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New limits on lepton flavour violating processes in the Littlest Higgs model with T-parity

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By understanding neutrino masses via an inverse seesaw mechanism, the Littlest Higgs model with T parity is enlarged. Promising predictions for lepton flavor violating processes within this setting (see Refs. [3,4]) are reviewed.

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1. Introduction

The SM theory successfully describes processes of all known elementary particles, but it faces a significant challenge due to the divergent radiative corrections arising from interactions with SM fields that affect the Higgs mass. Many models have been proposed over time to address this issue. One of such is the Little Higgs model [3,4], which explains the little hierarchy between the Higgs mass $M_h(\approx \upsilon = 246$ GeV) and the expected NP scale $f \sim 1$ TeV [5–7], with the Higgs boson being a pseudo-Nambu Goldstone boson of a spontaneously broken global symmetry. The Littlest Higgs model with T parity (LHT) [3,4,8–12] is among the most promising frameworks within this set of models.

2. The model

The Littlest Higgs with T-parity (LHT) is a non-linear σ model built out of the coset space $SU(5)/SO(5)$. This model is grounded on a global $SU(5)$ symmetry, yielding the existence of 14 Nambu-Goldstone bosons. Nonetheless, the global symmetry is disrupted by the vacuum expectation value ($\sim \mathcal{O}$(TeV)), represented by a 5×5 symmetric tensor [13,14]

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbb{I} \\ 0 & 1 & 0 \\ \mathbb{I} & 0 & 0 \end{pmatrix}. \qquad (1)$$

The gauge group is taken to be $G_1 \times G_2 = [SU(2)\times U(1)]^2$, subgroup of the $SU(5)$ global symmetry (see Fig. 1).

$$\begin{array}{ccccc} SU(5) & \xrightarrow{f} & SO(5) & & \\ \bigcup & & \bigcup & & \\ [SU(2)\times U(1)]^2 & \xrightarrow{f} & SU(2)_w \times U(1)_Y & \xrightarrow{v_h} & U(1)_{\text{EM}} \end{array}$$

FIGURE 1. The global SU(5) contains two copies of local $[SU(2)\times U(1)]^2$ that are diagonally broken to one $SU(2)\times U(1)$, within $SO(5)$.

The symmetry breaking $[SU(2)\times U(1)]^2 \to SU(2)_W \times U(1)_Y$ occurs at the energy scale f, which is in the ballpark of some TeVs.

T-parity is a fundamental symmetry in the majority of little Higgs models that rely on a product group structure. Its remarkable feature is that it ensures the Standard Model (SM) particles are even under this symmetry (T-even), while the new particles that exist at the TeV scale are odd (T-odd). The application of T-parity leads to a direct prohibition of any tree-level contributions from the heavy gauge bosons to observables that involve only SM particles as external states. Consequently, corrections to Electroweak Precision Observables (EWPO) start at loop level [13,15–38].

A natural action of T-parity on the gauge fields is

$$G_1 \leftrightarrow G_2, \qquad (2)$$

exchanging both gauge groups $SU(2)_i \times U(1)_i$. Then, T invariance imposes that the gauge couplings associated to both factors be equal

$$g_1 = g_2 = \sqrt{2} g_W, \qquad g_1' = g_2' = \sqrt{2} g', \qquad (3)$$

where g_W is the $SU(2)_W$ coupling constant and g' is the $U(1)_Y$ coupling constant.

The Scalar Lagrangian for the light and heavy gauge bosons sector is

$$\begin{aligned} \mathcal{L}_S = &\frac{1}{2}\left[\frac{g_W^2 \upsilon_h^2}{4}\left(1-\frac{\upsilon_h^2}{6f^2}\right)\right] W_L^+ W_L^- \\ &+ \frac{1}{2}\left[f^2 g_W^2\left(1-\frac{\upsilon_h^2}{4f^2}\right)\right] W_H^+ W_H^- \\ &+ \frac{1}{2}\left[f^2 g_W^2\left(1-\frac{\upsilon_h^2}{4f^2}\right)\right] (Z_H)^2 \\ &+ \frac{1}{2}\left[\frac{f^2 g'^2}{5}\left(1-\frac{5\upsilon_h^2}{4f^2}\right)\right] (A_H)^2 \\ &+ \frac{1}{2}\left[\frac{g_W^2 \upsilon_h^2}{4\cos^2\theta_W}\left(1-\frac{\upsilon_h^2}{6f^2}\right)\right] Z_L^2. \end{aligned} \qquad (4)$$

There are no mixings between SM and heavy bosons, as expected.

