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Multiplicity-Momentum Correlations in Relativistic Nuclear Collisions

Mary Cody[1]
Sean Gavin[2]
Brendan Koch[1,3]
Mark Kocherovsky[1]
Zoulfekar Mazloum[2]
George Moschelli[1]

[1]*Department of Natural Sciences, Lawrence Technological University, 21000 West Ten Mile Road, Southfield, MI 48075*

[2]*Department of Physics and Astronomy, Wayne State University, Detroit, MI, 48202*

[3]*Duke University Medical Center, Durham, NC, USA*

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We introduce a two-particle correlation observable that measures multiplicity-momentum correlations and may facilitate an estimate of the level of equilibration of the medium created in relativistic nuclear collisions. We calculate that multiplicity-momentum correlations should vanish in equilibrium in the Grand Canonical Ensemble, therefore non-zero measured values may indicate that the system has not reached local thermal equilibrium. Information about the level of equilibration of the system is important because many state-of-the-art models assume local equilibration either directly or through the use of an equation of state that makes this assumption. We make estimates of multiplicity-momentum correlations using PYTHIA/Angantyr and find positive values comparable in magnitude to well-measured correlations of transverse momentum fluctuations. We then outline a formalism that can use multiplicity-momentum correlations and correlations of transverse momentum fluctuations to quantify the level of partial thermalization of the system.

Keywords: Nuclear collision

1 Introduction

We present a new observable that measures multiplicity-momentum correlations and may facilitate an estimate of the level of equilibration of the medium created in relativistic nuclear collisions. This observable completes a set of mathematically related two-particle number and momentum density correlations; the relationship can be used both as a validation tool, and also as a method for interpreting one observable in therms of the physics of the others. In this work we only outline the mathematical relationship and some consequences. For a more detailed discussion, see Ref. [1].

In Ref. [1], we show that multiplicity-momentum correlations should vanish in equilibrium in the Grand Canonical Ensemble. Non-zero values of these correlations then may indicate that the collision system is only partially thermalized. Following Refs. [2–4], we take a progressive step toward generating a formalism that utilizes two-particle correlations to quantify the level of thermalization of the collision system.

In Sec. 2, we introduce multiplicity-momentum correlations, $\mathcal{D}$, and outline their connection to three other two-particle correlation observables: multiplicity fluctuations, $\mathcal{R}$, transverse momentum correlations, $\mathcal{C}$, and net correlations of transverse momentum fluctuations, $\langle \delta p_{t1} \delta p_{t2} \rangle$. Multiplicity fluctuations, have been linked to centrality fluctuations and studied as a possible signal for Quark-Gluon Plasma (QGP) [5–20]. Transverse momentum correlations have been used to estimate the shear viscosity to entropy density ratio and the shear relaxation time of the collision medium [21–29]. Transverse momentum correlations, in the form of a covariance of two different particle's traverse momentum fluctuation away from the global average, have been examined as a signature of critical fluctuations associated with the phase change between QGP and hadronic matter; they have also been linked to event-by-event temperature fluctuations [30–43]. In past work, we argue that measured $\langle \delta p_{t1} \delta p_{t2} \rangle$ values result from initial state correlations modified by radial flow [44]. We also argue that these correlations can signal the level of thermalization reached by the collision medium [2, 45, 46]. Specifically, Ref. [2], is our starting point for using multiplicity-momentum correlations to quantify partial thermalization in the system.

In Sec. 3 we show some estimates of multiplicity-momentum correlations, as well as the other correlation observables, using the PYTHIA/Angantyr model [47, 48]. We find $\mathcal{D}$ to have a positive value with a magnitude on the same order as that of $\langle \delta p_{t1} \delta p_{t2} \rangle$. This leads us to explore the possibility of using $\mathcal{D}$ as a signature of partial thermalization in Sec. 4.

