Suplemento de la Revista Mexicana de Física **3** 0308072 (2022) 1–5

Study of $\chi_{bJ}(nP) \to \omega\Upsilon(1S)$ at Belle

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Received 16 January 2022; accepted 14 March 2022

We report preliminary results from a study of hadronic transitions of the $\chi_{bJ}(nP)$ states of bottomonium at Belle. The P-wave states are reconstructed in transitions to the $\Upsilon(1S)$ with the emission of an ω meson. The transitions of the $n = 2$ triplet states provide a unique laboratory in which to study nonrelativistic quantum chromodynamics, as the kinematic threshold for production of an ω and $\Upsilon(1S)$ lies between the $J = 0$ and $J = 1$ states. A search for the $\chi_{bJ}(3P)$ states is also reported.

Keywords: Bottomonium at Belle; nonrelativistic quantum chromodynamics.

DOI: https://doi.org/10.31349/SuplRevMexFis.3.0308072

1. Introduction

The large datasets accumulated by the B-Factories [1] around the turn of the century have facilitated a wealth of new measurements in the study heavy quarkonium ($Q\bar{Q}$, where $Q = c, b$), the bound state of a heavy quark and its corresponding anti-quark. Lately, studies of the hadronic transitions among $Q\bar{Q}$ states have come into vogue [2–11]. These transitions, especially those proximal to the kinematic threshold for the decay, provide a unique probe of the physics of soft gluon emission and hadronization [12].

Recently, BESIII reported first observation of the near-threshold transition $\chi_{c1}(3872) \to \omega J/\psi$. Although the χ_{c1} is a narrow state $(1.19\pm0.21$ MeV [13]), that lies about 8 MeV-below the kinematic threshold for production of a J/ψ and an ω meson, the observed branching fraction is reportedly as large as the discovery channel $\chi_{c1}(3872) \to \pi^+\pi^- J/\psi$, with a relative branching ratio of 1.1 ± 0.4 [13,14]. An earlier study by BaBar suggests that the decay may proceed through the low-energy tail of the ω lineshape, which is characteristically broad ($\Gamma = 8.68 \pm 0.13$) MeV [4,13]. The analogous $\omega\Upsilon(1S)$ final-state threshold in the bottomonium ($b\bar{b}$) sector lies between the $j = 0$ and $J = 1$ states of the $\chi_{bJ}(2P)$ triplet, with the $j = 0$ lying about $10.5\ MeV$ below threshold.

First observed in 2004 by CLEO in a sample of $(5.81 \pm 0.12) \times 10^6$ $\Upsilon(3S)$ mesons, the transitions $\chi_{bJ}(2P) \to \omega\Upsilon(1S)$ were seen to have large branching fractions on the order of 1% [15]. Since their discovery, no confirmation of these measurements has been made. Although no evidence of a sub-threshold $j = 0$ signal was seen in CLEO data, Monte Carlo (MC) simulation of $\chi_{b0}(2P)$ transitions to an S-wave $\omega\Upsilon(1S)$ indicate that the decay may be observed, though in such transitions the ω lineshape is distorted due to the presence of the nearby kinematic threshold.

In what follows, we report preliminary results from an inclusive search for the hadronic transitions $\chi_{bJ}(nP) \to \omega\Upsilon(1S)$, where $n = 2, 3$ and $J = 0, 1, 2$. Measurements of the hadronic branching fractions and cascade branching ratio,

$$r_{J/1} = \frac{\mathcal{B}\left(\Upsilon(3S) \to \gamma\chi_{bJ}(2P) \to \gamma\omega\Upsilon(1S)\right)}{\mathcal{B}\left(\Upsilon(3S) \to \gamma\chi_{b1}(2P) \to \gamma\omega\Upsilon(1S)\right)}, \quad (1)$$

and reported. The latter is compared with the expectation from the QCD multipole expansion (QCDME) model [16], which we have calculated using the current world averages [13,19]. An upper limit is also set on the dominant cascade branching fraction $\mathcal{B}\left(\Upsilon(4S) \to \gamma\chi_{b1}(3P) \to \gamma\omega\Upsilon(1S)\right)$.

