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# Is $P_{cs}(4459)$ one state or two?

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The LHCb collaboration reported recently a charm-strange pentaquark state $P_{cs}(4459)$ found in the $J/\psi\Lambda$ invariant mass distribution. Using a coupled channel unitary approach combined with the local hidden gauge formalism, we investigate the $\bar{D}^{(*)}\Xi_c^{(*,\prime)}$ interactions, together with the $J/\psi\Lambda$ and other coupled channels, with the constraints of the heavy quark spin symmetry. We dynamically reproduce the $P_{cs}(4459)$ state in the coupled channel interactions, which is a degenerate state of $\bar{D}^*\Xi_c$ and analogous to the $P_c(4450)$ state before. Furthermore, we make more predictions for future experiments.

*Keywords:* Pentaquark state; heavy quark spin symmetry; coupled channel.

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## 1. Introduction

Several years after the state $\Theta^+(1540)$ was ruled out from the Particle Data Book, the authors of Refs. [1, 2] predicted in 2010 some pentaquark-like resonances with hidden charm and hidden charm strangeness around the energy range of 4200 MeV – 4600 MeV. Later, similar predictions were made in Refs. [3–10] for the hidden charm sector. Several years later, two pentaquark-like resonances were found by the LHCb Collaboration in 2015 in the decay $\Lambda_b^0 \to J/\psi K^- p$ [11, 12], one of which was named as $P_c(4380)^+$ with a large width and the other one $P_c(4450)^+$ with a small width. In a further work of Ref. [13], these findings were confirmed by a model-independent analysis of the data. Furthermore, these two states were also found in the $\Lambda_b^0 \to J/\psi p\pi^-$ decay [14]. In 2019, the LHCb Collaboration updated the results of these two $P_c$ states with more data in the Run-2, which in fact were three narrow structures [15], $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$. Note that the former broad one $P_c(4380)$ could neither be confirmed nor refuted in the new results. The same conclusion had been reached before in Ref. [16] based on the analysis of the old data.

One thing should be mentioned that Refs. [1, 2] also made the predictions of hidden charm strangeness states $P_{cs}$ as the partners of $P_c$ states under the SU(4) flavour symmetry, of which the decay properties were investigated in details in Ref. [17]. These predictions were revisited in Ref. [18] by taking into account the heavy quark spin symmetry (HQSS) [19–21]. Moreover, the possible pentaquark-like states with the hidden charm strangeness were also predicted in other theoretical models [22–24]. As suggested in Refs. [18, 22, 25], some of these predicted pentaquark-like states could be searched for in the decay process $\Xi_b^- \to J/\psi\Lambda K^-$. Besides, looking for these states in the $\Lambda_b$ decays was also proposed in Refs. [26,27]. Recently, the LHCb Collaboration reported a resonance structure of $P_{cs}(4459)$ in the $J/\psi\Lambda$ invariant mass distributions of the decay $\Xi_b^- \to J/\psi\Lambda K^-$ in Ref. [28], which caught much theoretical attentions [29–38]. Furthermore, more discussions on the progress of the pentaquark states and the other exotic states can be found in the reviews of Refs. [39–50] both in theories and experiments.

In view of the new finding of the LHCb Collaboration for the $P_{cs}(4459)$ state [28], and based on the results of Refs. [1, 2, 18], we revisited the interactions of $\bar{D}^{(*)}\Xi_c^{(*,\prime)}$ and their coupled channels as discussed in Ref. [51]. We will introduce our formalism in the next section. And then, we show the results of the coupled channel interactions with the formalism of HQSS. Finally, it is our conclusion.

## 2. Formalism

As done in Ref. [8], considering the lowest order constraints of HQSS [19–21], we utilize the coupled channel unitary approach (CCUA) [52–54] to investigate the interactions of $\bar{D}^{(*)}\Xi_c^{(*,\prime)}$ and their coupled channels, which are nine channels $\eta_c\Lambda$, $J/\psi\Lambda$, $\bar{D}\Xi_c$, $\bar{D}_s\Lambda_c$, $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c$, $\bar{D}_s^*\Lambda_c$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Xi_c^*$ in the $J^P = \frac{1}{2}^-$, $I = 0$ sector, and six channels $J/\psi\Lambda$, $\bar{D}^*\Xi_c$, $\bar{D}_s^*\Lambda_c$, $\bar{D}^*\Xi_c'$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Xi_c^*$ in the $J^P = \frac{3}{2}^-$, $I = 0$ sector. In addition, a single channel of $\bar{D}^*\Xi_c^*$ with $J^P = \frac{5}{2}^-$ was not taken into account in the present work since it could not couple to the $J/\psi\Lambda$ channel in the $s$-wave; see more details in Ref. [18]. Note that the $\bar{D}^*\Xi_c^*$

channel with spin $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ was considered in Ref. [18] under the constraint of the HQSS, but not in Refs. [1, 2].