In Sec. 4, we discuss how observables $\langle \delta p_{t1} \delta p_{t2} \rangle$ and $\mathcal{D}$ can be written in terms of a "survival probability" that quantifies the difference between model expectations of correlations arising from locally equilibrated matter and from the unaltered initial state. To constrain estimates of the survival probability, we look to the four observables, $\mathcal{R}$, $\mathcal{C}$, $\mathcal{D}$, and $\langle \delta p_{t1} \delta p_{t2} \rangle$, for different dependencies on the survival probability. We advocate for simultaneous experimental measurement of the four observables to support this effort.

2 Two-Particle Multiplicity and Momentum Correlations

In this section we introduce multiplicity-momentum correlations that are observable as

$$\mathcal{D} = \frac{\left\langle \sum_{i=1}^{N_k} \sum_{j=1, j\neq i}^{N_k} \delta p_{t,i} \right\rangle}{\langle N\rangle^2} = \frac{\left\langle (N_k - 1) \sum_{i=1}^{N_k} \delta p_{t,i} \right\rangle}{\langle N\rangle^2}, \quad (1)$$

where

$$\delta p_{t,i} = p_{t,i} - \langle p_t\rangle. \quad (2)$$

Here the angled brackets represent an average over events. For any quantity X, the event average is defined as $\langle X\rangle = N_{events}^{-1} \sum_{k=1}^{N_{events}} X_k$. In (1), indices i, j for particles 1 and 2 run over N_k particles in event k. Then, $\langle N\rangle$ is the average number of particles per event, $\langle P_T\rangle$ is the average total transverse momentum per event, and $\langle p_t\rangle = \langle P_T\rangle / \langle N\rangle$ is the average transverse momentum per particle. With some algebra, one can show that (1) becomes

$$\mathcal{D} = \frac{Cov(P_T, N) - \langle p_t\rangle Var(N)}{\langle N\rangle^2}. \quad (3)$$

Here $Cov(P_T, N) = \langle P_T N\rangle - \langle P_T\rangle\langle N\rangle$ is the covariance of total transverse momentum and multiplicity per event and the multiplicity variance is $Var(N) = \langle N^2\rangle - \langle N\rangle^2$. For details see Ref. [1].

Equation (3) indicates that multiplicity-momentum correlations vanish if the only source of the correlation is simply due to multiplicity fluctuations. These trivial multiplicity-momentum correlations are subtracted by the last term on the right side of (3).

We show in Ref. [1] that $\mathcal{D}$ is zero in equilibrium in the Grand Canonical Ensemble. This suggests that non-zero values of $\mathcal{D}$ indicate that the collision system freezes out before it thermalizes. In Sec. 3 we make estimates of $\mathcal{D}$ in pp and AA collisions using the PYTHIA/Angantyr model and find that $\mathcal{D}$ is non-zero and positive. We propose that this positive value is consistent with the increase in $\langle p_t\rangle$ with multiplicity as seen in simulation and experiment.

Positive values of $\mathcal{D}$ suggest that multiplicity-momentum correlations may make a non-trivial contribution to other two-particle transverse momentum correlations that have been previously used to search for the QCD critical point and the onset of quark-gluon plasma. Following Ref. [1], we find the result

$$(1 + \mathcal{R})\langle\delta p_{t1}\delta p_{t2}\rangle - \mathcal{C} + 2\langle p_t\rangle\mathcal{D} + \langle p_t\rangle^2\mathcal{R} = 0, \quad (4)$$

where multiplicity fluctuations are defined by

$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}, \quad (5)$$

transverse momentum correlations are defined by

$$\mathcal{C} = \frac{\left\langle \sum_{i=1}^{N_k} \sum_{j\neq i}^{N_k} p_{t,i} p_{t,j} \right\rangle - \langle P_T\rangle^2}{\langle N\rangle^2}, \quad (6)$$

and the net correlation of transverse momentum fluctuations are

$$\langle\delta p_{t1}\delta p_{t2}\rangle = \frac{\left\langle \sum_{i=1}^{N_k} \sum_{j=1,j\neq i}^{N_k} \delta p_{t,i}\delta p_{t,j} \right\rangle}{\langle N(N-1)\rangle}. \quad (7)$$

