

Hypernuclei based on chiral interactions

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We present Λ separation energies for light hypernuclei based on chiral hyperon-nucleon interactions up to next-to-leading order. In the first part, we consider several sources of uncertainties with a focus on using different realizations of chiral hyperon-nucleon interactions to estimate three-baryon forces that enter at next-to-next-to leading order. We also demonstrate that the similarity renormalization group evolution of the hyperon-nucleon interactions induces a strong variation of the separation energies. The energies are however strongly correlated which allows one to define a preferred similarity renormalization group parameter for which hypernuclear binding energies can be predicted reliably. With these insights, we present in the second part three examples of recent applications of chiral interactions to hypernuclei. In the first application, we study the predictions for $A = 4$ and $A = 7$ hypernuclei based on the version of the hyperon-nucleon interaction that yields a large hypertriton binding energy as suggested by the recent experiment of the STAR collaboration. The first predictions for $A = 4 - 6$ strangeness $S = -2$ hypernuclei are discussed in the second application. Finally, in the third application, we use the charge-symmetry breaking of $A = 4$ Λ separation energies to constrain the Λ -neutron interaction.

Keywords: Hyperon-nucleon interaction; effective field theory; hypernuclei; charge-symmetry breaking.

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1. Introduction

Hyperons play an important role in nuclear as well as astrophysics [1]. Especially, the possibly important contribution of hyperons to neutron stars has been of high interest recently [2–5] and is commonly referred to as hyperon puzzle since a softening of the nuclear equation of state due to hyperons is in contradiction to the observation of neutron stars with masses larger than two solar masses [6–8].

In order to progress, it is important to understand the interactions of hyperons with nucleons or hyperons since, *e.g.*, the presence of hyperons in neutron matter sensitively depends on the properties of the underlying interactions [9].

Besides this phenomenological interest, hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions are also conceptually interesting. Compared to nucleon-nucleon (NN) interactions, particle conversion processes (like Λ - Σ or Λ - Ξ conversion) induce interesting dependencies on isospin. In some cases, multi-pion exchanges become even longer-ranged than the leading one-boson exchange. Studying YN and YY interactions also give experimental access to the impact of explicit chiral symmetry breaking of baryon-baryon (BB) interactions.

On the other hand, information on these interactions is rather limited since scattering data involving hyperons is scarce. This situation triggered several experimental programs. Thereby, a direct measurement of scattering observables, *e.g.* differential cross sections, is demanding and will not be possible for most YN and YY systems. Nevertheless, first new results have recently appeared and are providing new constraints for the development of interactions [10, 11]. Another important source of information is hypernuclei, see *e.g.* [12]. Therefore, also the spectroscopy and the binding

energies of hypernuclei are in the focus of several experimental programs at J-PARC, FAIR, MAMI and JLab [13–17].

Despite the scarcity of the data, BB interactions have been developed since many years. In the past, the models were mostly based on one-boson exchanges and, in addition, flavor $SU(3)$ symmetry was employed to relate some parameters to nucleon-nucleon (NN) scattering [18–23]. In order to avoid any model assumptions and to enable reliable estimates of the theoretical uncertainty, chiral effective field theory (ChEFT) has been used in recent years to develop new BB interactions [24–26]. For the NN system, the data up to the pion production threshold can be described essentially perfectly [27, 28]. Up to next-to-leading order (NLO), the extension to YN and YY systems has also been achieved [29–33]. Again, flavor $SU(3)$ symmetry has been exploited to minimize the number of short distance parameters, so called low energy constants (LECs). Still, a unique determination of these LECs is not possible because of the scarce data [33].

