A classroom alternative to simulate radioactive decay of nuclei

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In this work we present a classroom alternative to simulate radioactive decay of nuclei. It consists of a game in which the students in a classroom are invited to participate in a kind of lottery. The students (around 45 in a typical classroom) take 20 "tickets" numbered at random from 1 to 50. As a whole we have 900 initial "tickets". Then one student is asked to give a random number between 1 and 50 (for example 34) and the teacher gives a range of numbers from 34 to 38 or 30 to 34. The students cross the numbers in their list that coincide with the numbers given by the teacher. These numbers are the "lucky" ones and represent "nuclei" which decay, then someone gives another number and the process is repeated and the number of "lucky" tickets put in a list. Repetition of the process gives a sequence of numbers which are the number of "nuclei" surviving as the time goes on. The nuclei decaying are considered as stable ones. A plot of the surviving "nuclei" as a function of time (number of times the students are asked to cross the lucky numbers) gives a typical exponential decay. When one considers the case of nuclei A decaying into nuclei B decaying to nuclei C and this one is a stable one, using the corresponding differential equations one obtains the normal curves of nuclei as a function of time.

Keywords: Radioactive decay; classroom; lottery.

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1. Introduction

There are many methods that simulate the radioactive decay of various unstable nuclei, these methods range from mechanical [1] to electronics [2] and computers [3]. In this work, a method is proposed that consists of a kind of lottery game in which a not-so-large number of students participate -about 45 of a group of undergraduates, for example-. Each one of the students is asked to write a set of random numbers between 1 and 50, let's say about 20, together we will have 900 numbers that represent the unstable nuclei. Someone is then asked to choose a range of numbers, for example from 11 to 15, and they are asked to cross off all the numbers that match that range on their list.

There will be about 90 that cross out, which represents the nuclei that decayed. You now have 810 surviving nuclei. With the numbers that were not crossed out, now someone decides to cross out the numbers in the range of 46 to 50, now there will be about 81, and 729 survive. The procedure is repeated now with the numbers 21 to 25, with which some 73 are crossed out and 657 survive. Each time you ask to cross out, a time interval is defined to record the "nuclear radiation". Repeating this procedure several more times with different ranges of size 5 will give you 590, approximately, then 531, 478, etc.

When we repeat the procedure with another range of numbers, 4 instead of 5; the list of surviving nuclei follows another pattern and gives the behavior of a different nuclei decaying radioactively.

2. Experiment

The experiment was made in a classroom with 45 students and the list of numbered "tickets", was something like these: 12, 1, 3, 6, 23, 45, 5, 7, 10, 34, 20, 32, 28, 30, 19, 48, 37, 11, 43, 31, other student of course had a different sequence of numbers between 1 and 50. The amount of surviving nuclei could be just the numbers not crossed out or a new list is designed with a number of tickets equal to the surviving "nuclei". It is preferred this last procedure because in this way the randomness is assured and the possible bias is avoided. With the 45 students playing the lottery we had 900 "nuclei" which eventually decay to another stable "nuclei".

3. Results

Figure 1 shows the number of surviving "nuclei" as a function of time t (in this case every time we asked to cross out the new numbers is considered as a unit of time)

The equation giving the number of surviving nuclei as a function of time was

$$N_{\mathrm{A}}(t) = 902.14e^{-0.106t}, \tag{1}$$

where $N_A(t)$ is the number of surviving nuclei at time t.

The daughter nuclei as a function of time is given by

$$N_{\mathrm{B}}(t) = 902.14(1 - e^{-0.106t}), \tag{2}$$

where $N_B(t)$ is the number of stable nuclei B.