In the gauge sector, before electroweak symmetry breaking (EWSB), the SM (light) gauge bosons are W_L^a and B_L, which are massless and T-even, while the massive heavy gauge bosons (T-odd) are W_H^a and B_H. After high energy symmetry breaking, their masses are [16]

$$M_{W_H^a} = g_W f, \qquad M_{B_H} = \frac{g' f}{\sqrt{5}}, \tag{5}$$

where $e = g_W s_W = g' c_W$.

After EWSB, the light gauge sector includes the $W_L^{\pm}$, Z_L, and A_L bosons, corresponding to the SM gauge bosons with masses [13, 16] (ρ factor is conserved)

$$\begin{aligned} M_{W_L^{\pm}} &= \frac{g_W \upsilon_h}{2}\left(1-\frac{\upsilon_h^2}{6f^2}\right)^{1/2} \approx \frac{g_W \upsilon_h}{2}\left(1-\frac{\upsilon_h^2}{12f^2}\right), \\ M_{Z_L} &= \frac{g_W \upsilon_h}{2\cos\theta_H}\left(1-\frac{\upsilon_h^2}{6f^2}\right)^{1/2} = \frac{M_{W_L^{\pm}}}{\cos\theta_W}, \\ M_{A_L} &= 0. \end{aligned} \tag{6}$$

Heavy bosons masses are [13, 16]

$$\begin{aligned} M_{W_H^{\pm}} &= M_{Z_H} = f g_W \left(1-\frac{\upsilon_h^2}{4f^2}\right)^{1/2} \approx f g_W \left(1-\frac{\upsilon_h^2}{8f^2}\right), \\ M_{A_H} &= \frac{f g'}{\sqrt{5}}\left(1-\frac{5\upsilon_h^2}{4f^2}\right)^{1/2} \approx \frac{f g'}{\sqrt{5}}\left(1-\frac{5\upsilon_h^2}{8f^2}\right). \end{aligned} \tag{7}$$

We concentrate in the following on the fermion sector. Therein, we introduce a right-handed $SO(5)$ multiplet Ψ_R [22] and incorporate two $SU(5)$ incomplete quintuplets $\Psi_{1,2}$

$$\Psi_1 = \begin{pmatrix} i\psi_1 \\ 0 \\ 0 \end{pmatrix}, \ \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ i\psi_2 \end{pmatrix}, \ \Psi_R = \begin{pmatrix} \tilde{\psi}_R^c \\ \chi_R \\ \psi_R \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} \psi_i &= -\sigma^2 \begin{pmatrix} \nu_{L_i} \\ \ell_{L_i} \end{pmatrix} \qquad (i=1,2), \\ \psi_R &= -i\sigma^2 \begin{pmatrix} \nu_{HR} \\ \ell_{HR} \end{pmatrix}. \end{aligned} \tag{9}$$

For each SM lepton doublet, two doublets $\psi_{1(2)}$ have been introduced. T-parity is defined to act on the left-handed (LH) leptons as

$$\Psi_1 \longleftrightarrow \Omega \Sigma_0 \Psi_2, \tag{10}$$

with

$$\begin{aligned} \Omega &= \mathrm{diag}(-1,-1,1,-1,-1), \\ \Sigma_0 &= \begin{pmatrix} 0 & 0 & \mathbf{1}_{2\times2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times2} & 0 & 0 \end{pmatrix}. \end{aligned} \tag{11}$$

Mirror (T-odd) and partner leptons obtain $\mathcal{O}(f)$ masses by means of [37]

$$\begin{aligned} \mathcal{L}_{Y_H} &= -\kappa f \left(\overline{\Psi}_2 \xi + \overline{\Psi}_1 \Sigma_0 \xi^{\dagger}\right) \Psi_R \\ &\quad + \kappa_2 \tilde{\Psi}_R^T \Psi_R + \text{h.c.}, \end{aligned} \tag{12}$$

where $\xi = e^{i\Pi/f}$, being Π the Goldstone bosons matrix, and κ_2 is a Dirac mass. T-odd leptons thus acquire masses after EWSB, given by [17, 22]

$$m_{\ell_H^i} = \sqrt{2}\kappa_{ii} f \equiv m_{Hi}, \quad m_{\nu_H^i} = m_{Hi}\left(1-\frac{\upsilon^2}{8f^2}\right), \tag{13}$$

where κ_{ii} are the mass matrix κ eigenvalues. Mass eigenstates are defined as the T-odd combination, namely

$$\psi_H = \frac{1}{\sqrt{2}}\left(\psi_1 + \psi_2\right), \tag{14}$$

while the T-even combination is

$$\psi_{SM} = \frac{1}{\sqrt{2}}\left(\psi_1 - \psi_2\right). \tag{15}$$

The latter is massless until EWSB takes place.