To illustrate the importance of $\mathcal{D}$, examine Eq. (4) when solved for $\langle\delta p_{t1}\delta p_{t2}\rangle$,

$$\langle\delta p_{t1}\delta p_{t2}\rangle = \frac{\mathcal{C} - 2\langle p_t\rangle\mathcal{D} - \langle p_t\rangle^2\mathcal{R}}{(1+\mathcal{R})}. \quad (8)$$

The difference $\mathcal{C} - \langle p_t\rangle^2\mathcal{R}$ naively represents the construction of $\langle\delta p_{t1}\delta p_{t2}\rangle$ as transverse momentum correlations with multiplicity fluctuations removed. This assumption has been historically made when interpreting $\langle\delta p_{t1}\delta p_{t2}\rangle$ as event-by-event temperature fluctuations, and using it to search for critical fluctuations. Through Eq. (4), we now see that the definition of $\langle\delta p_{t1}\delta p_{t2}\rangle$ also removes multiplicity-momentum correlations. This supports the interpretation of as temperature fluctuations. Conversely, if one were to solve (4) for $\mathcal{C}$, one would find

$$\mathcal{C} = \langle p_t\rangle^2\mathcal{R} + 2\langle p_t\rangle\mathcal{D} + (1+\mathcal{R})\langle\delta p_{t1}\delta p_{t2}\rangle. \quad (9)$$

Here, we see that all of $\mathcal{R}$, $\mathcal{D}$, and $\langle\delta p_{t1}\delta p_{t2}\rangle$ contribute to transverse momentum correlations. This is significant since $\mathcal{R}$, $\mathcal{C}$, and $\langle\delta p_{t1}\delta p_{t2}\rangle$ have all been measured deferentially in relative pseudorapidity and relative azimuthal angle and show long-range "ridge" correlations. Thus far, $\mathcal{D}$ has not been measured, though we now see it is a key component to understanding ridge correlations.

Experiments can now use Eq. (4) as a validation tool by measuring (1), (5), (6), and (7) simultaneously. Additionally, any theoretical model that seeks to describe the ridge, or any individual observable appearing in (4), now has the additional constraint that it must describe all observables simultaneously as well.

3 Results from Simulation

The centrality dependence of two-particle correlations is potentially subject to a significant bias based on the method of determining centrality and on the contribution of multiplicity fluctuations to the observable [49]. To reduce this bias when determining centrality with multiplicity, we follow Ref. [43] and use the so-called "sub-group" method.

Observables are calculated using charged particles in the mid-rapidity $|\eta| < 0.5$ region and centrality is found using particles in the remaining experimental acceptance. This method allows for the use of one particle wide multiplicity bins but relies on the fact that the pseudorapidity distribution of particles is relatively flat in the entire experimental acceptance. For comparison to STAR, charged particles in the region $0.5 < |\eta| < 1.0$ are used for centrality and labelled N_{acc}. For comparison to ALICE, charged particles in the region $0.5 < |\eta| < 0.8$ are used for N_{acc}.