For the scattering amplitudes ($T$) in the CCUA, we solve the coupled channel Bethe-Salpeter equation with the on-shell prescription, given by

$$T = [1 - V\,G]^{-1}\,V, \tag{1}$$

where $G$ is made of the meson-baryon loop functions and $V$ constructed by the potentials of the coupled channel interactions. Note that $G$ is a diagonal matrix, of which the elements are the loop functions, and we take the explicit formula of dimensional regularization [54]. Thus, the subtraction constant $a_\mu$ in the loop functions is the only free parameter, of which the value will be discussed later. For the potentials, the elements of $V$ matrix can be referred to Ref. [18] for more details.

One should know that the HQSS only gives the constraint on the coupled channel interactions, where the strengths of the interactions are not specified. Therefore, as done in Ref. [8], we exploit an extension of the local hidden gauge (LHG) approach [55–58] to determine the interaction potentials by the mechanism of exchanging vector mesons between the interactions of mesons and baryons. As found in Refs. [59, 60], the LHG formalism worked well in the hidden charm sector, since it gave a general consistent explanation for many observed hadronic molecular candidates. Thus, with the LHG formalism, the low energy constants of the HQSS can be determined as [18],

$$\mu_1 = \mu_3 = \mu_{24} = \mu_{34} = 0, \tag{2}$$

$$\mu_2 = \frac{\mu_{23}}{\sqrt{2}} = \mu_4 = \lambda = -F,\; F = \frac{1}{4f^2}(p^0 + p'^{\,0}), \tag{3}$$

$$\mu_{12} = -\frac{\mu_{13}}{\sqrt{2}} = \frac{\mu_{14}}{\sqrt{3}} = -\sqrt{\frac{2}{3}}\,\frac{m_V^2}{m_{D^*}^2}\,F, \tag{4}$$

with $f_\pi = 93$ MeV and $m_V = 800$ MeV, where $p^0$ and $p'^{\,0}$ are the energies of the incoming and outgoing mesons in a certain channel. Note that the reduction factor $m_V^2/m_{D^*}^2$ is explicitly used in the non-diagonal elements because of the transition processes with the exchange of a $D^*$ meson. The zero values for $\mu_{24}$ and $\mu_{34}$ are owing to the neglect of pion exchange in our formalism; see more discussions in Ref. [8].

## 3. Results

As discussed above, the subtraction constant $a_\mu$ in the meson-baryon loop functions is a free parameter in our formalism. Thus, it is an open issue in our framework how to determine its value. Note that the value of this parameter will affect the lowest strength of attractive potential to form a bound state, which will be discussed later. One can recall that due to no experimental data available, a central value of $a_\mu(\mu = 1\text{GeV}) = -2.3$ was used in Refs. [1, 2] for the predictions, which was chosen in order to match the cutoff in the loop function with the masses of the exchanged vector mesons $\rho$ and $\omega$. The only way in practice is that one can use some experimental data to fix its accurate value. This is really done in Ref. [61], where a value of $a_\mu(\mu = 1\text{GeV}) = -2.09$ was determined by the newest experimental results of Ref. [28] for the findings of three $P_c$ states. This value was confirmed in a further work of Ref. [62] with the fitting of the experimental $J/\psi\, p$ invariant mass distributions.

Naturally, one can use this value of $a_\mu(\mu = 1\text{GeV}) = -2.09$ to make the predictions for the $P_{cs}$ states, which was the motivation of Ref. [18], and where the results of $|T|^2$ are shown in Figs. 1 and 2 for the sectors $J = 1/2$ and $J = 3/2$, respectively. In Fig. 1 for the $J = 1/2$ sector, one can see that there are five resonance structures, which appear in the amplitudes of the channels $\bar{D}\Xi_c$, $\bar{D}^*\Xi_c$, $\bar{D}\Xi'_c$, $\bar{D}^*\Xi'_c$ and $\bar{D}^*\Xi^*_c$. The corresponding poles of these peaks in the second Riemann sheets are obtained at $(M + i\,\Gamma/2) = (4276.59 + i7.67)$ MeV, $(4429.89 + i7.92)$ MeV, $(4436.70 + i1.17)$ MeV, $(4580.96 + i2.44)$ MeV, and $(4650.86 + i2.59$ MeV. Note that these poles are all small widths, and two higher ones are