2. Data samples and detector

We analyze data samples corresponding to an integrated luminosity of $3\ \ fb^{-1}$ and $513\ \ fb^{-1}$ accumulated near the $\Upsilon(3S)$ and $\Upsilon(4S)$ resonances, respectively, by the Belle detector [17] at the KEKB asymmetric-energy e^+e^-collider [18]. We also study a sample, referred to as the off-resonance sample, collected about 60 MeV below the $\Upsilon(4S)$ resonance, totalling $56\ \ fb^{-1}$. Following the proposal by our colleague S. Eidelman [20], these datasets are combined to maximize the number of $\Upsilon(3S)$ events. The number of $\Upsilon(3S)$ events in the combined dataset is determined from a reconstruction of $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)[\ell^+\ell^-]$ to be $(27.9 \pm 1.0) \times 10^6$ mesons [19]. Decays of $\Upsilon(3S)$ mesons in data accumulated at energies above the $\Upsilon(3S)$ resonance are assumed to come from initial state radiation (ISR) by the e^+e^- pair [13,20]. To study event selection criteria, MC events are generated with $EVTGEN$ [21], and detector response is simulated with GEANT3 [25].

3. Event selection

We devise a set of event selection criteria to optimize the retention of signal events while suppressing backgrounds from mis-reconstructed $\pi^0 \to \gamma\gamma$ decays, resonant $b\bar{b}$ decays, and non-resonant (continuum) production of other quark species. The figure of merit $S/\sqrt{S+B}$, where S and B denote the number of signal and background events, is employed for all optimizations. Slight differences exist in the event selection criteria depending on the dataset and decay channel.

Where appropriate, these differences are labeled according to the dataset and radial quantum number (n) of the $\chi_{bJ}(nP)$ triplet.

Good charged tracks, originating near the interaction point, are classified according to their momentum in the center-of-mass (CM) frame as leptons ($p_{\text{CM}} > 4.0$ GeV) or pions ($p_{\text{CM}} < 0.43$ GeV and 0.75 GeVfor $n = 2$ and 3, respectively).[i] $\Upsilon(1S)$ candidates are formed from lepton pairs if their invariant mass lies within the range $M(\ell^+\ell^-) \in [9.0, 9.8]$ GeV.

Utilizing particle identification information from various subdetectors, muon identification $(\mathcal{R}_\mu)$ and electron identification $(\mathcal{R}_e)$ likelihood ratios are ascribed to each track [26]. Leptons reconstructed in the $\Upsilon(4S)$ dataset are required to have a value of $\mathcal{R}_e$ or $\mathcal{R}_\mu$ that exceeds 0.2, in order to suppress continuum backgrounds of the form $e^+e^- \to q\bar{q}$, where $q = u, d, s, c$. Additionally, the leptons in $\Upsilon(4S)$ data must also satisfy $p_{\text{CM}} < 5.25$ GeV in order to suppress QED continuum events, which peak near 5.29 GeV. These requirements are not imposed in the $\Upsilon(3S)$ dataset due to the relative size of the production cross sections for these background classes and that of our signal. To improve the purity in our search for $\chi_{bJ}(3P) \to \omega\Upsilon(1S)$, the electron mode is rejected with a selection of $\mathcal{R}_\mu > 0.2$, and a more restrictive mass window of $M(\ell^+\ell^-) \in [9.2, 9.6]$ GeV is applied.

Contamination from photon conversion to an e^+e^- pair in detector material are suppressed by demanding that the cosine of the opening angle between oppositely-charged pions be less than 0.95. To reject events with misreconstructed tracks, events containing multiple pairs of oppositely charged pions are rejected.

Photons are reconstructed from isolated clusters in the electromagnetic calorimeter [19]. To suppress beam-related backgrounds, photons are required to have an energy greater than 50 MeV, 100 MeV, and 150 MeV in the barrel, backward endcap, and forward endcap regions, respectively.