However, nowadays reliable predictions for light hypernuclear systems are possible based on these realistic interactions including particle conversions and tensor forces. The comparison to hypernuclear data is already used and will allow to further improve such interactions in the future, *e.g.*, the binding energy of the lightest hypernucleus, the ${}^3_{\Lambda}\text{H}$, has been employed in many YN interactions to determine the relative strength of the spin $S = 0$ and $S = 1$ ΛN interaction that cannot be obtained by the available spin-independent ΛN scattering data. In the future, more hypernuclear data will be used to further constraint interactions. In this contribution, we will present some first steps into this direction. We start in Sec. 2 with an estimate for possible three-baryon force (3BF) contributions to hypernuclear observables. Such contributions are higher order in the chiral expansion of the BB interaction. At this point, they limit the accuracy of our predictions which is

important for a better understanding of the later results. Related to this is the size of interactions that are induced by so-called similarity renormalization group (SRG) evolutions of hypernuclear interactions. In Sec. 3, we discuss a strong linear correlation of binding energies related by such SRG evolutions. This allows us to use SRG to facilitate calculations using the Jacobi no-core shell model [34–36]. After these general remarks, we explicitly study how an increased hypertriton binding energy changes the spin dependence of YN interactions and how this affects other hypernuclear binding energies in Sec. 4. In the next section, we turn to first predictions for $S = -2$ hypernuclei and relate the binding energy for ${}_{\Lambda\Lambda}{}^6\text{He}$ to the one for ${}_{\Lambda\Lambda}{}^5\text{He}/{}_{\Lambda\Lambda}{}^5\text{H}$ and ${}_{\Lambda\Lambda}{}^4\text{He}$. Finally, in Sec. 6, we use the charge symmetry breaking (CSB) splitting of ${}^4_{\Lambda}\text{H}/{}^4_{\Lambda}\text{He}$ to pin down the difference of the Λ -proton (Λp) and Λ -neutron (Λn) interaction and predict the Λn scattering length. We conclude and give an outlook in Sec. 7.

2. Estimating 3BFs

All calculations presented here are based on chiral NN, YN and YY interactions. If not stated otherwise, the semilocal momentum space (SMS) regularized chiral interaction at order N^4LO with a cutoff of 450 MeV [28] is used for the NN interaction. In contrast, the highest order of the YN and YY interaction used is NLO. Since 3BFs contribute only starting from order N^2LO [37–39], we do not need to take 3BFs into account. However, it is well known that the contribution of three-nucleon forces (3NFs) are somewhat more visible than that of NN interactions of order N^2LO . This is probably related to the fact that light nuclei binding energies are correlated to the ${}^3\text{H}$ binding energy. The addition of the leading 3NF is therefore generally used to fix this binding energy to the experimental value which automatically improves other binding energies [40]. In practice, the model dependent contribution of 3NFs to the binding energy of ${}^3\text{H}$ is of the order of 500 keV which is approximately 10% of the binding energy of this nucleus with respect to break up in ${}^2\text{H}$ and a neutron.

An interesting question is how important such 3BFs are for the binding energies of hypernuclei. The difference of the energies of the nuclear core of a hypernucleus and the hypernucleus, the so-called Λ separation energies, can be as small as 130 keV for ${}^3_{\Lambda}\text{H}$. For this weakly bound system, particles are generally far apart so that one may naively expect small contributions of 3BFs to the separation energy. On the other hand, subtle contribution can change such small separation energies significantly. As mentioned in the introduction, the binding energy of ${}^3_{\Lambda}\text{H}$ is often used to determine the spin dependence of YN interactions which can only work reliably if 3BF contributions are insignificant (or well understood). The most direct estimate of such 3BF contributions will be a direct calculation based on the formulation of the leading 3BFs [37]. This is still work in progress, therefore, we rely here on an indirect approach. We fix the Λn singlet

and triplet scattering length for one cutoff of the YN interaction and predict hypertriton binding energies for the other cutoffs. The differences of the hypertriton binding energy obtained are of the order of 40 keV [33] which is comparable to the experimental uncertainty of 50 keV for the same observable [41, 42].