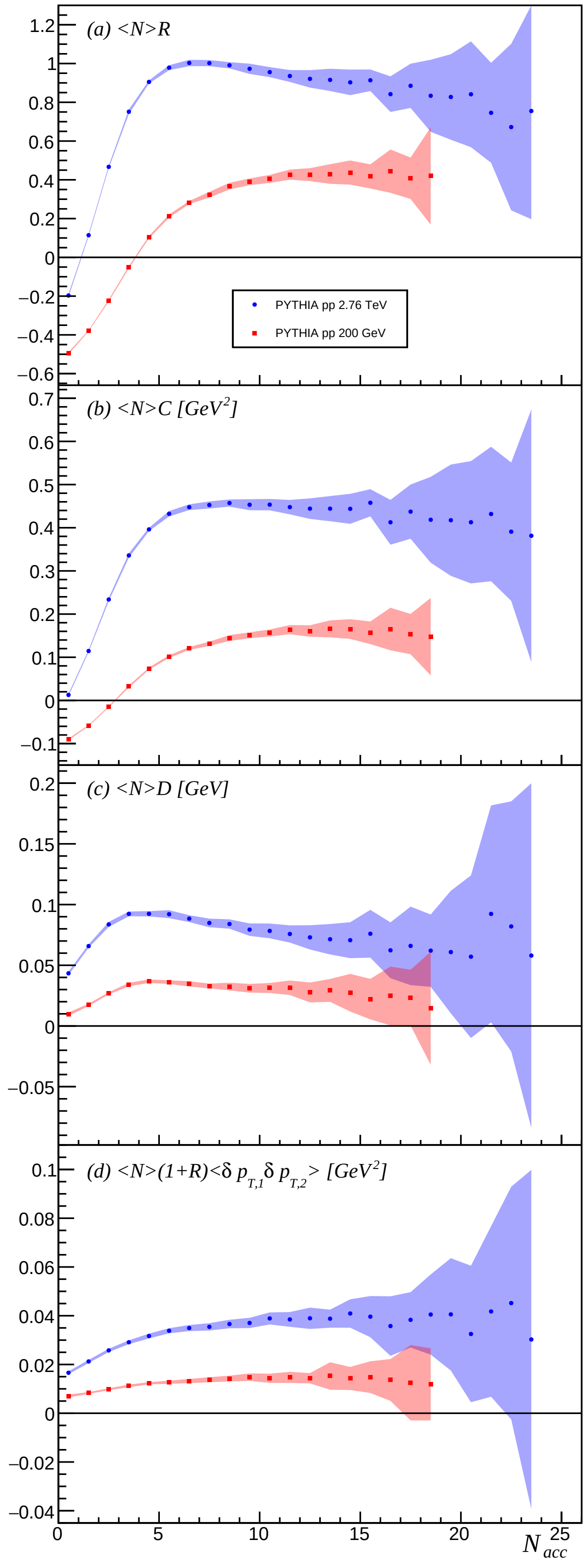

FIGURE 1. Calculation of observables (1), (5), (6), and (7) scaled by mid-rapidity multiplicity $\langle N\rangle$ using PYTHIA *pp* collisions.