The same occurs for T-odd quarks masses, where κ_{ii}^q is used instead of κ_{ii} and d_H-quark is replaced with ℓ_H and u_H-quark is changed by ν_H. Additionally, κ_2^q is replaced with κ_2 in the partner quarks mass matrix, of u and d types.

3. Neutrino masses in the LHT and new contributions to LFV processes

We have incorporated Majorana neutrinos with an inverse seesaw mechanism to the LHT model. Its main aspects are shown next (see [37, 39] for details).

We recall that the T-odd fermions are given $\mathcal{O}(f)$ masses by the Lagrangian [13, 37]

$$\begin{aligned} \mathcal{L}_{Y_H} &= -\kappa f \left(\overline{\Psi}_2 \xi + \overline{\Psi}_1 \Sigma_0 \xi^{\dagger}\right) \Psi_R \\ &\quad + h.c. \approx \sqrt{2}\kappa f \, \overline{\psi}_{HL} \psi_{HR}, \end{aligned} \tag{16}$$

where $\xi = \exp(i\Pi/f) \approx \mathbb{I}$. Furthermore, concerning Ψ_R:

$$\Psi_R = \begin{pmatrix} \psi_R^c \\ \chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad \Psi_R \xrightarrow{T} \Omega \Psi_R, \tag{17}$$

where the lepton singlets are denoted by χ_R, and have a large vector-like mass. This one comes from combining them with a LH singlet χ_L through a direct mass term without new couplings to the Higgs. Then, χ_R mass term is

$$\mathcal{L}_M = -M \overline{\chi}_L \chi_R + h.c.. \tag{18}$$

Since χ_L is $SU(5)$ singlet, a small Majorana mass for it is permitted. We assume that lepton number breaks by small Majorana masses (μ) in the heavy LH neutral sector. Consequently, we have:

$$\mathcal{L}_\mu = -\frac{\mu}{2} \overline{\chi_L^c} \chi_L + h.c.. \tag{19}$$

The resulting (T-even) neutrino mass matrix reduces to the inverse see-saw one [37]

$$\mathcal{L}_M^{\nu} = -\frac{1}{2}\left(\overline{\nu_l^c}\ \overline{\chi_R}\ \overline{\chi_L^c}\right) \mathcal{M}_\nu^{T-even} \begin{pmatrix} \nu_L \\ \chi_R^c \\ \chi_L \end{pmatrix} + h.c., \tag{20}$$

where

$$\mathcal{M}_\nu^{T-even} = \begin{pmatrix} 0 & i\kappa^* f \sin\left(\frac{\upsilon}{\sqrt{2}f}\right) & 0 \\ i\kappa^\dagger f \sin\left(\frac{\upsilon}{\sqrt{2}f}\right) & 0 & M^\dagger \\ 0 & M^* & \mu \end{pmatrix}, \tag{21}$$

with each entry a (3×3) matrix including the 3 lepton families. The κ entries are given by the Yukawa Lagrangian in Eq. (16) and M stands for the direct heavy Dirac mass matrix from Eq. (18). Finally, μ is the mass matrix of small Majorana masses in Eq. (19).

Taking into account the inverse see-saw hierarchy $\mu \ll \kappa f \ll M$, mass eigenvalues for M are $\sim$ 10 TeV, of the same order as $4\pi f$, where f is in the TeV range. This is necessary to satisfy the current Electroweak Precision Data (EWPD) assuming the κ eigenvalues of order 1 (we have considered lighter, $\mathcal{O}$(4 TeV), Majorana neutrino masses in [40]). In contrast, the μ eigenvalues are expected to be significantly smaller, likely in the GeV range.

After diagonalizing the (Majorana) mass matrix, (21), the Lagrangian reads

$$\mathcal{L}_M^\nu = -\frac{1}{2}\left(\sum_{i=1}^{3}(\mathcal{M}_\nu^l)_i \overline{\nu_{Li}^l}\nu_{Ri}^l + \sum_{j=4}^{9}(\mathcal{M}_\chi^h)_j \overline{\Psi_{Lj}^h}\Psi_{Rj}^h\right), \tag{22}$$

with $\mathcal{M}_\nu^l$ a (3×3) matrix and $\mathcal{M}_\chi^h$ a (6×6) matrix.