Additionally, we devised two different realizations of YN interactions that are essentially phase shift equivalent: NLO13 [31] and NLO19 [33]. Despite their similar predictions for YN observables, their non-observable potential matrix elements differ significantly. Especially, the strength of the Λ - Σ transition potential is different. Therefore, any differences in predictions for systems larger than $A = 2$ can be seen as an indication of 3BF effects. For the hypertriton, the energies again differ only by 40 keV [33]. Therefore, at this point, the results indicate that 3BFs can be neglected for the determination of the YN spin dependence at this point.

With the two realizations at hand, it is now also possible to quantify the 3BF contributions to other observables. In Fig. 1, we show as an example the single particle potential of Λ (left) and Σ (right) in nuclear matter depending on the Fermi momentum. For the Σ single particle potential, one observes that predictions for NLO13 and NLO19 are very similar. We therefore can expect small contributions from 3BFs. For the Λ , the situation is different. At Fermi momenta above 1.0 fm^{-1} , the predictions become more and more different indicating significant contributions from 3BFs. For lower densities, the predictions are still similar. This is consistent with the expectation that 3BFs are less important for very light hypernuclei like the hypertriton.

In summary, the two different realizations of chiral interactions, NLO13 and NLO19, allow one to better quantify uncertainties of predictions due to missing higher order interactions. We will make use of this possibility below.

3. SRG evolution for hypernuclear interactions

Our tool for predictions of hypernuclei with $A > 4$ is the Jacobi NCSM [34, 35]. In this approach, the Schrödinger equation is solved using a harmonic oscillator (HO) basis. Because of the short-distance repulsion of realistic BB interactions, directly using these interactions requires prohibitively large model spaces. Therefore, for the NCSM and also other many-body techniques, the interactions need to be softened at short distances without changing the description of YN (or YY) observables. Nowadays, the standard tool for this is a similarity renormalization group (SRG) evolution of the interactions [43]. Such an evolution induces not only two-baryon but also multi-baryon interactions. Unfortunately, as is well known, especially the induced 3BFs are very important for Λ -hypernuclei [44]. This is surprising since the corresponding 3NFs are comparable in size to the 3NFs predicted by ChEFT [45]. For YNN interactions, the induced interactions are clearly larger than expected so that they can gen-

