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Ruling out some predictions of deeply-bound light-heavy tetraquarks using lattice QCD

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We discuss our lattice QCD calculations of a number of tetraquark channels with at least one heavy quark where some phenomenological models, already fully constrained by fits to the ordinary meson and baryon spectrum, predict deep binding. We find no evidence of deeply-bound tetraquarks, except in previously established strong-interaction stable $I = 0$, $J^P = 1^+$, $ud\bar{b}\bar{b}$ and $I = 1/2$, $J^P = 1^+$ $\ell s\bar{b}\bar{b}$ (where $\ell = u/d$) channels, allowing us to rule out models predicting deep binding. Preliminary results from an updated analysis of doubly-bottom tetraquarks are also presented.

Keywords: Tetraquarks; lattice QCD; hadron structure.

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1. Introduction

Over the last several years it has been firmly established by lattice QCD calculations that there are tetraquark channels that are strong-interaction-stable [1-5]. To date, these have been exclusively in the $I = 0$, $J^P = 1^+$, $ud\bar{b}\bar{b}$ and $I = 1/2$, $J^P = 1^+$ $\ell s\bar{b}\bar{b}$ channels. Outside of lattice QCD, however, literature exists that suggests the existence of other strong-interaction-stable channels. Using lattice QCD we have explored a number of channels where such predictions exist. In our lattice calculations we use as large a basis of interpolating operators as is feasible. Among these operators, and expected to be important for phenomenological reasons, is one in which the light diquarks are in a $\bar{3}_F$, spin 0 and colour $\bar{3}_C$ configuration while the anti-diquarks are in a colour 3_C configuration. In this case, when the two antiquarks are of the same flavour only the $J^P = 1^+$ channel is accessible, while $J^P = 0^+$ is available when they differ.

In Fig. 1 we demonstrate binding energy results from heavy quark symmetry, nonchiral models, chiral models and from QCD sum rules for $I = 0$, $ud\bar{c}\bar{b}$ tetraquarks with $J^P = 0^+$ and $J^P = 1^+$. Of particular note are the differences between the nonchiral models that typically find tetraquark energies in the region of the relevant two-meson thresholds and chiral models that predict binding of around 200 MeV. In both cases the models are already fully constrained by fits to the ordinary meson and baryon spectrum but they give wildly different results. In Ref. [2] we give a more comprehensive summary of results from all channels considered.

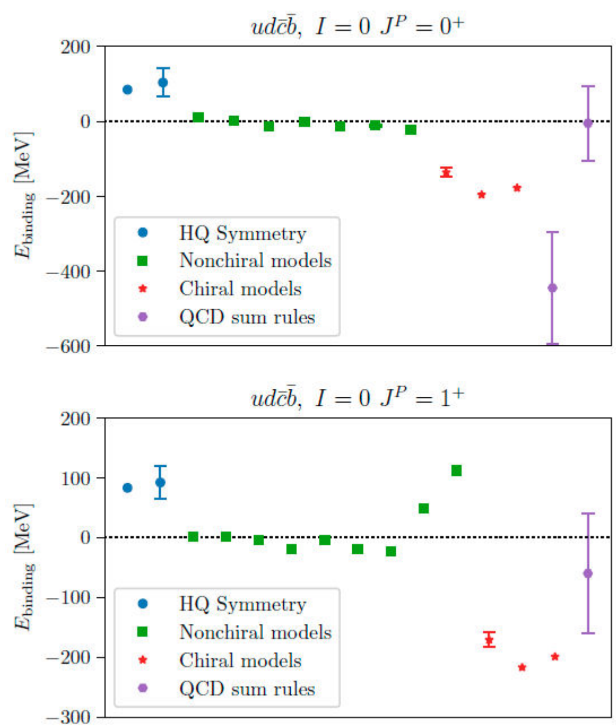

FIGURE 1. Various determinations of $I = 0$, $ud\bar{c}\bar{b}$ masses in the $J^P = 0^+$ (top) and $J^P = 1^+$ (bottom) channels. We show results from heavy quark symmetry, nonchiral models, chiral models, and from QCD sum rules.

TABLE I. Lattice volume, pion mass and number of configurations in each ensemble. There is a single lattice spacing: $a^{-1} = 2.194(10)$ GeV.