Applying the $(\mu \ll \kappa f \ll M)$ hierarchy, the eigenstates of light and heavy Majorana neutrinos transform as [37]

$$\sum_{j=1}^{3} U_{ij}\nu_{Lj}^l = \sum_{j=1}^{3}[\mathbf{1}_{3\times3} - \frac{1}{2}(\theta\theta^\dagger)]_{ij}\nu_{Lj} - \sum_{j=7}^{9}\theta_{ij}\chi_{Lj},$$

$$\chi_{Li}^h = \sum_{j=7}^{9}[\mathbf{1}_{3\times3} - \frac{1}{2}(\theta^\dagger\theta)]_{ij}\chi_{Lj} + \sum_{j=1}^{3}\theta_{ij}^\dagger\nu_{Lj}, \tag{23}$$

where θ mixes light and heavy Majorana neutrinos,

$$\theta = -if \sin\left(\frac{\upsilon}{\sqrt{2}f}\right)\kappa M^{-1}, \tag{24}$$

with U denoting the $U_{\rm PMNS}$ matrix. Therefore, the mass matrix $\mathcal{M}_\nu^l$ for light (active) neutrinos is

$$(\mathcal{M}_\nu^l)_{ij} = \theta_{ik}^*\mu_{kl}\theta_{jl}^\dagger. \tag{25}$$

The weak charged-current SM Lagrangians are modified to:

$$\mathcal{L}_W^l = \frac{g}{\sqrt{2}}W_\mu^+ \sum_{j=1}^{3}\sum_{i=1}^{3}\overline{\nu_i^l}W_{ij}\gamma^\mu P_L \ell_j + h.c.,$$

$$\mathcal{L}_W^{lh} = \frac{g}{\sqrt{2}}W_\mu^+ \sum_{j=1}^{3}\sum_{i=7}^{9}\overline{\chi_i^h}\theta_{ij}^\dagger\gamma^\mu P_L \ell_j + h.c., \tag{26}$$

where the W matrix has been defined as

$$W_{ij} = \left\{U^\dagger\left[\mathbf{1}_{3\times3} - \frac{1}{2}(\Theta\Theta^\dagger)\right]\right\}_{ij}. \tag{27}$$

And the SM neutral currents become

$$\mathcal{L}_Z^l = \frac{g}{2\cos\theta_W}Z_\mu \sum_{i,j=1}^{3}\overline{\nu_i^l}\gamma^\mu(X_{ij}P_L - X_{ij}^\dagger P_R)\nu_j^l,$$

$$\mathcal{L}_Z^{lh} = \frac{g}{2\cos\theta_W}Z_\mu \sum_{i,j=1}^{3}\overline{\chi_i^h}\gamma^\mu(Y_{ij}P_L - Y_{ij}^\dagger P_R)\nu_j^l + h.c.,$$

$$\mathcal{L}_Z^h = \frac{g}{2\cos\theta_W}Z_\mu \sum_{i,j=1}^{3}\overline{\chi_i^h}\gamma^\mu(S_{ij}P_L - S_{ij}^\dagger P_R)\chi_j^h, \tag{28}$$

whose neutral couplings turn out to be

$$X_{ij} = \sum_{k=1}^{3}\left(U^\dagger[\mathbf{1}_{3\times3} - (\theta\theta^\dagger)]\right)_{ik}U_{kj},$$

$$Y_{ij} = \sum_{k=1}^{3}\theta_{ik}^\dagger U_{kj}, \quad S_{ij} = \sum_{k=1}^{3}\theta_{ik}^\dagger\theta_{kj}. \tag{29}$$

Particle content of this enlarged LHT model is summarized in Table II.

4. Bounds on LFV processes

We present various LFV processes here: $\ell \to \ell'\gamma$ and $\ell \to \ell'\ell''\bar{\ell}'''$ (with all possible channels shown in Table I).

All of them involve the effective interaction of a neutral vector boson with a pair of on-shell fermions, through a loop with neutrinos (heavy/Majorana contributions are non-negligible).