Neutral pion candidates are formed from combinations of photons that satisfy $M(\gamma\gamma) \in [0.11, 0.15]$ GeV. To suppress combinatorial background from spurious photon combinations, we require that the π^0 candidate satisfy $p_{\text{CM}} \in [0.08, 0.43]$ GeV. The invariant mass of each candidate is constrained to the nominal π^0 mass [13] with a kinematic fit, and the best-candidate π^0 is selected according to the smallest mass-constrained fit χ^2. The ω candidate is reconstructed as the combination of the π^0 and the $\pi^+\pi^-$ pair, satisfying $M_\omega \in [0.71, 0.83]$ GeV.

Charged di-pion transitions among $b\bar{b}$ states may mimic our final state $2\gamma 2\pi 2\ell$. This pollution is due to the transitions $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$, $\Upsilon(4S) \to \pi^+\pi^-\Upsilon(2S)$, and $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$, which may be produced directly, via ISR, or through feed-down decays of other $b\bar{b}$ states. To veto these backgrounds, we define a shifted mass difference $\Delta M_{\pi\pi} = M(\pi^+\pi^-\ell^+\ell^-) - M(\ell^+\ell^-) + M(\Upsilon(1S))$, where the broad resolution of the di-lepton invariant mass is removed by subtracting the reconstructed mass of the leptons and adding back the nominal $\Upsilon(1S)$ mass [13]. Contamination from $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ events is suppressed with $\Delta M_{\pi\pi} < 10.32$ GeV. Conveniently, the $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ and $\Upsilon(4S) \to \pi^+\pi^-\Upsilon(2S)$ backgrounds nearly overlap as $M(\Upsilon(4S)) - M(\Upsilon(2S)) \approx M(\Upsilon(2S)) - M(\Upsilon(1S))$. The FOM optimization yields $\Delta M_{\pi\pi} \notin (10.017, 10.290)$ GeV for the $\Upsilon(3S)$ and off-resonance $\Upsilon(4S)$ datasets and $\Delta M_{\pi\pi} \notin (10.014, 10.030)$ GeV for the on-resonance $\Upsilon(4S)$ dataset.

The resulting selection efficiencies are approximately 8.5% for the $\chi_{bJ}(2P)$ channels and 5.4% for the $\chi_{bJ}(3P)$ channels.

4. Signal extraction

The $\chi_{bJ}(nP)$ signal channels are discriminated with the shifted mass difference

$$\Delta M_\chi = M(2\gamma 2\pi 2\ell) - M(\ell^+\ell^-) + M(\Upsilon(1S)), \qquad (2)$$

where $M(2\gamma 2\pi 2\ell)$ is the invariant mass of the final state, $M(\ell^+\ell^-)$ is the reconstructed $\Upsilon(1S)$ mass, and $M(\Upsilon(1S))$ is the nominal mass from Ref. [13]. The distribution of signal events is narrowly peaked at the corresponding $\chi_{bJ}(nP)$ mass. We extract signal yields from a simultaneous unbinned extended maximum-likelihood fit to the $\chi_{bJ}(2P)$ (ΔM_χ) and ω mass (M_ω) distributions. The projections of this fit are shown in Fig. 1. The extracted signal yields are summarized in Table I.

All signal shapes are described by double-sided Crystal Ball (DSCB) functions [27], which consist of a Gaussian core complemented by power-law tails on either side. The $j = 0$ lineshape in M_ω is impacted by the proximity of the $\omega\Upsilon(1S)$ kinematic threshold, and so is parameterized as the product of a DSCB and a sigmoid function. The backgrounds are modeled by cubic and quadratic functions in ΔM_χ and M_ω, respectively.

The statistical significance of each signal hypothesis, including systematic uncertainties, is calculated using the profile likelihood method [28], and is summarized in Table I. A fluctuation in excess of 3.2σ is observed that is consistent with the $J = 0$ hypothesis, constituting the first evidence for a sub-threshold transition $\chi_{b0}(2P) \to \omega\Upsilon(1S)$.

From the radiative branching fractions in Ref. [29], we project few $\chi_{bJ}(3P) \to \omega\Upsilon(1S)$ signal events with negligible contributions from the $J = 0$ and $J = 2$ channels.

TABLE I. Extracted signal yields for various transitions and the associated significances, including systematic uncertainties, expressed in terms of standard deviations (σ).