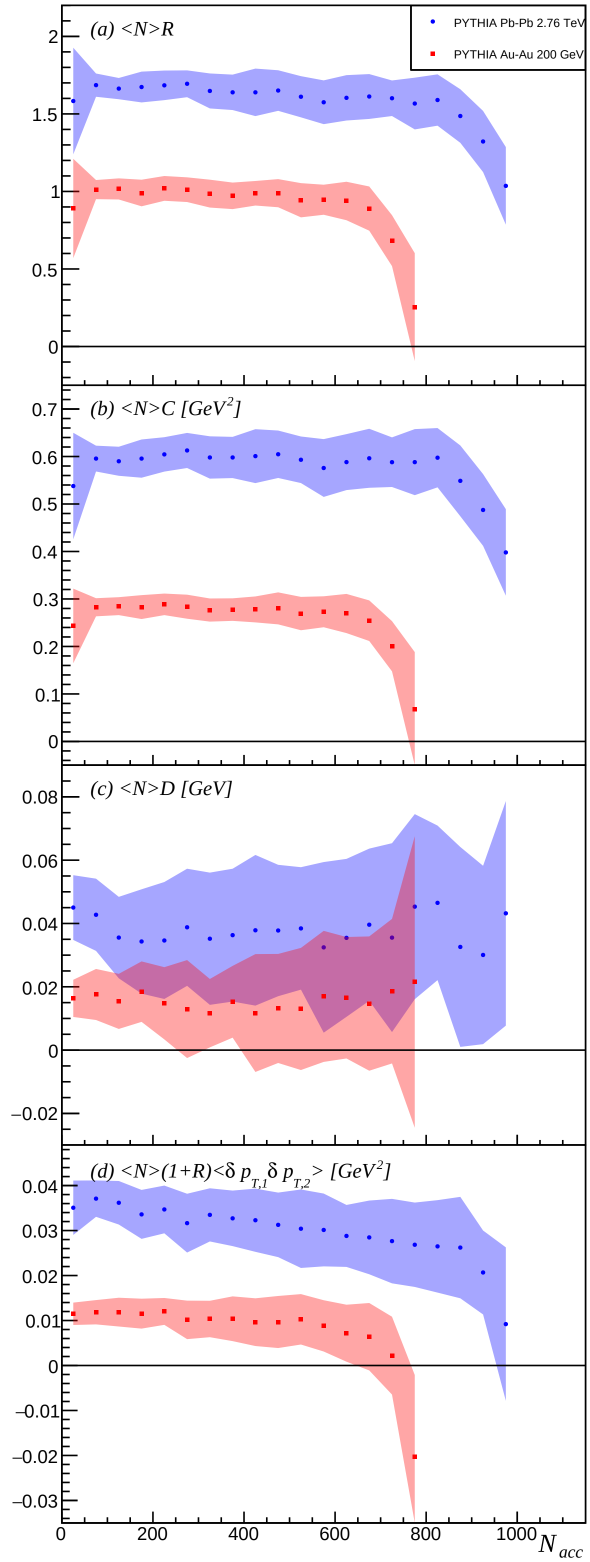

FIGURE 2. Calculation of observables (1), (5), (6), and (7) scaled by mid-rapidity multiplicity $\langle N\rangle$ using PYTHIA/Angantyr AA collisions.

For pp collision systems we calculate observables (1), (5), (6), (7), and the relation (4) in 30 sub-groups of events. We then calculate the sub-group average and standard deviation to represent the value and uncertainty of each observable. For AA collisions, we use the same method with the exception that we subsequently calculate the average and standard deviation of the values of every 50 multiplicity bins. This average is the reported value and the standard deviation determines the uncertainty band.

In Fig. 1, we plot (1), (5), (6), and (7) calculated with PYTHIA pp events. Each observable is scaled in a manner that should naively remove centrality dependence. For example, $\mathcal{R}$, (5), is expected to trend like $1/\langle N\rangle$ since $Var(N) \propto \langle N\rangle$ [1, 14]. Therefore, $\langle N\rangle\mathcal{R}$ should be flat with N_{acc}. Because of its different normalization, $\langle \delta p_{t1}\delta p_{t2}\rangle$ requires an extra factor of $(1+\mathcal{R}) = \langle N(N-1)\rangle/\langle N\rangle^2$ in order for it to match the $\mathcal{R}$, $\mathcal{C}$, and $\mathcal{D}$ multiplicity centrality dependence.

Importantly, if each event yields a particle multiplicity that is completely independent of every other event, the the multiplicity distribution would be Poisson, meaning $Var(N) = \langle N\rangle$, and $\mathcal{R} = 0$. Non-zero $\mathcal{R}$, indicates that different events are linked by the physics of the particle production mechanism. Similarly, if the total multiplicity of an event is small, $Var(N) \to 0$, then $\mathcal{R} \to -1/\langle N\rangle$. This explains the drop to negative values in Fig. 1(a). Similarly, these negative values exist in very peripheral AA collisions and this accounts for the lower value of the most peripheral point of Fig. 2(a). The transverse momentum correlation, $\mathcal{C}$, is the p_t weighted version of $\mathcal{R}$ and therefore has very similar centrality dependence in both Figs. 1 and 2.

Figures 1(c,d) and 2(c,d) show PYTHIA/Angantyr results for $\langle N\rangle\mathcal{D}$ and $\langle N\rangle(1+\mathcal{R})\langle\delta p_{t1}\delta p_{t2}\rangle$ in pp and AA collisions respectively. Notice that $\langle N\rangle\mathcal{D}$ is larger than $\langle N\rangle(1+\mathcal{R})\langle\delta p_{t1}\delta p_{t2}\rangle$ in all cases. We also calculate (4) for all systems and results are consistent with zero within a maximum numerical error of $|3|\times 10^{-17}$ for all collision systems. Both observables are relatively flat with N_{acc} which is consistent with $1/\langle N\rangle$ behavior. We look for deviations from this trend in mid-peripheral to central collisions to signal novel physics effects. As discussed in Sec. 2, the non-zero value of $\mathcal{D}$ may signal the level of partial thermalization of the system. In the next section, we briefly outline how the level of thermalization can be quantified by and $\mathcal{D}$.