Particle Content of LHT with Majorana neutrinos	
Nambu–Goldstone bosons	
SM Higgs	$\left(-i\pi^+/\sqrt{2},\ (\upsilon + h + i\pi^0)/2\right)$
Longitudinal modes of the heavy gauge fields	$\omega^\pm, \omega^0, \eta$
Complex $SU(2)_L$ triplet	$\begin{pmatrix} i\Phi^{--} & i\Phi^-/\sqrt{2} \\ i\Phi^-/\sqrt{2} & (i\Phi^0 + \Phi^P)/\sqrt{2} \end{pmatrix}$
Gauge bosons	
SM gauge bosons (T-even)	$\{W_L^\pm, Z_L, \gamma\}$
Heavy gauge bosons (T-odd)	$\{W_H^\pm, Z_H, A_H\}$
Fermions (i = 1,2,3)	
SM Fermions (T-even)	$\{\ell_L^i, \nu_L^i, u_L^i, d_L^i\}$
Mirror/Heavy/T-odd Fermions	$\{\ell_H^i, \nu_H^i, u_H^i, d_H^i\}$
Partner Fermions	$\{\ell_i^c, \nu_i^c, u_i^c, d_i^c\}$
Majorana neutrinos $(i = 1, 2, 3)$	
Heavy Majorana neutrinos	χ_{Li}^h

FIGURE 2. The full content of particles of LHT with Majorana neutrinos.

TABLE I. Three different types of decay channels in the $\ell \to \ell'\ell''\bar{\ell}'''$ processes.

Type	Flavor	$\ell \to \ell'\ell''\bar{\ell}'''$		
1	$\ell \neq \ell' = \ell'' = \ell'''$	$\mu \to ee\bar{e}$	$\tau \to ee\bar{e}$	$\tau \to \mu\mu\bar{\mu}$
2	$\ell \neq \ell' \neq \ell'' = \ell'''$		$\tau \to e\mu\bar{\mu}$	$\tau \to \mu e\bar{e}$
3	$\ell \neq \ell' = \ell'' \neq \ell'''$		$\tau \to ee\bar{\mu}$	$\tau \to \mu\mu\bar{e}$

4.1. $\ell \to \ell'\gamma$ decays

The $\mu \to e\gamma$ branching ratio considering active and heavy (Majorana) neutrinos is

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\left|\theta_{ej}\theta^{\dagger}_{\mu j}F^{\chi}_{M}(x) + W_{ej}W^{\dagger}_{\mu j}F^{\nu}_{M}(y)\right|^2, \quad (30)$$

with $x = M_W^2/M_j^2$, $y = m^2_{\nu_i}/M_W^2$ and W_{ij} given by the Eq. (27).

Noteworthy, the ν_H contribution to $\mathrm{Br}(\mu \to e\gamma)$ is

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\left|\frac{\upsilon^2}{4f^2}V^{ie*}_{H\ell}V^{i\mu}_{H\ell}F_W(x)\right|^2, \quad (31)$$

which is neglected because of $\upsilon^2/4f^2 << 1$.

Finally, we get

$$\mathrm{Br}(\mu \to e\gamma) \approx \frac{3\alpha}{8\pi}\left|\theta_{ej}\theta^{\dagger}_{\mu j}\right|^2. \quad (32)$$

$Br(\mu \to e\gamma) < 4.2 \times 10^{-13}$, $\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$, $\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ (at 90% C.L.) [41,42] bind

$$|\theta_{ej}\theta^{\dagger}_{\mu j}| < 0.14 \times 10^{-4}, \quad |\theta_{ej}\theta^{\dagger}_{\tau j}| < 0.95 \times 10^{-2},$$
$$|\theta_{\mu j}\theta^{\dagger}_{\tau j}| < 0.011. \quad (33)$$

Our result for $|\theta_{ej}\theta^{\dagger}_{\mu j}|$ matches the one in Ref. [37] but the results for $|\theta_{ej}\theta^{\dagger}_{\tau j}|$ and $|\theta_{\mu j}\theta^{\dagger}_{\tau j}|$ do not. For tau decays, a ~ 0.17 (accounting for its semileptonic decays) factor is missing in that reference.

5. Limits on LFV processes driven by heavy Majorana neutrinos

5.1. Global analysis

In this section, we will delve into a comprehensive analysis of 10 processes: LFV Z decays $Z \to \bar{\mu}e$, $Z \to \bar{\tau}e$, and $Z \to \bar{\tau}\mu$; LFV Type I $\mu \to ee\bar{e}$, $\tau \to ee\bar{e}$ and $\tau \to \mu\mu\bar{\mu}$; LFV Type II $\tau \to e\mu\bar{\mu}$ and $\tau \to \mu e\bar{e}$; $\mu - e$ conversion in nuclei $^{48}_{22}$Ti and $^{197}_{79}$Au, with $\mathcal{O}$(TeV) Majorana neutrinos[i].