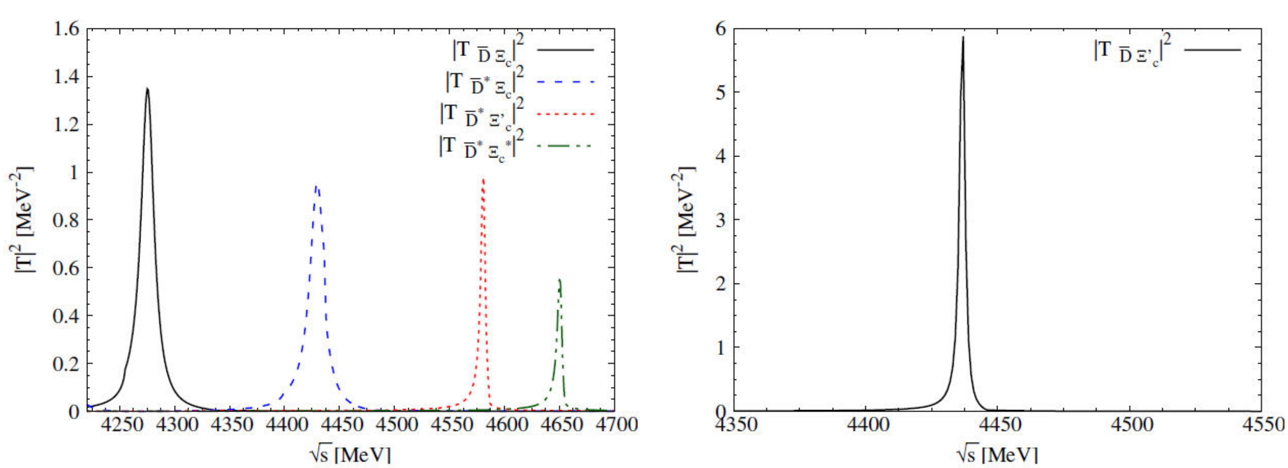


FIGURE 1. Results of the modulus squared of the amplitudes for the sector $J = 1/2,\ I = 0$.

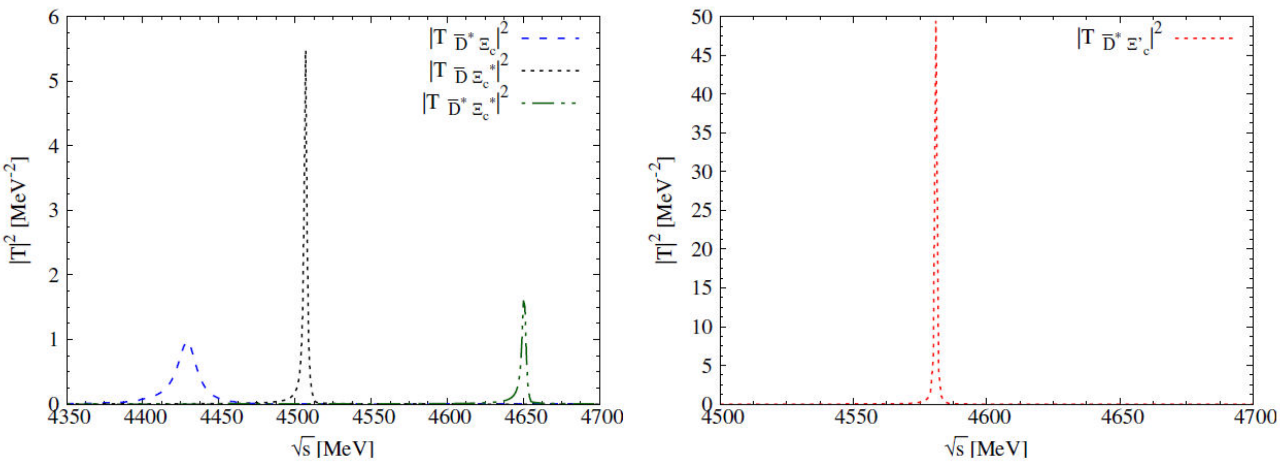


FIGURE 2. Same as Fig. 1 for the sector $J = 3/2,\ I = 0$.