$L^3 \times T$	m_π [MeV]	N_{cnfg}	a^{-1} [GeV]
$32^3 \times 64$	700	399	2.194(10)
	575	400	
	415	400	
	299	800	
$48^3 \times 64$	192	122	
	165	88	

2. Lattice and calculation details

We use Wilson-clover lattice gauge ensembles with the effect of $2+1$ flavours of sea quarks. We have a number of light quark masses so that pion masses, m_π, range from 700 MeV down to 165 MeV. We use gauge configurations with two volumes: $L^3 \times T = 32^3 \times 64$ ensembles that were generated by the PACS-CS Collaboration [7]; and $L^3 \times T = 48^3 \times 64$ ensembles that were generated by our collaboration. There is a single lattice spacing of $a^{-1} = 2.194$ GeV. Details are given in Table I. Only the ensemble with $m_\pi = 192$ MeV is used for most of the results discussed below, although all ensembles are involved in the preliminary results from our doubly-bottom tetraquark update.

Light and strange valence quarks also use the Wilson-clover formalism. Due to a slight mistuning of the strange sea quarks we have a partially quenched strange. Charm quarks, meanwhile, use the Tsukuba interpretation of the Relativistic Heavy Quark formalism [8,9] and bottom quarks use a tadpole-improved Nonrelativistic QCD (NRQCD) action with tree-level Wilson coefficients, *i.e.*, $c_i = 1$ [10].

In order to generate a matrix of tetraquark correlators, we choose as large a basis as possible from the set of operators,

$$\begin{aligned}
D(\Gamma_1,\Gamma_2) &= (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T),\\
E(\Gamma_1,\Gamma_2) &= (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T - \bar\theta_b C\Gamma_2\bar\omega_a^T),\\
M(\Gamma_1,\Gamma_2) &= (\bar\theta\Gamma_1\psi)(\bar\omega\Gamma_2\phi),\\
N(\Gamma_1,\Gamma_2) &= (\bar\theta\Gamma_1\phi)(\bar\omega\Gamma_2\psi),\\
O(\Gamma_1,\Gamma_2) &= (\bar\omega\Gamma_1\psi)(\bar\theta\Gamma_2\phi),\\
P(\Gamma_1,\Gamma_2) &= (\bar\omega\Gamma_1\phi)(\bar\theta\Gamma_2\psi),
\end{aligned} \tag{1}$$

which couple to the tetraquarks channels of interest with quark flavours ψ, ϕ, θ and ω. Complete details of which operators are used for each channel can be found in [6]. We then solve a generalized eigenvalue problem (GEVP) in order to construct "optimized" correlators

$$C_i(t) = \sum_{j,k} V_{ij}^\dagger(\tau) C_{jk}(t) V_{ki}(\tau). \tag{2}$$

The matrix V is made from column vectors that are the eigenvector solutions of

$$C_{ij}(t)v_j(t) = \lambda_i C_{ij}(t+t_0)v_j(t). \tag{3}$$

The 'diagonalization time', τ is chosen to improve the projection of the optimized correlator onto the ground state. The results presented in the next section make the choices $t_0 = 2$ and $\tau = 4$.

3. Box-sinks

We use Coulomb gauge-fixed wall sources in our calculations using the Fourier-accelerated conjugate gradient algorithm [11]. Our initial doubly-bottom tetraquark results used local sinks [3,12]. Wall-local correlators have negative amplitudes in at least the first excited states so their effective masses approach the ground state from below. If the ground state signal does not last until large t this can mean a poor plateau.

Effective masses that plateau from above can be obtained by using wall-sinks to create a "wall-wall" correlator, but this is statistically noisy. A wall-wall correlator is constructed by summing over the spatial sites of wall-local propagators, $S(x,t)$, at the sink,

$$S^W(t) = \sum_x S(x,t). \tag{4}$$

A third possibility is the use of "box-sinks", which is a significant improvement in our analyses. This approach is similar to the use of a wall-sink except the sum is restricted to a sphere of radius R:

$$S^B(t) = \sum_{r^2<R^2} S(x+r,t). \tag{5}$$

In this way, the choice $R^2 = 0$ is then equivalent to wall-local correlators and $R^2 = 3(L/2)^2$ corresponds to the wall-wall result. Appropriately tuning the radius results in correlators