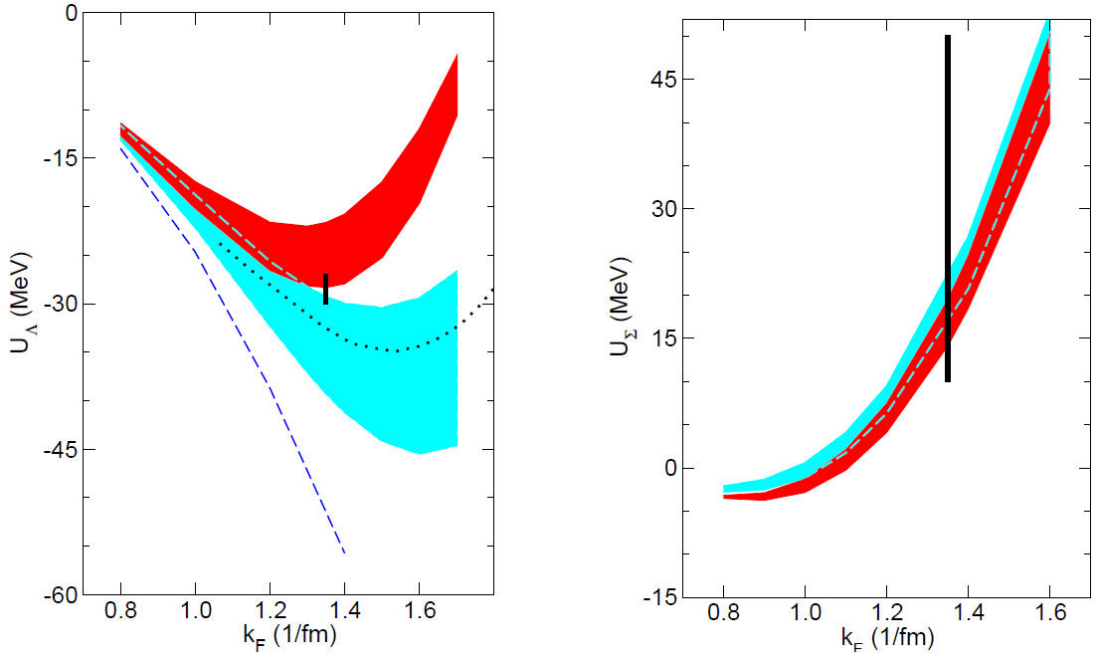


FIGURE 1. Single particle potential for momentum $p_Y = 0$ depending on the Fermi momentum in symmetric nuclear matter for Λ (left) and Σ (right). The red (dark) bands are results for NLO13 and the cyan (light) bands result for NLO19 with cutoffs in the range between 500 and 650 MeV. For a comparison, also results for Jülich '04 [21] (dashed blue line) and Nijmegen SC97f [20] (dotted black line) are given. The black bars indicate the empirical value [1].

erally not be omitted at order NLO. This manifests itself as a strong dependence on the SRG parameter λ_{YN} . Wirth *et al.* have therefore included SRG-induced YNN interactions in their NCSM calculations [44, 46, 47]. In order to reduce the computational complexity of the problem, we have so far not used induced YNN interactions in our Jacobi NCSM calculations. This became possible after we observed that there is a strong correlation of binding energies of light hypernuclei [35].

Two examples of this correlation are shown in Fig. 2. The linear correlation of separation energies is fulfilled with high

accuracy. This indicates that the induced 3BF can be parameterized by a single parameter. We have then observed that for each interaction, one can find a value for λ_{YN} for which the binding energy of ${}^5_\Lambda\text{He}$ is described in agreement with experiment. For NLO19(600) this special, “magic” parameter is $\lambda_{YN} = 0.836 \text{ fm}^{-1}$. For $A = 3$ and $A = 4$ hypernuclei, we are able to compare to calculations using the bare interactions without SRG evolution [33] and find that the results are in agreement with each other within the size of expected chiral 3BFs. Therefore, our predictions for the hypertriton are in good agreement with experiment. For the $J = 0^+$ state of

