathbf{x}) \to \exp\left[-ig\int\limits_{-\infty}^{t} \mathrm{d}t' A_0(t',\mathbf{0})\right]\psi(t,\mathbf{x}), \qquad (10)$$

we write the Lagrangian

$$\mathcal{L}_{\rm NRQCD'} = \psi^\dagger\left\{i\partial_0 + \mathbf{x}\cdot g\tilde{\mathbf{E}}(t,\mathbf{0}) + \frac{\nabla^2}{2M}\right\}\psi, \qquad (11)$$

where we have again discarded terms containing thermal gluons and factors of $1/M$. Eq. (11) is the basis of the following analyses. We note that $\tilde{\mathbf{E}}(t,\mathbf{0})$ is a dressed chromoelectric field as defined in Eq. (5).

We note that the above manipulations performed to arrive at Eq. (9) are analogous to those performed to write the EFT pNRQCD in terms of color singlet and color octet heavy-heavy composite fields. pNRQCD [5-7] is an EFT of the strong interaction obtained from NRQCD by the integrating out of the soft scale Mv where $v \ll 1$ is the relative velocity in a heavy quark-heavy antiquark bound state. The degrees of freedom of pNRQCD are heavy-heavy bound states in color singlet and color octet configurations and light quarks and gluons at the ultrasoft scale Mv^2. Singlet-octet and octet-octet transitions are encoded in chromoelectric dipole vertices. The further manipulations performed to arrive at Eq. (11) mirror those performed to arrive at the Lagrangians serving as the basis of the analyses of Refs. [8,9] which perform a similar analysis to that given below but for heavy quarkonium rather than a single heavy quark.

3. Master equation

The time evolution of the density matrix $\rho(t)$ of a heavy quark of mass M interacting with a non-Abelian plasma as described in Eq. (11) is given by

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i\,[h,\,\rho(t)] - \Sigma(t)\rho(t) - \rho(t)\Sigma^\dagger(t) + \Xi(\rho(t);t), \qquad (12)$$

where $h = \mathbf{p}^2/(2M)$ is the heavy quark Hamiltonian and Σ and Ξ encode interactions with the medium and are defined as

$$\Sigma = x^i A_i^\dagger, \qquad (13)$$

$$\Xi(\rho(t)) = A_i^\dagger\rho(t)x^i + x^i\rho(t)A_i, \qquad (14)$$

where

$$A_i = \frac{g^2}{6N_c}\int\limits_{\infty}^{0}\mathrm{d}s\, e^{-ihs}x^i e^{ihs}\langle\tilde{E}^{a,j}(0,\mathbf{0})\tilde{E}^{a,j}(s,\mathbf{0})\rangle. \quad (15)$$

Analogously with the treatment of App. D of Ref. [9], the evolution equation given in Eq. (12) can be rewritten as a master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i\,[H,\rho(t)] + \sum_{nm}\left(L_i^n\rho(t)L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger}L_i^n,\rho(t)\right\}\right), \qquad (16)$$

where

$$H = h + \mathrm{Im}(\Sigma), \quad L_i^0 = x^i, \quad L_i^1 = A_i^\dagger, \qquad (17)$$

$$h = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \qquad (18)$$

Setting the exponentials of Eq. (15) to 1, the medium interactions take the form

$$A_i|_{LO} = \frac{x^i}{2}\left(\kappa - i\gamma\right), \qquad (19)$$

where κ is the heavy quark momentum diffusion coefficient defined in Eq. (4) and γ is its dispersive counterpart first identified in Refs. [8,9] and defined as

$$\gamma = -i\frac{g^2}{6N_c}\int\limits_0^\infty \mathrm{d}t\left\langle\left[\tilde{E}^{i,a}(t,\mathbf{0}),\tilde{E}^{i,a}(0,\mathbf{0})\right]\right\rangle. \qquad (20)$$

In this case, the master equation reduces to the Lindblad equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i\,[H,\rho(t)] + \left(C_i\rho(t)C_i^\dagger - \frac{1}{2}\left\{C_i^\dagger C_i,\rho(t)\right\}\right), \qquad (21)$$

where

$$C_i = \sqrt{\kappa}x^i. \qquad (22)$$

An equivalent Lindblad equation describing the in-medium evolution of heavy quarkonium was derived in Refs. [8,9] and solved solved in Refs. [10,11] using the `QTraj` code [12].