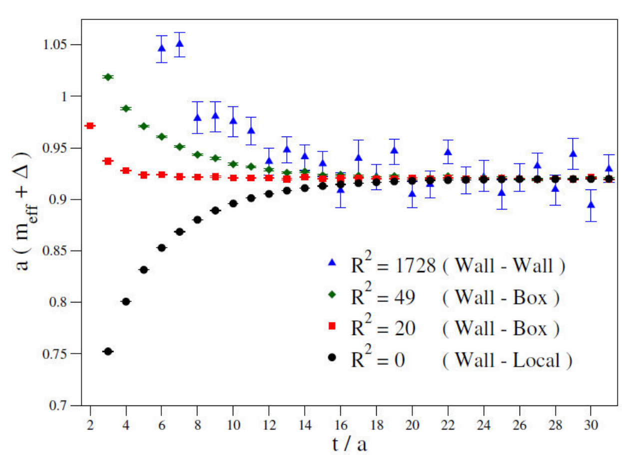

FIGURE 2. Effective masses for correlators describing the same B_c meson with different box-sink sizes R^2 in lattice units.

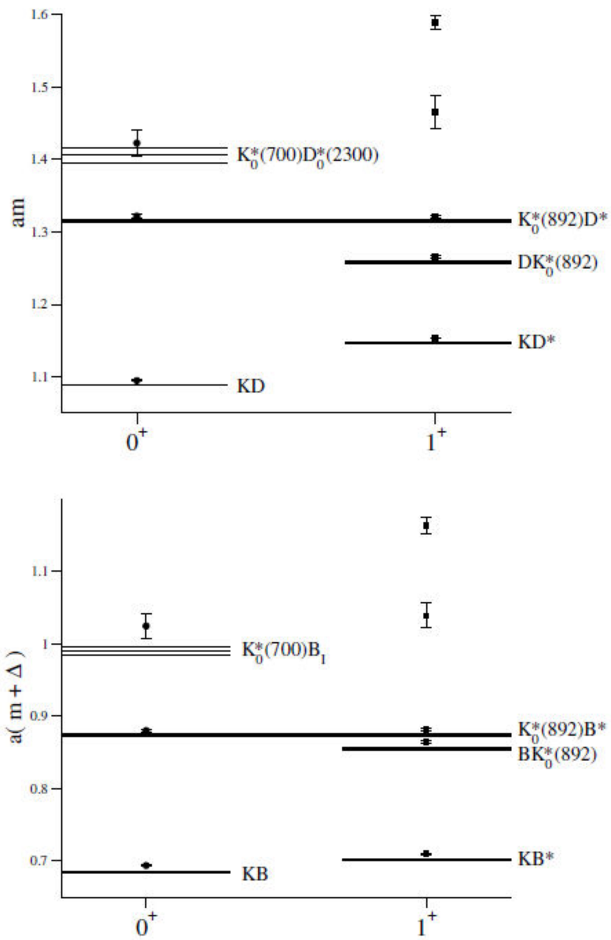

FIGURE 3. Fit results for the $I = 0$, $J^P = 0^+$ and 1^+ $ud\bar{s}\bar{c}$ (top) and $ud\bar{s}\bar{b}$ channels (bottom) alongside corresponding two-meson thresholds.

whose ground state effective masses approach from above while the statistical noise is kept under control.

The use of the box-sink construction is appropriate for both the mesons and tetraquarks in our work. In Fig. 2 we demonstrate the effect of the box-sink approach. Effective masses from the same pseudoscalar B_c correlators with different box-sink radii ($R^2 = 20$ and 49) are compared with wall-local and wall-wall correlators. At large t, each of the correlators reach the same plateau value as one would expect. In this case the correlator with a box-sink size of $R^2 = 20$ is the most appropriate choice since it reaches a plateau at an earlier t while the statistical errors on the points remain small.

4. Results

We present results from our study of a number of tetraquark channels in Figs. 3, 4 and 5. All results are given in lattice units. Other than in the the left plot of Fig. 3 where the energies are absolute, the energies are relative to some unknown offset due to the NRQCD b quarks. Energy differences are preserved, however, and therefore so too are the binding energies in which we are interested.