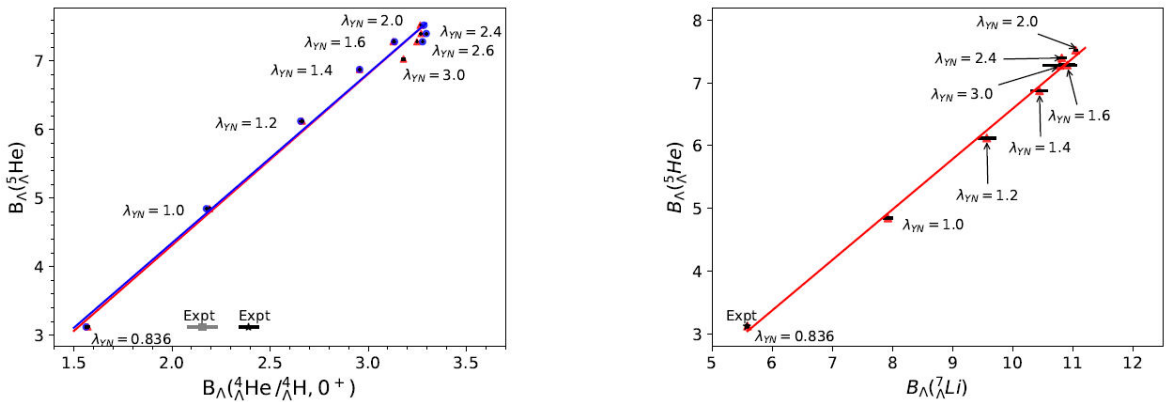


FIGURE 2. Correlation of Λ separation energies with the one of ${}^5_\Lambda\text{He}$ for the Idaho-N3LO(500) NN interaction [48] and the NLO19(600) YN interaction. Left: ground states of ${}^4_\Lambda\text{He}$ (blue circles)/ ${}^4_\Lambda\text{H}$ (red triangles). Right: ${}^7_\Lambda\text{Li}$ (red triangles). The numerical uncertainty is indicated by black error bars. The lines are linear fits to the numerical calculations. The experimental values (black stars and boxes) are taken from [42, 49]. Energies are given in MeV. The SRG parameters are indicated as annotation and given in fm^{-1} .

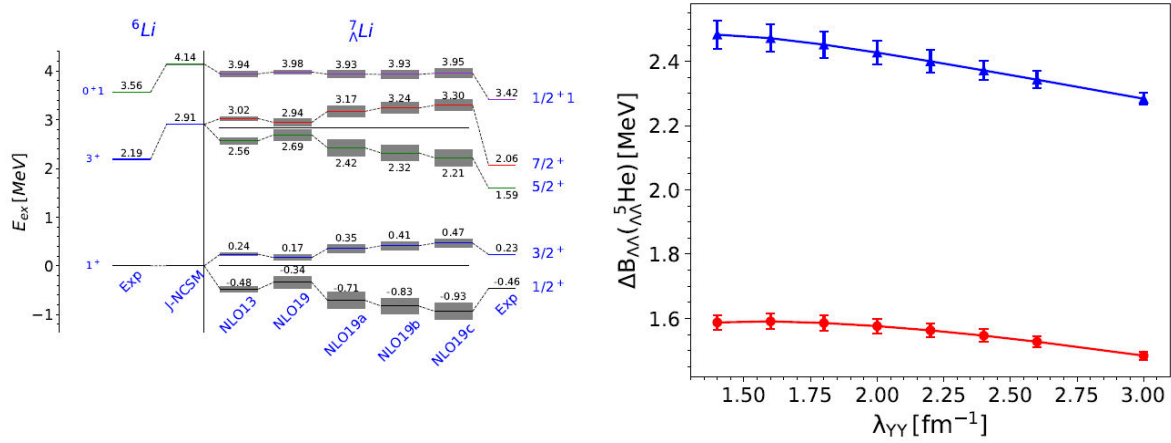


FIGURE 3. Left: Level scheme of ${}^7_{\Lambda}\text{Li}$ based on SRG evolved YN interactions at NLO. The levels are related to the levels of the ${}^6\text{Li}$ core. For all calculations, λ_{YN} has been chosen such that ${}^5_{\Lambda}\text{He}$ is correctly predicted. The grey bands show the dependence on the chiral cutoffs. Right: $\Delta B_{\Lambda\Lambda}$ excess energy for ${}^5_{\Lambda\Lambda}\text{He}$ depending on the SRG parameter λ_{YY} . The upper blue symbols are for the LO, the lower red symbols are for the NLO interaction for a 600 MeV cutoff. Lines are added to guide the eye. Error bars indicate the numerical uncertainty.

$A = 4$ hypernuclei, the agreement is less good. However, for these hypernucleus, sizable chiral 3BFs can be expected [33]. The agreement for the $J = 1^+$ is somewhat better although similarly large chiral 3BFs might contribute. For more complex nuclei, we have to rely on predictions based on SRG evolved interactions. As can be seen on the right hand side of the figure, there is a favorable agreement with experiment for ${}^7_{\Lambda}\text{Li}$. Note, however, that the experimental results based on emulsion and counter experiments are sometimes in contradiction to each other [50].

In the following, we will always use these magic cutoffs for predictions for single Λ hypernuclei when SRG evolved interactions are necessary.