4 Partial Thermalization

In Ref. [2], we show that the linearized Boltzmann equation, in the relaxation time approximation, can be modified with a random Brownian motion-like noise term that models stochastic changes in the phase space parton distribution $f(\mathbf{p},\mathbf{x},t)$. This equation can be solved using the method of characteristics to find

$$\begin{aligned} f &= f_0(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)S(\tau,\tau_0) \\ &+ f^e(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)(1-S(\tau,\tau_0)), \end{aligned} \tag{10}$$

where t is a function of the proper time τ in accord with $dt/d\tau = E/p\cdot u$. In (10), f_0 is the initial state phase space distribution at the formation time τ_0, and f^e represents what the equilibrium distribution would be given the values of the temperature, flow velocity, and chemical potential of the system at the time t. The quantity $\mathbf{x}-\mathbf{v_p}t$ accounts for how partons would “free-stream” given a drift velocity $\mathbf{v_p}$ that arises from the fluid cell momentum $\mathbf{p}$ at any phase space point.

The survival probability

$$S(\tau,\tau_0) = \exp\left\{-\int_{\tau_0}^{\tau}\nu(\tau')d\tau'\right\}, \tag{11}$$

indicates the probability that particles escape the collision volume without suffering any collisions. A vaue of $S=1$ represents free-streaming particles and as thermalization proceeds $S\to 0$. In Eq. (11), ν^{-1} is the characteristic relaxation time that dictates the rate at which $f\to f^e$.

To apply this result to two-particle correlations we define the correlation function

$$G_{12} = \langle f_1 f_2\rangle - \langle f_1\rangle\langle f_2\rangle - \langle f_1\rangle\delta(1-2) \tag{12}$$

for phase space distributions f_1, and f_2 of particles 1 and 2. Here we define $\delta(1-2) \equiv \delta(\mathbf{x}_1-\mathbf{x}_2)\delta(\mathbf{p}_1-\mathbf{p}_2)$ so that the quantity $\langle f_1 f_2\rangle - \langle f_1\rangle\delta(1-2)$ is the phase space density of particle pairs with auto-correlations removed. In the absence of any correlations, the pair density is equal to $\langle f_1\rangle\langle f_2\rangle$, which would result in $G_{12}=0$.

In Ref. [2], we derive a differential equation for the evolution of G_{12} based on the Boltzmann-Langevin formulation and Itô calculus. Again we use the method of characteristics to find the solution which takes the form

$$G_{12} = G^e_{12} + (X^0_{12} + X^0_{21})S + \Delta G^0_{12}S^2. \tag{13}$$

Defining the fluctuation of the phase space distribution away from the equilibrium distribution to be $\delta f = f - f^e$, the mixed correlation function X_{12} is the covariance $\langle\delta f_1 f^e_2\rangle - \langle\delta f_1\rangle f^e_2$, and $\Delta G = \langle\delta f_1\delta f_2\rangle - \langle\delta f_1\rangle\langle\delta f_2\rangle - \langle\delta f_1\rangle\delta(1-2)$. In Eq. (13), terms weighted by the survival probability are determined by initial conditions and the local equilibrium function has arguments $G^e_{12} = G^e(\mathbf{p}_1,\mathbf{x}_1-\mathbf{v}_{\mathbf{p}_1}t,\mathbf{p}_2,\mathbf{x}_2-\mathbf{v}_{\mathbf{p}_2}t)$.

By integrating appropriate moments of (13), we further show in Ref. [2] that net correlations of transverse momentum fluctuations $\langle\delta p_{t1}\delta p_{t2}\rangle$, depends on the survival probability (11). We find

$$\langle\delta p_{t1}\delta p_{t2}\rangle = \langle\delta p_{t1}\delta p_{t2}\rangle_0 S^2 + \langle\delta p_{t1}\delta p_{t2}\rangle_e(1-S^2). \tag{14}$$

Notice that (14) depends on S^2.