Transition	Signal Yield	Significance
$\chi_{b0}(2P) \to \omega\Upsilon(1S)$	$33.1^{+11.1}_{-10.8}$	3.2σ
$\chi_{b1}(2P) \to \omega\Upsilon(1S)$	309 ± 24	15.0σ
$\chi_{b2}(2P) \to \omega\Upsilon(1S)$	62 ± 16	3.9σ
$\chi_{b1}(3P) \to \omega\Upsilon(1S)$	$3.2^{+3.6}_{-2.8}$	1.1σ

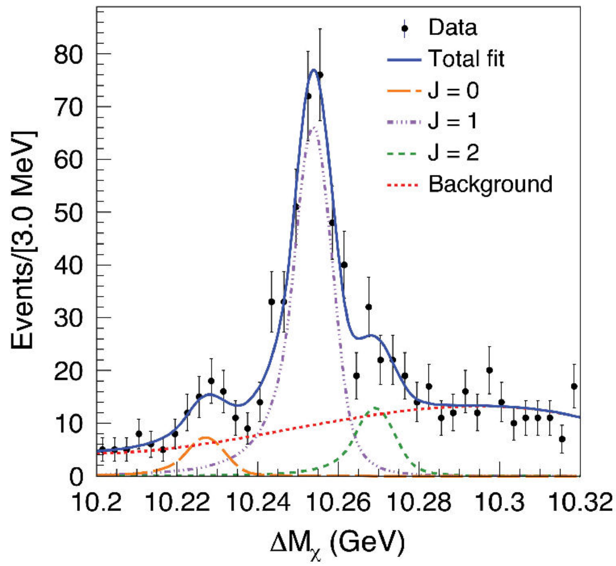

FIGURE 1. Fit to the ΔM_χ (Left) and M_ω (Right) distributions for $\chi_{bJ}(2P) \rightarrow \omega\Upsilon(1S)$ candidates reconstructed in data. The solid blue curve shows the total fit and the dotted red curve indicates the background. In both panels, the long dashed orange curve is the $J = 0$ signal. In the left panel, the dash-dotted violet curve is the $J = 1$ signal, and the dashed green curve is the $J = 2$ signal. In the right panel, the dash-dotted gray curve shows the combined $J = 1$ and 2 signal.

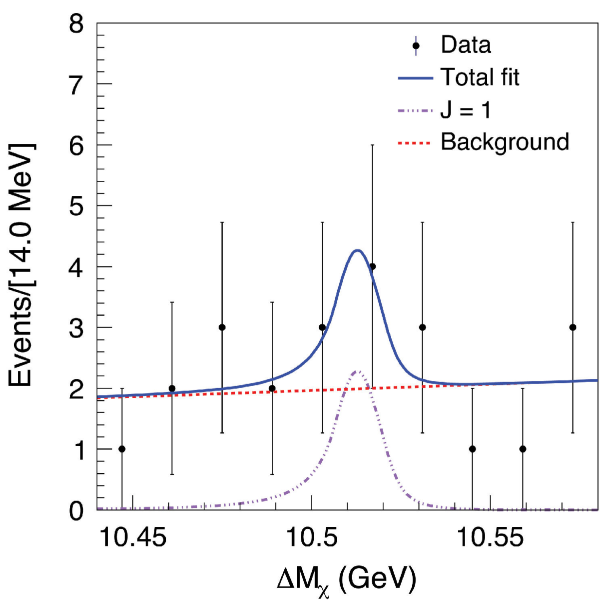

FIGURE 2. Fit to the ΔM_χ distribution for $\chi_{b1}(3P) \rightarrow \omega\Upsilon(1S)$ candidates reconstructed in data. The legend is similar to that of Fig. 1.