To conduct the analysis, we employed a state-of-the-art simulation tool that runs all 10 processes concurrently. One of the key features of these events is that they share the same free parameters: three heavy neutrino masses $\mathrm{M_i}$ with $i = 1, 2, 3$ and the neutral couplings given by $(\theta S\theta^{\dagger})$ matrices.

TABLE II. Mean values for branching ratios, conversion rates and three heavy neutrino masses compared to current upper limits.

LFV Z decays	Our mean values	Present limits [PDG]
$\mathrm{Br}(Z \to \bar{\mu}e)$	1.20×10^{-14}	3.7×10^{-7}
$\mathrm{Br}(Z \to \bar{\tau}e)$	1.46×10^{-8}	4.9×10^{-6}
$\mathrm{Br}(Z \to \bar{\tau}\mu)$	1.09×10^{-8}	0.6×10^{-5}
LFV Type I		
$\mathrm{Br}(\mu \to ee\bar{e})$	1.85×10^{-14}	1.0×10^{-12}
$\mathrm{Br}(\tau \to ee\bar{e})$	4.16×10^{-9}	2.7×10^{-8}
$\mathrm{Br}(\tau \to \mu\mu\bar{\mu})$	4.24×10^{-9}	2.1×10^{-8}
LFV Type II		
$\mathrm{Br}(\tau \to e\mu\bar{\mu})$	3.60×10^{-9}	2.7×10^{-8}
$\mathrm{Br}(\tau \to \mu e\bar{e})$	2.48×10^{-9}	1.8×10^{-8}
$\mu - e$ conversion rate		
$\mathcal{R}$(Ti)	6.21×10^{-14}	4.3×10^{-13}
$\mathcal{R}$(Au)	7.82×10^{-14}	7.0×10^{-12}
Heavy neutrino masses		
$\mathrm{M_1}$ (TeV)	17.186	
$\mathrm{M_2}$ (TeV)	17.185	
$\mathrm{M_3}$ (TeV)	17.187	

After conducting numerous simulations, we decided to focus on a mass range of 15 to 20 TeV for M_i, as this interval provides the most solutions respecting current upper limits.

Our analysis (see Table II) revealed that the form factors of these events are influenced by both the charged $(\theta\theta^{\dagger})$ and the neutral couplings $(\theta S\theta^{\dagger})$, with non-negligible interference between them.

The modulus of the $(\theta S\theta^{\dagger})_{e\mu}$ elements are all smaller than 7.5×10^{-10}, while for the other flavor combinations we get $|(\theta S\theta^{\dagger})_{e\tau}| < 5.13 \times 10^{-7}$ and $|(\theta S\theta^{\dagger})_{\mu\tau}| < 6.2 \times 10^{-7}$.

6. Bounds for wrong sign processes

We study here two tau decays known as wrong-sign processes: $\tau \to ee\bar{\mu}$ and $\tau \to \mu\mu\bar{e}$. This analysis is done by a Monte Carlo simulation where both processes are computed simultaneously assuming that the LNV couplings are free parameters, which we bind. Our nine free parameters for each process are: the masses of heavy neutrinos, $\mathrm{M_i}$, and the LNV couplings corresponding to each decay flavor transition.

We restrict the couplings as follows [46]:

$$|\theta_{\mu 1}\theta_{\tau 1}| + |\theta_{\mu 2}\theta_{\tau 2}| + |\theta_{\mu 3}\theta_{\tau 3}| < 0.32 \times 10^{-3},$$
$$|\theta_{e1}\theta_{e1}| + |\theta_{e2}\theta_{e2}| + |\theta_{e3}\theta_{e3}| < 0.01, \quad (34)$$

from the equations above, we limit each term

$$-0.32 \times 10^{-3} \leq (\theta_{\mu i}\theta_{\tau i})^{\dagger} \leq 0.32 \times 10^{-3}, \quad (35)$$

$$-0.01 \leq (\theta_{ei}\theta_{ei}) \leq 0.01, \quad (36)$$

TABLE III. Mean values for the free parameters and branching ratios in the wrong sign processes considering Majorana neutrinos.