4. Langevin equation

Following the procedure of Blaizot and Escobedo, we expand the exponentials of Eq. (15) to linear order giving the additional contribution

$$A_i|_{NLO} = -\frac{i\mathbf{p}}{4MT}\kappa. \tag{23}$$

In this case, the master equation no longer takes the form of a Lindblad equation. Working with A_i up to and including NLO contributions, we project the evolution equation onto eigenstates of the position of the heavy quark $\langle\mathbf{x}|$ and $|\mathbf{x}'\rangle$ which may be considered to be the position of the heavy quark before and after, respectively, interaction with the medium. The terms of the projected evolution equation are given by

$$i\langle\mathbf{x}|\,[h,\rho(t)]\,|\mathbf{x}'\rangle = \frac{i}{2M}\left(\nabla^2-\nabla'^2\right)\rho_{\mathbf{x}\mathbf{x}'}(t), \tag{24}$$

$$\langle\mathbf{x}|\left(\Sigma\rho(t)+\rho(t)\Sigma^\dagger\right)|\mathbf{x}\rangle = \Bigg[\frac{\kappa}{2}(\mathbf{x}^2+\mathbf{x}'^2)+\frac{i\gamma}{2}(\mathbf{x}^2-\mathbf{x}'^2) + \frac{\kappa}{4MT}\left(\mathbf{x}\cdot\nabla+\mathbf{x}'\cdot\nabla'\right)\Bigg]\rho_{\mathbf{x}\mathbf{x}'}(t), \tag{25}$$

and

$$\langle\mathbf{x}|\Xi(\rho(t);t)|\mathbf{x}'\rangle = \left[\kappa\,\mathbf{x}\cdot\mathbf{x}'+\frac{\kappa}{4MT}\left(\mathbf{x}\cdot\nabla'+\mathbf{x}'\cdot\nabla\right)\right] \times\rho_{\mathbf{x}\mathbf{x}'}(t), \tag{26}$$

where $\rho_{\mathbf{x}\mathbf{x}'}(t) = \langle\mathbf{x}|\rho(t)|\mathbf{x}'\rangle$. We move to the system of coordinates defined by

$$\mathbf{x}_+ = \frac{\mathbf{x}+\mathbf{x}'}{2}, \quad \mathbf{x}_- = \mathbf{x}-\mathbf{x}'. \tag{27}$$

In this coordinate system, the projected evolution equation takes the form

$$\frac{\mathrm{d}\rho_{\mathbf{x}\mathbf{x}'}(t)}{\mathrm{d}t} = \Bigg[\frac{i}{M}\nabla_+\cdot\nabla_- - \kappa\left(\frac{\mathbf{x}_-^2}{2}+\frac{1}{2MT}\mathbf{x}_-\cdot\nabla_-\right) - i\gamma\,\mathbf{x}_+\cdot\mathbf{x}_-\Bigg]\rho_{\mathbf{x}\mathbf{x}'}(t). \tag{28}$$

Wigner transforming, we arrive at the Fokker-Planck equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_+\right)\tilde{\rho}(t) = \Bigg[\frac{\kappa}{2}\nabla^2_{\mathbf{p}}+M\eta\nabla_{\mathbf{p}}\mathbf{v} + \gamma\,\mathbf{x}_+\cdot\nabla_{\mathbf{p}}\Bigg]\tilde{\rho}(t), \tag{29}$$

where $\mathbf{v}=\mathbf{p}/M$ is the heavy quark velocity and $\tilde{\rho}(t)$ is the Wigner transform of $\rho_{\mathbf{x}\mathbf{x}'}(t)$. This Fokker-Planck equation has the corresponding Langevin equation

$$\frac{dp_i}{dt} = -F_i - \eta\,p_i + \xi_i(t), \tag{30}$$

where

$$\mathbf{F} = -\gamma\,\mathbf{x}_+, \quad \langle\xi_i(t)\xi_j(t')\rangle = \kappa\,\delta_{ij}\delta(t-t'), \tag{31}$$

$$\eta = \frac{\kappa}{2MT}. \tag{32}$$

We observe that Eq. (30) contains the features of the general Langevin equation given in Eq. (1), namely the presence of a drag and a force term. Furthermore, the force-force correlator is κ in its dressed chromoelectric-electric correlator form first derived in Ref. [1] and defined in Eq. (4). We note that Blaizot and Escobedo in their NRQCD treatment of Ref. [2] derived a similar single heavy quark Langevin equation containing a drag and a force term although the precise form of these terms differs from those presented here due to their use of standard NRQCD. Additionally, in the modified NRQCD treatment presented in this proceeding, we find an additional term F_i in the Langevin equation not present in the standard Langevin equation or Blaizot and Escobedo's derivation.

5. Conclusions

In this proceeding, we present the derivation of a Langevin equation describing the in-medium evolution of a single heavy quark. The force-force correlator gives the heavy quark momentum diffusion coefficient in its dressed chromoelectric-electric correlator form. In a forthcoming work [13], we plan to investigate the nature of the additional force F_i (which may be spurious) occurring in the Langevin equation presented in Eq. (30), rigorously integrate out the scale $\sqrt{MT}$ from NRQCD to place the Langrangian of Eq. (11) on firmer EFT footing, and present a detailed and unified derivation of the single heavy quark Langevin equation presented in this work and the heavy quarkonium Langevin equation presented in Ref. [14].

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