4. Impact of an increased hypertriton binding energy

As mentioned in the introduction, the Λ separation energy of ${}^3_{\Lambda}\text{H}$ is important to determine the spin dependence of YN interactions. In NLO13 and NLO19, the long standing experimental value of $E_{\Lambda} = 130 \pm 50$ keV [41] has been used to this aim. Recently, the STAR collaboration published their measurement resulting in $E_{\Lambda} = 410 \pm 120$ keV [51]. Such a large value can only be accommodated by increasing the singlet ΛN scattering length significantly. Based on NLO19, we have therefore devised a series of interactions NLO19a, NLO19b and NLO19c that still describe all available low energy scattering data well but predict much larger singlet scattering lengths [52] leading to increased hypertriton binding energies comparable to the new STAR value. We then studied in detail for $A = 4$ and $A = 7$ how such a change affects the Λ separation energies for other hypernuclei. To our surprise, for $A = 4$, the agreement with experiment is even improved compared to the standard NLO13 and NLO19 predictions. The predictions for the spectrum of ${}^7_{\Lambda}\text{Li}$ are shown

on the left hand side of Fig. 3. Due to the missing 3NFs, the excitation energy of ${}^6\text{Li}$ is not correctly reproduced in these calculations. Since the splittings on the $1/2^+ - 3/2^+$ and $5/2^+ - 7/2^+$ doublets is not affected by this shortcoming, a comparison of these splittings to experiment is still useful. For the standard interactions, NLO13 and NLO19, the splittings are correctly reproduced although the interaction dependence is visible indicating again visible 3BF contributions to this quantity. Using the new interactions NLO19a,b and c, leads to a significantly increased splitting. The larger splitting is not supported by experiment, however, the overall deviations are small given the visible 3BF contributions. Therefore, the results for ${}^7_{\Lambda}\text{Li}$ do not contradict an increased hypertriton binding energy. It will be interesting to add chiral and induced 3BFs to such calculations in the future since smaller uncertainties can be expected in this case.

5. Light $S = -2$ hypernuclei

The second example are strangeness $S = -2$ hypernuclei. Based on chiral YY interactions [30,32,53], we recently predicted binding energies for ${}^6_{\Lambda\Lambda}\text{He}$, ${}^5_{\Lambda\Lambda}\text{He}$ and ${}^4_{\Lambda\Lambda}\text{H}$ [36]. The calculations were done using the Jacobi NCSM and therefore rely on SRG evolutions of the interactions. We ensured a realistic description on the single Λ core nuclei by choosing again the magic λ_{YN} . Fortunately, it turned out that the dependence of the binding energy on the corresponding parameter λ_{YY} for the $S = -2$ two-body interactions is small for these double Λ hypernuclei as can be seen on the right hand side of Fig. 3. The strength of the bond in the $\Lambda\Lambda$ system is commonly quantified by the $\Lambda\Lambda$ excess energy $\Delta B_{\Lambda\Lambda} = 2E({}^{A-1}_{\Lambda}X) - E({}^A_{\Lambda\Lambda}X) - E({}^{A-2}X)$. For ${}^6_{\Lambda\Lambda}\text{He}$, this energy is experimentally well established [54,55]. We found that the LO interaction overbinds these systems significantly but that the NLO interaction leads to a quite realistic excess energy. ${}^5_{\Lambda\Lambda}\text{He}$ and ${}^4_{\Lambda\Lambda}\text{H}$ have not been experimentally

TABLE I. Predictions for singlet a_s and triplet a_t scattering length in the Λp and Λn systems for different charge symmetry breaking interactions. The scattering lengths are given in fm.