Branching Ratios	Our mean values
$\mathrm{Br}(\tau \to ee\bar{\mu})$	1.8×10^{-9}
$\mathrm{Br}(\tau \to \mu\mu\bar{e})$	1.9×10^{-9}
Heavy neutrino masses	
M_1 (TeV)	17.170
M_2 (TeV)	17.166
M_3 (TeV)	17.166
LNV couplings	
$(\theta_{e1}\theta_{\tau1})^\dagger$	$(2.24 \pm 9.59) \times 10^{-7}$
$(\theta_{e2}\theta_{\tau2})^\dagger$	$(14.82 \pm 9.69) \times 10^{-7}$
$(\theta_{e3}\theta_{\tau3})^\dagger$	$(1.84 \pm 9.79) \times 10^{-7}$
$\lvert\theta_{ei}\theta_{\tau i}\rvert$	2.76×10^{-4}
$(\theta_{\mu1}\theta_{\mu1})$	$(0.75 \pm 2.98) \times 10^{-5}$
$(\theta_{\mu2}\theta_{\mu2})$	$-(8.78 \pm 2.99) \times 10^{-5}$
$(\theta_{\mu3}\theta_{\mu3})$	$(1.02 \pm 2.98) \times 10^{-5}$
$\lvert\theta_{\mu i}\theta_{\mu i}\rvert$	8.5×10^{-3}
$(\theta_{\mu1}\theta_{\tau1})^\dagger$	$(2.08 \pm 2.26) \times 10^{-6}$
$(\theta_{\mu2}\theta_{\tau2})^\dagger$	$-(0.39 \pm 2.27) \times 10^{-6}$
$(\theta_{\mu3}\theta_{\tau3})^\dagger$	$(0.55 \pm 2.25) \times 10^{-6}$
$\lvert\theta_{\mu i}\theta_{\tau i}\rvert$	6.52×10^{-4}
$(\theta_{e1}\theta_{e1})$	$(3.88 \pm 1.95) \times 10^{-5}$
$(\theta_{e2}\theta_{e2})$	$(4.59 \pm 1.96) \times 10^{-5}$
$(\theta_{e3}\theta_{e3})$	$-(5.04 \pm 1.95) \times 10^{-5}$
$\lvert\theta_{ei}\theta_{ei}\rvert$	5.65×10^{-3}

with $i = 1, 2, 3$. Their product must satisfy [46]

$$|\theta_{\mu i}\theta_{\tau i}||\theta_{ej}\theta_{ej}| < 0.32 \times 10^{-5}. \tag{37}$$

Heavy neutrino masses $\mathrm{M_i}$ $(i = 1, 2, 3)$ run from 15 to 20 TeV, according to our previous experience.Our results are summarized in Table III.

The mean values for the heavy neutrino masses from the studies in the previous section differ only slightly from the 'Wrong Sign' analysis, $\sim 0.12\%$ in all cases.

7. Lepton Flavour Violation in Hadron Decays of the Tau Lepton in LHT

7.1. $\tau \to \ell q\bar{q}$ ($\ell = e, \mu$)

The amplitude comprises two common topologies: i) penguin-like diagrams, which consist of $\tau \to \ell\gamma, Z$ followed by $\gamma, Z \to q\bar{q}$, and ii) box diagrams. In our calculation, we will assume that both light quarks and leptons (ℓ) are massless to simplify the results.

The total amplitude has two components: the first contribution arises from T-odd particles, while the second one comes from heavy Majorana neutrinos, then

$$\mathcal{M} = \mathcal{M}^{\mathrm{T-odd}} + \mathcal{M}^{\mathrm{Maj}}, \tag{38}$$

where each one is written

$$\begin{aligned} \mathcal{M}^{\mathrm{T-odd}} &= \mathcal{M}_\gamma^{\mathrm{T-odd}} + \mathcal{M}_Z^{\mathrm{T-odd}} + \mathcal{M}_{\mathrm{box}}^{\mathrm{T-odd}}, \\ \mathcal{M}^{\mathrm{Maj}} &= \mathcal{M}_\gamma^{\mathrm{Maj}} + \mathcal{M}_Z^{\mathrm{Maj}} + \mathcal{M}_{\mathrm{box}}^{\mathrm{Maj}}, \end{aligned} \tag{39}$$

we will use the 't Hooft-Feynman gauge along the calculation and will write $\ell = \mu$ for concreteness.

