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# Studying the quark gap equation at finite temperature, magnetic field and different numbers of flavor and color numbers

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In recent years, a momentum-independent symmetry preserving vector-vector contact interaction has been used to provide exploratory studies in QCD. It helps calculate different static and dynamic observables of all mesons and baryons. In this work, we revisit how the quark gap equation is affected by changing the number of quark colors and/or flavors, or by placing quarks in a thermal bath in the presence of an external magnetic field. In particular, we describe how the phenomenon of magnetic catalysis and its inverse can be studied.

*Keywords:* Contact interaction; flavor and color number; magnetic catalysis; inverse magnetic catalysis.

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## 1. Motivation

Quantum chromodynamics (QCD) is the theory of quarks, gluons and the interactions between them. We believe these interactions lead to their bound states known as hadrons. The behavior of QCD changes drastically according to the energy scales at which we probe strongly interacting particles. In the ultraviolet, where the well-known asymptotic freedom manifests itself, the running coupling becomes small enough to implement perturbation theory approach to QCD. In the infrared, emergent phenomena of confinement and dynamical chiral symmetry breaking (DCSB) take place. The running coupling increases to the extent that perturbation theory is unable to explain these uncanny phenomena.

These two peculiarities of QCD are influenced by the number of light quark flavors $N_f$ and the number of colors $N_c$ of the fundamental gauge group $SU(N_c)$. It should not come as a surprise as even in the ultraviolet, sign of the one-loop $\beta$-function depends upon the factor $-11N_c + 2N_f$. When studying real life QCD, we readily observe that the sign of this factor is negative as $N_c = 3$ outmatches $N_f$, and hence the strong interactions are asymptotically free [1–3]. If there were a universe with $N_f > (11/2)N_c$, this effect will be reversed and asymptotic freedom will be lost.

Additionally, the behavior of QCD is also affected in the presence of a heat bath or a magnetic field. At low temperatures, the observable degrees of freedom continue to be color-singlet hadrons, whereas at high temperatures, the interaction gets increasingly screened and weak, causing quarks to deconfine into a phase where the coupling decreases, quarks and gluons are deconfined and chiraly symmetry is duly restored. A quantitative study of this behavior has widely been carried out in literature, see for example Refs. [4–12] for lattice QCD and Refs. [13–21] for works based upon continuum techniques of Schwinger-Dyson equations (SDEs).

On the other hand, it is well known that the presence of strong magnetic fields generates quark masses dynamically for any value of the coupling strength. This phenomenon was first studied in quantum electrodynamics (QED) and was called magnetic catalysis [22–28]. In the presence of external magnetic fields, it is observed that near the cross-over temperature, chiral quark condensate develops a peculiar behaviour. It starts decreasing with increasing magnetic field. This effect has come to be known as inverse magnetic catalysis [30–33, 47].

In this manuscript, we sketch how different numbers of light flavor and color, temperature and external magnetic field affect the dynamically generated quark mass function. The detailed analysis and discussions is found in Ref. [34]. Here, we outline the general expressions and describe how the results can be obtained.

## 2. Changing numbers of flavor and color

We start by presenting the generalities of the contact interaction (CI) for the quark gap equation. Moreover, to include the anti-screening effects of the gluons and the screening effects of the light quarks loops, we extend this model to an arbitrary $SU(N_c)$ gauge theory with $N_f$ number of light quarks.

### 2.1. The contact interaction model

In this simple yet effective model, [35–39], it is assumed that the quark-gluon interaction is effectively the symmetry-preserving vector-vector CI leading to a constant gluon propagator:

$$g^2 D_{\mu\nu}(k) = \frac{4\pi\alpha_{\rm IR}}{m_g^2}\delta_{\mu\nu} \equiv \delta_{\mu\nu}\alpha_{\rm eff}, \tag{1}$$

$$\Gamma^a_\mu(p,q) = \frac{\lambda^a}{2}\gamma_\mu, \tag{2}$$

where $m_g = 800\,$MeV is a gluon mass scale generated dynamically in QCD [40], and $\alpha_{\rm IR}$ is the CI model parameter, which can be interpreted as the interaction strength in the infrared region [41, 42]. Replacing the full quark-gluon vertex by its bare counterpart completes the definition of the model. As a consequence, a constant mass function is obtained. Since it is necessary to solve divergent integrals, a regularization procedure should be adopted. To regularize the integrals in the CI model, the proper time regularization scheme [43] is adopted. This leads to a quark propagator of the type:

$$S_f^{-1}(p) = i\gamma\cdot p + M_f\,, \tag{3}$$

with a constant dressed quark mass.