loosely bound and locate at the energy positions nearby the certain thresholds. Indeed, the corresponding couplings of these poles for a certain channel indicate that their dominant channels are the ones $\bar{D}\Xi_c$, $\bar{D}^*\Xi_c$, $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c'$ and $\bar{D}^*\Xi_c^*$, respectively; see more results in Ref. [18]. Thus, these predicted states are assigned as the molecular states of $\bar{D}\Xi_c$, $\bar{D}^*\Xi_c$, $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c'$ and $\bar{D}^*\Xi_c^*$, correspondingly. Furthermore, as found in Ref. [18], two lightest states, $\bar{D}\Xi_c$ and $\bar{D}^*\Xi_c$, also couple strongly to the open channels $\bar{D}_s\Lambda_c$ and $\bar{D}_s^*\Lambda_c$, respectively, which lead to their widths of around 15 MeV. Besides, constrained by the HQSS, all these states can couple to the channels $\eta_c\Lambda$ and $J/\psi\Lambda$, which are most likely the decay channels of these states to be observed. Especially, the channel $J/\psi\Lambda$ is analogous to the one $J/\psi N$ in the hidden charm sector [8] where the new pentaquark peaks have been observed [15]; see more discussions in Ref. [18].

For the results of the $J = 3/2$ sector, there are also four states predicted, which can be clearly seen from the peaks of Fig. 2. Similarly, their corresponding poles in the second Riemann sheets are found at $(4429.52 + i7.67)$ MeV, $(4506.99 + i1.03)$ MeV, $(4580.96 + i0.34)$ MeV and $(4650.58 + i1.48)$ MeV, all of which the widths are narrow. Once again, from their corresponding couplings to a certain channel [18], these poles couple mostly to the channels $\bar{D}^*\Xi_c$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Xi_c'$ and $\bar{D}^*\Xi_c^*$, respectively. Thus, three higher poles are also loosely bound. One can see that the lightest state $\bar{D}^*\Xi_c$ also couples to the open channel $\bar{D}_s^*\Lambda_c$ strongly, which also leads to a sizable width of around 15 MeV. Moreover, the channel $J/\psi\Lambda$ again sufficiently couples to all these states, and thus, this channel is a good candidate for the observation of these predicted structures.

Therefore, it is not surprising that a resonance structure of $P_{cs}(4459)$ was found in the $J/\psi\Lambda$ invariant mass distributions of the $\Xi_b^-$ decay as reported by the LHCb Collaboration recently [28]. In view of the new finding for the $P_{cs}(4459)$ state, we can determine the value of free parameter $a_\mu$ with the mass of $P_{cs}(4459)$, which is tuned as $a_\mu(\mu = 1\ \text{GeV}) = -1.94$. Compared to the one of $a_\mu(\mu = 1\ \text{GeV}) = -2.09$ for the predictions [18], this value is closer to the "natural values" $a_\mu = -2$ [54]. Note that due to not enough statistics for the present data on the $J/\psi\Lambda$ invariant mass distributions, we can not obtain the value of $a_\mu$ by fitting the experimental data as done in Ref. [62] for the case of three $P_c$ states. With the new tuned value $a_\mu = -1.94$, the modulus squared of the amplitudes are similar to Figs. 1 and 2 above, and just the peak structures become more narrow and move to higher energies. Therefore, the loosely bound poles as discussed above have moved to the corresponding thresholds, which leads to no stable poles in the second Riemann sheets for these peaks, as shown in the results of Ref. [51]. There are only four poles found in the second Riemann sheets for two cases of spins $J = 1/2$ and $J = 3/2$. The first pole, $(4459.07 + i6.89)$ MeV, is a bound state of $\bar{D}^*\Xi_c$, which is in fact a degenerate state with two poles of spins $J = 1/2$ and $J = 3/2$, since the pion exchange is neglected in our formalism as discussed above. We assign this $\bar{D}^*\Xi_c$ molecular state as the observed $P_{cs}$ state. One can see that the width of this pole is in good agreement with the reported one of about 17 MeV, which just has a difference of 3 MeV. Compared to the results obtained above with $a_\mu = -2.09$, the mass of this pole has become about 30 MeV higher. In view of the obtained results, a conclusion can be made that the $P_{cs}(4459)$ state may be a degenerate state with spins $J = 1/2$ and $J = 3/2$, which is analogous to the case of $P_c(4450)$ state before. Note that the two-pole structure for the $\bar{D}^*\Xi_c$ bound state was also predicted in Ref. [24], where a mass difference of about 6 MeV was obtained. But, at the present data's statistics, the two-pole hypothesis for the $P_c(4450)$ state is not confirmed or refuted [28]. The second stable pole locates at $(4310.53 + i8.23)$ MeV, which is in the $J = 1/2,\ I = 0$ sector and a bound state of $\bar{D}\Xi_c$ with a 56 MeV binding energy. It will be interesting to see that the mass of such predicated states is close to or even smaller than the $P_c(4312)$ state while the $s$ quark is instead of $u$ or $d$ quark. This pole is also consistent with the prediction of Ref. [29] within the uncertainties. The third one is at $(4445.12 + i0.19)$ MeV,