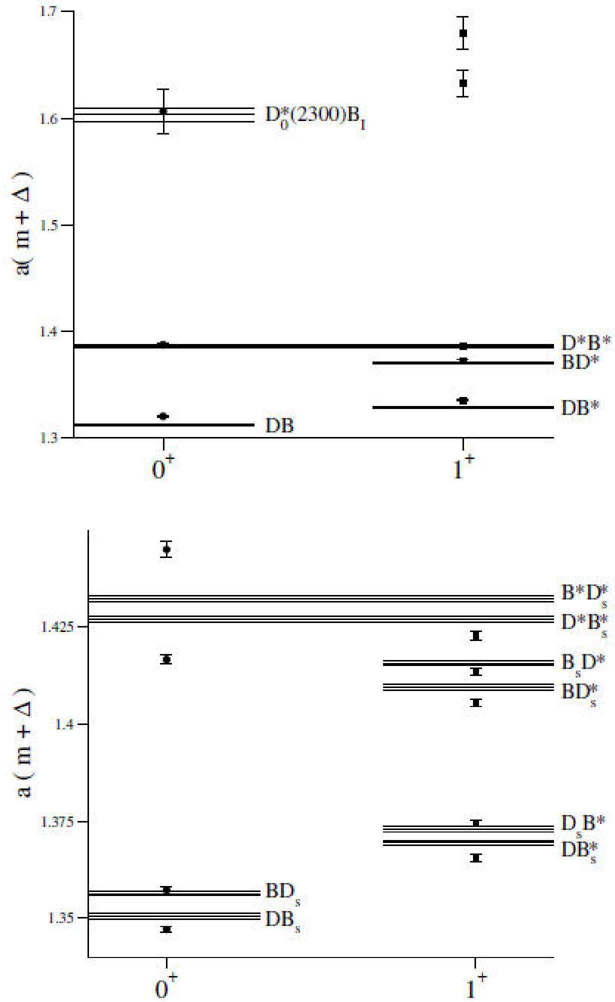

FIGURE 4. Fit results for the $I = 0$, $J^P = 0^+$ and 1^+ $ud\bar{c}\bar{b}$ (top) and $I = 1/2$, $J^P = 0^+$ and 1^+ $\ell s\bar{c}\bar{b}$ channels (bottom) alongside corresponding two-meson thresholds.

The ground state in each case is consistent with the appropriate two-meson threshold, with the exception of the $J^P = 0^+$ and $J^P = 1^+$ $\ell s\bar{c}\bar{b}$ channels where they are marginally (< 10 MeV) below threshold. However, even in that case we suspect that the lack of other nearby states indicate that this actually corresponds to the two-meson threshold and the slight deviation is a result of a finite volume or other systematic effect in our analysis.

An update of the $I = 0$, $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $I = 1/2$, $J^P = 1^+$ $\ell s\bar{b}\bar{b}$ channels is underway with improvements that include the use of the box-sink construction and additional lattice gauge ensembles. Figure 6 shows binding energy results for these channels using results from the three ensembles with $m_\pi < 300$ MeV. The linear extrapolation in m_π^2 – shown by the shaded band – confirms strong-interaction-stable tetraquarks in both channels in our calculations. This result is also found by other lattice groups [1,2,4,5]. The black points indicate the binding energies at the physical pion