$$M_f = m_f + \frac{\alpha_{\rm eff}^{N_c}(N_f)M_f^3}{8\pi^2 m_G^2}\Gamma(-1,\tau_{\rm UV}M_f^2,\tau_{\rm IR}M_f^2)\,, \tag{4}$$

where

$$\alpha_{\rm eff}^{N_c}(N_f) = (N_c - 1/N_c)\;\alpha_{\rm eff}(N_f)\,, \tag{5}$$

and $\Gamma(a,z_1,z_2)$ is the generalized incomplete Gamma function:

$$\Gamma(a,z_1,z_2) = \Gamma(a,z_1) - \Gamma(a,z_2)\,. \tag{6}$$

In the fundamental representation of $SU(N_c)$, the Gell-Mann matrices satisfy the identity $\sum_{a=1}^{8}\lambda^a\lambda^a = 2\,(N_c - 1/N_c)$. The parameters $\tau_{\rm IR}$ and $\tau_{\rm UV}$ are infrared and ultraviolet regulators, respectively. A nonzero value for $\tau_{\rm IR} \equiv 1/\Lambda_{\rm IR}$ implements confinement by ensuring the absence of quark production threshold [44]. On the other hand, since the CI model provided by Eq. (1) represents a nonrenormalizable theory, so $\tau_{\rm UV} \equiv 1/\Lambda_{\rm UV}$ cannot be removed, it becomes part of the model and sets the scale for all dimensional quantities. Additionally, the ultraviolet cutoff also plays an important role in studying heavier quarks: increasing $\Lambda_{\rm UV}$ mimics the short-distance effects as the quark mass increases [36, 45].

To study the effect of light quark flavors $N_f$ on the DCSB, the following flavor-dependence of the effective coupling is adopted which mimics already known results in QCD:

$$\alpha_{\rm eff}^{N_c}(N_f) = \alpha_{\rm eff}^{N_c}\sqrt{1 - \frac{(N_f-2)}{\lambda}}. \tag{7}$$

The gap equation, Eq. (4), is solved by using this coupling. For $N_c = 3$, different values of $\lambda$ are identified to obtain critical values of $N_f$ ($N_f^{\rm cr} = 7, 8, 9$). The values of $\lambda$ calculated are: $\lambda = 8.29$, 9.95 and 11.61, respectively. Above these values, the screening effects restore chiral symmetry. These numbers are in accordance with the findings of Ref. [46].

In addition, the number of quark colors $N_c$ anti-screens the interactions while $N_f$ screens them. For $N_f = 0$, the minimum value of $N_c$ required to trigger DCSB is found to be $N_c^{cr} \simeq 2.2$. If the number of massless quark flavors is increased, mellowing down the interaction strength, higher color group $SU(N_c)$ has to be invoked to set off DCSB.

## 3. Quark gap equation at finite temperature

With increasing temperature, it should be expected that the strong interaction diminishes, and, therefore, a smaller value for the critical number of massless quark flavors should be required to restore chiral symmetry for a given value of $N_c$. At finite temperature, the mass function is:

$$M_f = m_f + \frac{2\alpha_{\rm eff}(N_f)M_f T}{3\pi^{3/2}} \times \int_{\widetilde{\tau}_{\rm IR}^2}^{\tau_{\rm UV}^2} d\tau \frac{{\rm e}^{-M_f^2\tau}\Theta_2(0,{\rm e}^{-(2\pi T)^2\tau})}{\tau^{3/2}}\,, \tag{8}$$

with

$$\widetilde{\tau}_{\rm IR} = \tau_{\rm IR}\frac{M_f(0,2)}{M_f(T,N_f)}\,, \tag{9}$$

where $M_f(0,2)$ is identified with the dressed mass at $T = 0$, $N_f = 2$ and $N_c = 3$. In the chiral limit $m_f = 0$, it is assured that the confining scale vanishes at the chiral symmetry restoration temperature. This is a simple way of ensuring the coincidence of transitions to the phase of confinement and DCSB.

## 4. Quark Gap Equation in a magnetic field

In the following, we work with the average dynamical mass in the chiral limit,

$$M = \frac{1}{2}(M_f + M_g)\,, \tag{10}$$

whose gap equation is given by

$$M = \frac{1}{2}\,(m_f + m_g) + \frac{\alpha_{\rm eff}(N_f)}{3\pi^2}\frac{1}{2} \times \sum_{l=f,g}|\mathcal{Q}_l B|\int_{\tau_{\rm UV}^2}^{\widetilde{\tau}_{\rm IR}^2} d\tau\frac{M{\rm e}^{-M^2\tau}}{\tau\tanh(|\mathcal{Q}_l B|\tau)}\,, \tag{11}$$

where $\mathcal{Q}$ is the charge of the quark $f(g)$.