We seek to determine a method for extracting S from experiment. Since both $\langle\delta p_{t1}\delta p_{t2}\rangle_0$ and $\langle\delta p_{t1}\delta p_{t2}\rangle_e$ are model dependent, there is much uncertainty in using (14) with experimental data to determine S. However, given that we expect $\mathcal{D}$ is non-zero, we can calculate the dependence of $\mathcal{D}$ on the survival probability. Then, by measuring $\langle\delta p_{t1}\delta p_{t2}\rangle$ and $\mathcal{D}$ in the same experiment, model definitions of initial state correlations and equilibrium correlations must simultaneously match the two observables and yield the same value of S.

Following Refs. [3,4], we calculate

$$\mathcal{D} = \mathcal{D}_e + \int \delta p_{t1} \frac{\langle G_{12} - G_{12}^e \rangle}{\langle N \rangle^2} d\omega_1 d\omega_2, \tag{15}$$

where we abbreviate the differential phase space elements $d\omega = d\mathbf{x}d\mathbf{p}$, and the spatial integrals are on the Cooper-Frye freeze-out surface with $dx = p^\mu d\sigma_\mu / E$ [50]. We then find

$$\mathcal{D} = \mathcal{D}_o S + \mathcal{D}_e (1 - S). \tag{16}$$

Notice that (16) depends on only a single power of S where (14) depends on S^2. Therefore, experimental measurement of $\mathcal{D}$ will provide important constraints on our extraction of values of the survival probability. In future work we will investigate the S dependence of all observables (1), (5), (6), and (7). We advocate for simultaneous measurement of these observables – under the same conditions – to facilitate our estimates of the level of equilibration in relativistic nuclear collision systems.

5 Discussion and Summary

In this work we introduce multiplicity-momentum correlations, $\mathcal{D}$, Eq. (3), a relatively new observable that we first introduced in Ref. [1]. Our early estimates of $\mathcal{D}$, using PYTHIA/Angantyr, indicate a positive value similar in magnitude and larger than the related (and much studied) observable $\langle \delta p_{t1} \delta p_{t2} \rangle$. We have shown in Ref. [1] that one can expect $\mathcal{D}$ to vanish in equilibrium in the Grand Canonical Ensemble. Therefore, non-zero values of $\mathcal{D}$ may indicate that the collision system does not fully reach local thermal equilibrium before freeze-out.

Many hydrodynamic and other models rely either directly on the assumption of local equilibration or indirectly by using equations of state taken from lattice QCD calculations that, themselves, assume local equilibrium. If the experimental system never reaches this local equilibrium state, then these model comparisons to data may result in misinterpretations of the values of transport coefficients like viscosity and relaxation times.

To address this issue, we briefly outline a formalism for using two-particle correlation observables to quantify the level of partial thermalization of the system. We follow our earlier work, Ref. [2], where we showed that $\langle \delta p_{t1} \delta p_{t2} \rangle$ can be written in terms of a "survival probability" S, where S is the probability that a particle suffers no collisions in the evolution of the medium. Therefore $S = 1$ in the case where the initial state free-streams to the detector, and $S = 0$ in the case where the collision medium reaches local thermal equilibrium. This survival probability can be used as a quantitative measure of partial thermalization.

With estimates of the initial state correlations and models that include local equilibrium dynamics, one can use the result (14) in comparison to experimental data to extract the survival probability. This leads to a new problem that an extra parameter is available in the theory to facilitate fits to data. We propose a solution to this issue issue by looking for simultaneous comparisons to multiple observables with different dependencies on the survival probability. In (14), only powers of S^2 exist. We reproduce our methods from Ref. [1] using multiplicity-momentum correlations and find Eq. (16), which only depends on single powers of S. We advocate for simultaneous measurement of $\mathcal{D}$ and $\langle \delta p_{t1} \delta p_{t2} \rangle$ to provide a data set that can constrain our theoretical extraction of S.

To strengthen our approach, we define a mathematical relationship (4) between four two-particle correlation observables, (1), (5), (6), and (7). When all observables are measured simultaneously, (4) not only provides an experimental validation of results, but this set of four observables pose a challenge for any model to explain them simultaneously. Our motivation is to provide additional constraints on our extraction of S. In ongoing work, we determine the S dependence of all four variables [3,4].

6 Acknowledgements

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