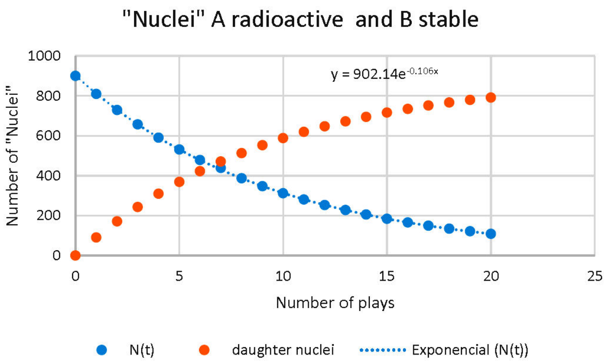

FIGURE 1. Number of nuclei A decaying to nuclei B stable, the number of nuclei B is also plotted as a function of time t.

When we treat the case of a sequence of nuclei from A to B to C, A is a nucleus which decays to a nucleus B which decays to nucleus C and this last one being stable one has to use the differential equations governing the process, these equations are found for example in Ref. [2] If nuclei A decays to nuclei B and it is governed by the equation

$$N_{\rm A}(t) = N_{\rm A} e^{-at}, \tag{3}$$

and nuclei B decays according to

$$N_{\rm B}(t) = N_{\rm B} e^{-bt}, \tag{4}$$

then when treated both processes altogether one obtains that

$$N_{\rm B}(t) = N_{\rm A} \left(\frac{a}{b-a}\right) \left(e^{-at} - e^{-bt}\right). \tag{5}$$

Finally, if nuclei B decays to nuclei C and this is stable, the equation now is:

$$N_{\rm C}(t) = N_{\rm A} \left(1 + \frac{a}{b-a} e^{-bt} - \frac{b}{b-a} e^{-at}\right). \tag{6}$$

In our experiment we found that a=0.106, b= 0.0834, and N_A =900 with these numbers the time evolution of the numbers of nuclei are as the plot that follows:

As can be seen from Fig. 2 when a sequence of nuclei from A to B to C and this last one is a stable one, the typical behavior of a series of nuclei decaying is observed.

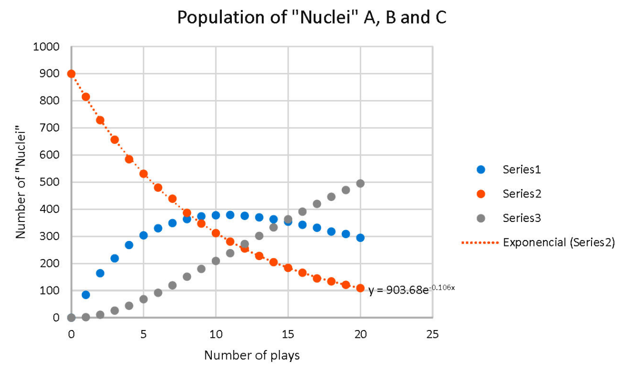

FIGURE 2. Typical behavior of nuclei A to B to C, Nuclei A decays to nuclei B which starts with a number 0 and increases to a maximum and then decay for times longer than around 11. On the other hand nuclei C increases continuously from 0 and eventually reaches a maximum for very large values of t.

4. Analysis and discussion

As can be seen in the figures, the lottery game mimics quite well the behavior of nuclei decaying with their radioactivity; the sequences of numbers representing the tickets for the lottery are quite at random and the decay is induced by the election of the lucky tickets. Depending on the groups of number that give the possible nuclei decaying we obtain different time constants for the decay. In our case when the group was of 5 candidates (out of 50), the time constant was 0.106 as used for the decay of nuclei A to nuclei B stable. For the decay of nuclei B to nuclei C we used a group of 4 numbers and that selection gave a time constant of 0.083. In conclusion the lottery game in a classroom can simulate the decay of radioactive nuclei, the students enjoy a not so common experience and learn the radioactive decay behavior of unstable nuclei and these results can be adapted to different unstable nuclei. With this game one can simulate decaying nuclei with different half lifetimes, from nanoseconds to millions of years, defining the unit of time properly.

Acknowledgments

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