Solving this gap equation in the chiral limit as a function of $eB$, we find that for large values of $eB$, the dependence of $M$ on $eB$ is given by $\sqrt{eB}$ which agrees with the results known in the literature.

Magnetic catalysis enhances DCSB, and this competes against $N_f$ to generate dynamical quark mass. Finally, it is found that larger $eB$ tends to increase the number of flavors $N_f^{\rm cr}$ needed for chiral symmetry restoration, contrary to the behavior of the critical number of fermions $N_f^{\rm cr}$ as a function of temperature where it gets reduced as $T$ is increased.

## 5. Phase diagram at finite $T$ and $B$

At finite temperature $T$ and in a magnetic field $eB$, the gap equation beyond the chiral limit reads:

$$M = \frac{1}{2}\left(m_f + m_g\right) + \frac{2\alpha_{\rm eff}(N_f)MT}{3\pi^{3/2}}\frac{1}{2}\sum_{l=u,d}|\mathcal{Q}_l B| \times \int \widetilde{\tau}_{\rm IR}^2 \tau_{\rm UV}^2 d\tau \frac{{\rm e}^{-M^2\tau}\Theta_2(0,{\rm e}^{-(2\pi T)^2\tau})}{\tau^{1/2}\,\tanh(|\mathcal{Q}_l B|\tau)}, \quad (12)$$

where

$$\widetilde{\tau}_{\rm IR} = \tau_{\rm IR}\frac{M(0,0,2)}{M(T,eB,N_f)}. \quad (13)$$

for quark flavor $l = f,\, g$. In Ref. [34], we used $m_u = m_d = m = 7$ MeV.

To observe the pattern of magnetic catalysis, we compute the thermal gradient of the dynamical mass for increasing values of $eB$ and observe that the maximum increases with larger $eB$. Thus, it can be seen that increasing temperature requires larger magnetic field to catalyze DCSB and confinement.

If the effect of the magnetic field is correctly taken into account in the functional form of the interaction strength, we observe that $\alpha_s$ decreases with increasing $eB$ in a certain range of values of the magnetic field. This effect suppresses the formation of chiral quark condensate and produces the inverse magnaetic catalysis.

This effect can be incorporated if a magnetic field dependence in the coupling constant is embedded. We adopt the following $eB$-dependent interaction strength fitted to reproduce the temperature of chiral transition using LQCD calculations [47]:

$$\alpha_{\rm eff}^{N_c}(N_f,x) = \alpha_{\rm eff}^{N_c}(N_f,0)\left(\frac{1+ax^2+bx^3}{1+cx^2+dx^4}\right), \quad (14)$$

where the $N_f$ dependence of the coupling, $\alpha_{\rm eff}^{N_c}(N_f,0)$, is given by Eq. (7), $x = eB/\Lambda_{\rm QCD}^2$, with $\Lambda_{\rm QCD} = 300$ MeV. The parameters $a, b, c$ and $d$ can be found in Ref. [48]. on $N_f^{\rm cr}$. We now observe the expected behavior. Increasing temperature requires lower magnetic field to trigger DCSB, the inverse magnetic catalysis.

Additionally, motivated by the assumption that effective coupling with magnetic fields drives physics with asymptotic freedom, we can also adopt an improved functional form of the coupling instead of Eq. (14), incorporating the temperature dependence as well. This refined approach was adopted in [49, 50], defining the coupling as:

$$\alpha_{\rm eff}^{N_c}(N_f,x,y) = \alpha_{\rm eff}^{N_c}(N_f,0,0)\frac{1-\gamma xy}{1+\alpha\ln(1+\beta x)}, \quad (15)$$

where $y = T/\Lambda_{\rm QCD}$. The parameters $\alpha$ and $\beta$ are fixed to obtain a reasonable description of the lattice average of up and down quark condensates at $T = 0$. $\gamma$ is obtained from a similar fit at the highest temperatures. Inverse magnetic catalysis persists with this improved model of the running coupling.

## 6. Conclusions

The flavor dependence in our CI model was integrated to mimic the latest SDE and lattice results. These results show that chiral symmetry is restored and a deconfinement phase appeared above $N_f \approx 7-10$, [46, 51–55]. This observation indicates that increasing the number of light quark flavors restores chiral symmetry, while increasing the number of colors tends to confine quarks as expected.

Naturally, temperature also has an effect of restoring chiral symmetry, implying that even a small value of $N_f$ is sufficient to restore chiral symmetry at a give temperature. Similarly, the presence of an external magnetic field effect produces magnetic catalysis. Finally in a heat bath in the presence of an external magnetic field with an adequate choice of the strong coupling, inverse magnetic catalysis is realized in a certain temperature interval.

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