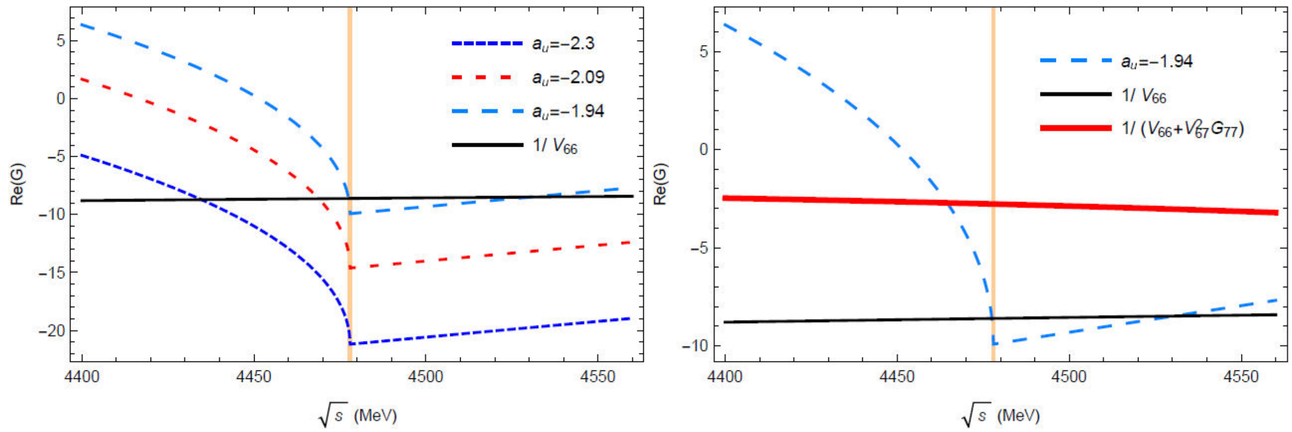


FIGURE 3. Results for the inverse potential $1/V_{66}$ and $1/(V_{66} + V_{67}^2 G_{77})$ (solid lines) compared with the real parts of the loop function (dashed lines) for the $\bar{D}^*\Xi_c$ channel with different $a_\mu$, where the vertical line corresponds to the location of the threshold.

which is also in the $J = 1/2,\ I = 0$ sector and a bound state of $\bar{D}\Xi_c'$ with a small binding energy of just 0.23 MeV. Note that, as found in Ref. [51], the two poles of the $\bar{D}^*\Xi_c$ state and the one of $\bar{D}\Xi_c$ state also couple strongly to the channels $\bar{D}_s^*\Lambda_c$ and $\bar{D}_s\Lambda_c$, respectively, also discussed above, which lead to quite large branching fractions on these decay channels compared to the $J/\psi\Lambda$ channel. Thus, these two decay channels will be good candidates for searching for two states $\bar{D}^*\Xi_c$ and $\bar{D}\Xi_c$ in the future experiments.