As a result, only the $J = 1$ signal component is included in the fit to data. With a small number of signal events anticipated, the largest source of irreducible background arises from QED continuum events, which we model with a linear shape. Studies of off-resonance $\Upsilon(4S)$ data and sidebands in on-resonance $\Upsilon(4S)$ data verify this parameterization. To stabilize the fit to data, shown in Fig. 2, the nominal $\chi_{b1}(3P)$ mass is fixed from Refs. [13, 30] and the calibration in the overall scale and resolution are determined from the control channel $\Upsilon(3S) \rightarrow \pi^+\pi^-\Upsilon(1S)$ [19]. The signal yield in the fit is $3.2^{+3.6}_{-2.8}$ events.

5. Systematic uncertainties

The sources of systematic uncertainty are described in detail in Ref. [19]. The dominant source of uncertainty in the measurement of $\mathcal{B}(\chi_{bJ}(2P) \rightarrow \omega\Upsilon(1S))$ arises from the uncertainties on the external branching fractions, which contribute uncertainties of 10.4%, 9.4%, and 12.4% for the $J = 0, 1,$ and 2 channels respectively. The $2P$ branching fractions are calculated by normalizing to the number of $\Upsilon(3S)$ reconstructed via $\Upsilon(3S) \rightarrow \pi^+\pi^-\Upsilon(1S)$. This results in the cancellation of several uncertainties, including those assessed for data-MC differences in tracking (1.4%) and particle identification (1.1%), which do not otherwise cancel in the $3P$ measurement. A momentum-dependent systematic uncertainty for π^0 reconstruction of 1.7% is assessed and included. The uncertainty due to the signal extraction procedure is studied with a suite of toy MC studies to probe the impact of the choice of fit window, background parameterizations, fixed shape parameters, and to search for possible bias. These studies yield an uncertainty of $^{+10.1\%}_{-12.6\%}$ for the $3P$ measurement, which is the dominant systematic uncertainty for that channel. These uncertainties are combined in quadrature to obtain the total systematic uncertainty on each measurement.

6. Results

With no significant $\chi_{bJ}(3P)$ signal observed, the $\chi_{bJ}(2P)$ reconstructed in $\Upsilon(4S)$ data are attributed to radiative decays of $\Upsilon(3S)$ mesons produced via ISR. The branching fractions for the ω transition are calculated from the signal yield (N_J) and efficiency (ϵ_J) as

TABLE II. Measured branching fractions (or upper limits) measured for each transition. The branching ratios $r_{0/1}$ and $r_{2/1}$ are also presented. The quoted uncertainties are statistical and systematic.

Quantity	Measurement (%)	90% CL UL (%)
$\mathcal{B}\left(\chi_{b0}(2P) \to \omega\Upsilon(1S)\right)$	$0.56\,^{+0.19}_{-0.18} \pm 0.08$	
$\mathcal{B}\left(\chi_{b1}(2P) \to \omega\Upsilon(1S)\right)$	$2.38 \pm 0.18\,^{+0.23}_{-0.24}$	
$\mathcal{B}\left(\chi_{b2}(2P) \to \omega\Upsilon(1S)\right)$	$0.46 \pm 0.12\,^{+0.06}_{-0.07}$	
$r_{0/1}$	$0.110^{+0.037}_{-0.036} \pm 0.010$	
$r_{2/1}$	$0.200^{+0.062}_{-0.058}\,^{+0.007}_{-0.017}$	
$\mathcal{B}\left(\Upsilon(4S) \to \gamma\chi_{b1}(3P) \to \gamma\omega\Upsilon(1S)\right)$	$\left(4.9\,^{+5.5}_{-4.3}\,^{+0.5}_{-0.6}\right) \times 10^{-4}$	$< 1.4 \times 10^{-3}$

$$\mathcal{B}(\chi_{bJ}(2P) \to \omega\Upsilon(1S)) = \frac{N_J}{\epsilon_J N_{\Upsilon(3S)} \Pi\mathcal{B}}, \qquad (3)$$

where $N_{\Upsilon(3S)}$ is the number of $\Upsilon(3S)$ events and $\Pi\mathcal{B}$ is the product of $\mathcal{B}\big(\Upsilon(3S) \to \gamma\chi_{bJ}(2P)\big)$, $\mathcal{B}\big(\omega \to \pi^+\pi^-\pi^0\big)$, and $\mathcal{B}\big(\pi^0 \to \gamma\gamma\big)$. The resulting branching fractions are reported in Table II. These measurements are consistent with the CLEO results [15] within 2σ.