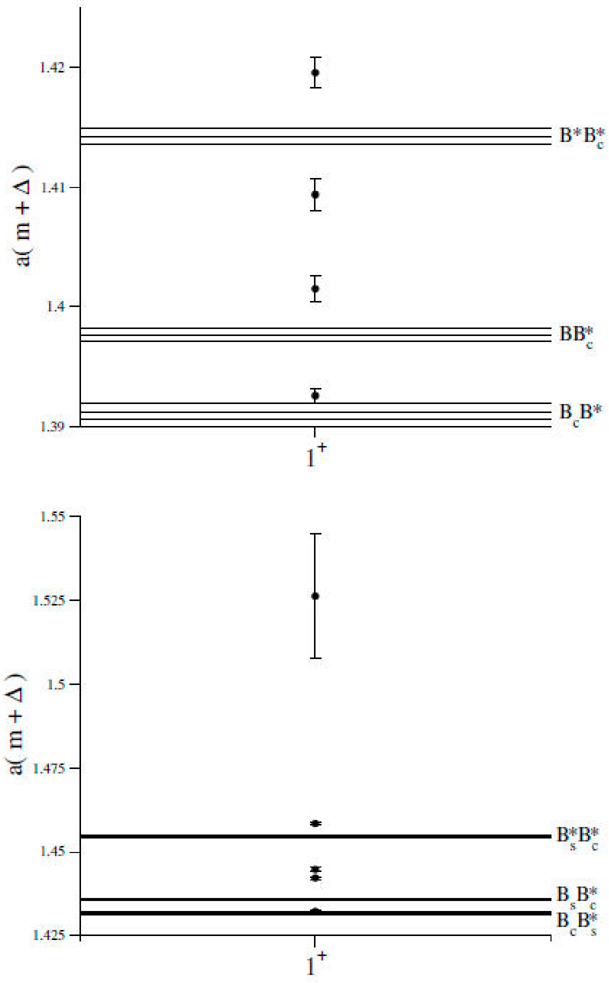

FIGURE 5. Fit results for the $I = 0$, 1^+ $uc\bar{b}\bar{b}$ (top) and $sc\bar{b}\bar{b}$ channels (bottom) alongside corresponding two-meson thresholds.

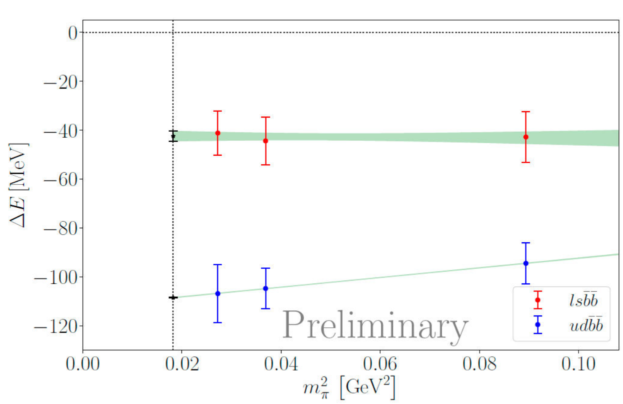

FIGURE 6. Linear extrapolation of the binding energies of the $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ channels to physical light quark mass for the ensembles with $m_\pi < 300$ MeV.

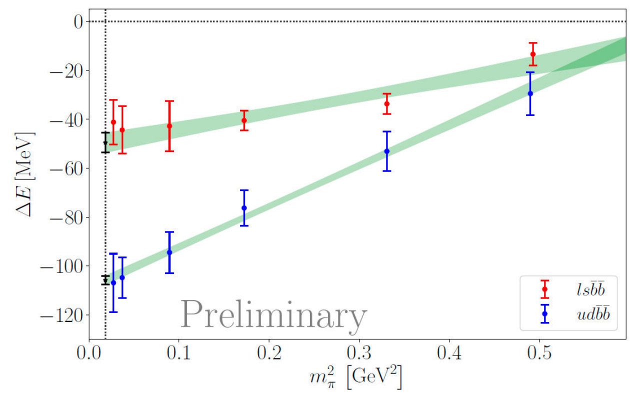

FIGURE 7. Linear extrapolation of the binding energies of the $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ channels to physical light quark mass. Data from all ensembles listed in Table I are included.

mass. The deep binding of the $ud\bar{b}\bar{b}$ tetraquark means that that channel is also stable with respect to the electromagnetic interaction. Although our primary result is the fit including only the three smallest m_π values, Fig. 7 shows the same linear fit form including all ensembles given in Table I, demonstrating that including these heavier pions in our fit has only a small effect on the physical binding energy value.

5. Conclusions

Exploring a number of tetraquark channels that had been predicted to be strong-interaction-stable in at least some of the literature, we find no evidence of deep binding. Models that do predict deep binding can therefore be ruled out. It remains possible that in some of these channels there is shallow binding that cannot be resolved in our work. These results leave only the doubly-bottom $I = 0$, $J^P = 1^+$ $ud\bar{b}\bar{b}$ and $I = 1/2$, $J^P = 1^+$ $\ell s\bar{b}\bar{b}$ tetraquark channels as established by lattice QCD as being strong-interaction-stable so far.

We are in the process of updating our doubly-bottom tetraquark results. Making use of the box-sink construction we can extract an improved ground state signal. Preliminary results demonstrate reduced binding energies compared to our initial work, but which are still clearly strong-interaction-stable. We have also generated new gauge ensembles that will allow a better extrapolation to the physical pion mass. Our updated results will provide a more robust quantitative assessment of the binding energy in these doubly-bottom channels.

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