force	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$
NLO13(500)	-2.60	-1.65	-3.27	-1.56
NLO13(550)	-2.59	-1.55	-3.29	-1.47
NLO13(600)	-2.59	-1.57	-3.29	-1.49
NLO13(650)	-2.59	-1.54	-3.27	-1.45
NLO19(500)	-2.65	-1.58	-3.20	-1.47
NLO19(550)	-2.64	-1.52	-3.21	-1.41
NLO19(600)	-2.63	-1.47	-3.23	-1.36
NLO19(650)	-2.62	-1.46	-3.23	-1.37

observed yet which triggers the question whether these hypernuclei are bound. Based on more simplified interactions, it was found that ${}_{\Lambda\Lambda}^5\text{He}$ is bound whereas ${}_{\Lambda\Lambda}^4\text{H}$ is most likely not bound [56–59]. Our calculations confirm these results now for a realistic interaction that fulfills all constraints due to the scarce $S = -2$ data. In contrast to Ref. [57], we find that the excess energy is larger for $A = 5$ than for $A = 6$. This might be related to the conversion processes $\Lambda\Lambda\text{-}\Sigma\Sigma\text{-}\Xi\text{N}$ that is part of the chiral interactions but has not been considered in [57]. Therefore, an observation of ${}_{\Lambda\Lambda}^5\text{He}$ and its excess energy would provide important new insights into $S = -2$ BB interactions.

6. CSB in ΛN interactions

Finally, as the third example, we discuss CSB on the YN interaction. The observation of the rather large difference of the Λ separation energies of ${}_{\Lambda}^4\text{H}$ and ${}_{\Lambda}^4\text{He}$ indicated a specifically large CSB of YN interactions. Dalitz and van Hippel proposed that $\Sigma^0\text{-}\Lambda$ mixing effectively leads to a CSB one-pion exchange contribution to the ΛN interaction [60]. This mechanism is already part of, *e.g.*, the Nijmegen SC97 interactions [20]. It can, however, not fully explain the CSB in $A = 4$ hypernuclei [61]. Clearly, the predictions for CSB based on this mechanism are model-dependent. This can be seen from recent results for the LO chiral interactions that lead to sizably larger predictions [62–64]. From the ChEFT perspective, it is clear that there are two CSB contact interactions in the same order as the CSB one-pion exchange. In Ref. [65], we have used the experimentally known values for the splitting of the separation energies of ${}_{\Lambda}^4\text{H}$ and ${}_{\Lambda}^4\text{He}$ in the 0^+ and 1^+ states to determine these two contact interactions. As a results, we were able to predict the Λn and Λp singlet and triplet scattering lengths for the first time based on data. Again, we make use of the NLO13 and NLO19 version of

the YN interactions based on various cutoffs between 500 and 650 MeV. The predicted scattering length turn out to be highly independent of the realization of the chiral interaction. For the currently accepted experimental values for the two splittings, we find that the triplet scattering length is only slightly changed by the CSB interaction. The singlet one is more affected leading to a larger Λn scattering length as can be seen in Table I. Given that a direct measurement of Λn scattering is impossible in the foreseeable future and that this interaction is of utmost importance for the properties of Λ in neutron matter, new experiments that reduce uncertainties of the splittings for ${}_{\Lambda}^4\text{H}$ and ${}_{\Lambda}^4\text{He}$ are quite important. Our calculations show that the theoretical uncertainties for the scattering lengths are small. In contrast, the experimental uncertainties lead to significant uncertainties of our predictions for the scattering lengths. These are not reflected in Table I.

7. Conclusion and outlook

In conclusion, we have shown that hypernuclei can provide important additional constraints for YN and YY interactions. The available YN and YY data alone do not completely determine these forces. Thereby, we have used realistic, chiral interactions that are flexible to accommodate new data and allow at the same time to quantify uncertainties. Using the Jacobi NCSM, we are able to predict Λ separation energies and $\Lambda\Lambda$ excess energies for hypernuclei up to $A = 7$.

The calculations still need to be further refined. First of all, chiral 3BFs should be included and could be used to improve the description of $A = 4$ separation energies. At the same time, SRG induced 3BFs need to be incorporated. Both developments are currently in progress. With such improved predictions, the study of CSB can extended to p -shell hypernuclei. Also a possible sensitivity of p -shell hypernuclei to p -wave YN interactions should be studied. With the upcoming new experimental data for light hypernuclei, we can expect a much better understanding of BB interactions in the future.

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