From the results obtained above, one can see that an increased value of parameter $a_\mu$ causes all of the poles become less bound and move to higher masses. Therefore, when we tune the parameter $a_\mu = -2.09$ as $a_\mu = -1.94$, some of the loosely bound poles move to the corresponding thresholds, such as the possible bound states $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Xi_c^*$ with $J = 1/2,\ I = 0$, and $\bar{D}^*\Xi_c'$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Xi_c^*$ with $J = 3/2,\ I = 0$. One can recall that the corresponding bound states found in Refs. [1, 2] became more bound, for example the bound state of $\bar{D}^*\Xi_c$ with a pole at 4370 MeV, which located even below the threshold of the $\bar{D}_s^*\Lambda_c$ channel. But, taking the $\bar{D}^*\Xi_c$ channel for example, which was denoted as the channel 6 in the lower indexes, we show the results of the real parts of the loop function $G_{66}$ with different values of $a_\mu$ compared with the inverse potential $1/V_{66}$ in the left part of Fig. 3. Note that the imaginary part of the loop function below the threshold of the bound channel is zero for the case of bound state. From the left panel of Fig. 3, one can see that the potential of $V_{66}$ cannot contribute such bound pole of 4370 MeV, which can be seen in the cross points for the poles. Note that the vertical line indicates the corresponding threshold. One can be refer to Refs. [63, 64] for more discussions about the pole affected by the free parameters of the loop functions. In fact, the reason of such bound pole as we found is the strong coupling between the channels $\bar{D}^*\Xi_c$ (channel 6) and $\bar{D}_s^*\Lambda_c$ (channel 7). Thus, due to $V_{77} = \mu_3 = 0$, see Eq. (2), the diagonal potential $V_{66}$ will be enhanced by an off-diagonal transition potential $V_{67}$ via the coupled channel effect, which finally strengthens the attractive interaction by an "effective" potential of $V_{66} + V_{67}^2 G_{77}$. This enhancement effect can be shown in the right panel of Fig. 3, where we show the results of the real parts of the loop function $G_{66}$ compared with $1/V_{66}$ and $1/(V_{66} + V_{67}^2 G_{77})$ using $a_\mu = -1.94$. From the right subfigure of Fig. 3, one can see that the intersection point of the pole becomes more bound for the "effective" potential of $V_{66} + V_{67}^2 G_{77}$. Therefore, it is easy to find such a bound pole of 4370 MeV for the "effective" potential with $a_\mu = -2.3$ used in Refs. [1, 2]. Analogously, the same effect lead to such a bound state of $\bar{D}\Xi_c$ [1, 2] with the contribution from the channel $\bar{D}_s\Lambda_c$.

## 4. Conclusion

In the present work, we revisited the $s$-wave interactions of the $J/\psi\Lambda$ channel and its coupled channels, using the coupled channel unitary approach combined with the local hidden gauge formalism and the heavy quark spin symmetry. In our formalism, since there is a free parameter of $a_\mu$ in the loop functions, we made predictions for the $P_{cs}$ states by taking its value from the investigation of three $P_c$ states. With the observation of the $P_{cs}(4459)$ state, we can determine its value from the mass of the $P_{cs}(4459)$ state. Thus, from our updated results, a pole $(4459.07 + i6.89)$ MeV bounded by the channel $\bar{D}^*\Xi_c$ can be assigned as the observed $P_{cs}(4459)$ state. Since this pole is in fact two nearly degenerate states with spin parities $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, the $P_{cs}(4459)$ state maybe has a two-pole structure, analogous to the previous $P_c(4450)$ peak observed by LHCb too, which can be looked for with higher statistics data in the future. Besides, there is strong evidence of a $J^P = \frac{1}{2}^-$ bound state of $\bar{D}\Xi_c$ with a stable pole at $(4310.53 + i8.23)$ MeV. Furthermore, there may be the possible loosely bound states $\bar{D}\Xi_c'$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Xi_c^*$ in the $J = 1/2,\ I = 0$ sector and $\bar{D}^*\Xi_c'$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Xi_c^*$ in the $J = 3/2,\ I = 0$ sector, which are very dependent on the value of the free parameter of $a_\mu$. Using the present data with not enough statistics, the existence of these possible states is put into question for the threshold effects. Although the interaction potentials from the local hidden gauge formalism are attractive, it depends on the model parameter and the neglected momentum dependent terms, as well as pion exchange potential, whether they are bound states or virtual states, as found in Ref. [59]. To search for these molecular states in future experiments, especially for the $\bar{D}\Xi_c$ state and two $\bar{D}^*\Xi_c$ states, we propose two decay channels $\bar{D}_s\Lambda_c$ and $\bar{D}_s^*\Lambda_c$, respectively, since their predicted decay branching fractions are much larger than the ones of the $J/\psi\Lambda$ channel. On the other hand, two other decay channels, $J/\psi\Lambda$ and $\eta_c\Lambda$, are also suggested for future experiments, which can be distinguished by the different spin nature of these bound states. We hope that future experiments can check our predictions to understand the nature of the $P_{cs}$ states.

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