We also reparameterize the fit in terms of the total signal yield and the ratios $\mathrm{P}_{0/1}$ and $\mathrm{P}_{2/1}$ between the $j = 0$, 1 and $J = 2$, 1 yields, respectively. Correcting the results for the efficiencies, we obtain the values of $r_{J/1} = \mathrm{P}_{J/1}\,(\epsilon_1/\epsilon_J)$ shown in Table II. In each ratio $r_{J/1}$, only the systematic uncertainties assigned for signal extraction.

We compare our measurement of $r_{2/1}$ with the QCDME expectation [16], which we have calculated using current world averages [13]: $r_{2/1}^{\text{QCDME}} = 0.77 \pm 0.16$ [19]. This reveals a tension with QCDME at the 3.3σ level.

We have also searched for the transition $\chi_{bJ}(3P) \to \omega\Upsilon(1S)$ produced in radiative decays of the $\Upsilon(4S)$. The branching fraction of the cascade transition, is calculated in terms of the signal yield (N), the reconstruction efficiency (ϵ), and the number of $\Upsilon(4S)$ events $(N_{\Upsilon(4S)})$ as:

$$\mathcal{B}\left(\Upsilon(4S) \to \gamma\chi_{bJ}(3P) \to \gamma\omega\Upsilon(1S)\right) = \frac{N}{\epsilon N_{\Upsilon(4S)} \Pi\mathcal{B}'}, \qquad (4)$$

where $\Pi\mathcal{B}'$ is the product of $\mathcal{B}\big(\omega \to \pi^+\pi^-\pi^0\big)$, $\mathcal{B}\big(\pi^0 \to \gamma\gamma\big)$, and $\mathcal{B}\left(\Upsilon(1S) \to \mu^+\mu^-\right)$. The result is included in Table II. An upper limit on the cascade branching fraction is obtained by convolving the profile likelihood Gaussian function whose width equals systematic uncertainty and integrating over positive values of the branching fraction. The result is an upper limit of 1.4×10^{-5} set at 90% confidence level (CL).

7. Conclusions

We report preliminary measurements using the combined $\Upsilon(3S)$ and $\Upsilon(4S)$ data samples collected by the Belle detector that constitute first evidence for the sub-threshold transition $\chi_{b0}(2P) \to \omega\Upsilon(1S)$ produced in radiative $\Upsilon(3S)$ decays with a branching fraction of $(0.56^{+0.19}_{-0.18} \pm 0.08)\%$ at a significance of 3.2σ. Moreover, we measure the hadronic branching fractions $\mathcal{B}\left(\chi_{b1}(2P) \to \omega\Upsilon(1S)\right) = (2.38 \pm 0.18^{+0.23}_{-0.24})\%$ and $\mathcal{B}\left(\chi_{b2}(2P) \to \omega\Upsilon(1S)\right) = (0.46 \pm 0.12^{+0.06}_{-0.07})\%$, which constitute the first confirmation of the $J = 1$ and 2 transitions since their discovery [15]. The ratios of the cascade branching fractions $(r_{J/1})$ are also measured. Comparison of the resulting measurement of $r_{2/1}$ with the value from QCDME reveals a 3.3σ tension. Finally, a search is performed for $\chi_{bJ}(3P) \to \omega\Upsilon(1S)$ produced in radiative decays of the $\Upsilon(4S)$. With no significant signal found, an upper limit is set on the cascade branching fraction $\mathcal{B}\left(\Upsilon(4S) \to \gamma\chi_{b1}(3P) \to \gamma\omega\Upsilon(1S)\right) < 1.4 \times 10^{-5}$ at 90% CL.

Acknowledgements

The author thanks the organizers of the 19th International Conference on Hadron Spectroscopy and Structure in memoriam Simon Eidelman for the opportunity to speak, his colleagues in the Belle Collaboration for their various contributions to this analysis, especially his mentors who have shepherded this work through the years, and the United States Department of Energy Office of High Energy Physics for their support of this work.

i. We use units in which the speed of light is $c = 1$.

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