7.2. $\tau \to \mu P$

These decays, where $P = \{\pi^0, \eta, \eta'\}$, are mediated only by axial-vector current (Z^0 gauge boson) and box diagrams. Thus, the amplitude is given by

$$\mathcal{M}_{\tau\to\mu P} = \mathcal{M}_Z^P + \mathcal{M}_{\mathrm{Box}}^P. \tag{40}$$

Each contribution reads

$$\begin{aligned} \mathcal{M}_Z^P &= -i\frac{g^2}{2c_W}\frac{F}{M_Z^2}C(P)\sum_i \overline{\mu}(p')\left[\not{Q}F_L^Z P_L\right]\tau(p), \\ \mathcal{M}_{\mathrm{Box}}^P &= -ig^2 F\sum_i B_i(P)\overline{\mu}(p')\left[\not{Q}P_L\right]\tau(p), \end{aligned} \tag{41}$$

where $C(P)$ and $B_j(P)$ functions are given as follows

$$\begin{aligned} C(\pi^0) &= 1, \\ C(\eta) &= \frac{1}{\sqrt{6}}\left(\sin\theta_\eta + \sqrt{2}\cos\theta_\eta\right), \\ C(\eta') &= \frac{1}{\sqrt{6}}\left(\sqrt{2}\sin\theta_\eta - \cos\theta_\eta\right). \end{aligned} \tag{42}$$

The box amplitude includes the following $B_j(P)$ factors

$$\begin{aligned} B_j(\pi^0) &= \frac{1}{2}(B_d^j - B_u^j), \\ B_j(\eta) &= \frac{1}{2\sqrt{3}}\Big[(\sqrt{2}\sin\theta_\eta - \cos\theta_\eta)B_u^j \\ &\quad + (2\sqrt{2}\sin\theta_\eta + \cos\theta_\eta)B_d^j\Big], \\ B_j(\eta') &= \frac{1}{2\sqrt{3}}\Big[(\sin\theta_\eta - 2\sqrt{2}\cos\theta_\eta)B_d^j \\ &\quad - (\sin\theta_\eta + \sqrt{2}\cos\theta_\eta)B_u^j\Big], \end{aligned} \tag{43}$$

where the B_q^j functions are the form factors from box diagrams and the angle $\theta_\eta \simeq -18°$.

The branching ratio reads

$$\begin{aligned} \mathrm{Br}(\tau \to \mu P) &= \frac{1}{4\pi}\frac{\lambda^{1/2}(m_\tau^2, m_\mu^2, m_P^2)}{m_\tau^2\Gamma_\tau} \\ &\quad \times \frac{1}{2}\sum_{i,f}|\mathcal{M}_{\tau\to\mu P}|^2, \end{aligned} \tag{44}$$

where $\Gamma_\tau \approx 2.267 \times 10^{-12}$ GeV and $\lambda(x,y,z) = (x+y-z)^2 - 4xy$.

Thus,

$$\sum_{i,f} |\mathcal{M}_{\tau\to\mu P}|^2 = \frac{1}{2m_\tau} \sum_{k,l} \left[(m_\tau^2 + m_\mu^2 - m_P^2)(a_P^k a_P^{l*} + b_P^k b_P^{l*}) + 2m_\mu m_\tau (a_P^k a_P^{l*} - b_P^k b_P^{l*}) \right], \quad (45)$$

with $k,l = Z,B$. Defining $\Delta_{\tau\mu} = m_\tau - m_\mu$ and $\Sigma_{\tau\mu} = m_\tau + m_\mu$, we get

$$a_P^Z = -\frac{g^2}{2c_W}\frac{F}{2}\frac{C(P)}{M_Z^2}\Delta_{\tau\mu}(F_L^Z + F_R^Z), \qquad b_P^Z = \frac{g^2}{2c_W}\frac{F}{2}\frac{C(P)}{M_Z^2}\Sigma_{\tau\mu}(F_R^Z - F_L^Z),$$
$$a_P^B = -\frac{g^2 F}{2}\Delta_{\tau\mu}B_j(P), \qquad b_P^B = -\frac{g^2 F}{2}\Sigma_{\tau\mu}B_j(P). \quad (46)$$

7.3. $\tau \to \mu PP$

We will explore the decays into pseudoscalar pairs, denoted as $P\overline{P} = \pi^+\pi^-, K^+K^-, K^0\overline{K^0}$. $\gamma-$ and $Z-$penguin diagrams as well as box diagrams contribute to them. The overall amplitude for these decays can be expressed:

$$\mathcal{M}_{\tau\to\mu PP} = \mathcal{M}_\gamma^{P\overline{P}} + \mathcal{M}_Z^{P\overline{P}} + \mathcal{M}_{\text{Box}}^{P\overline{P}}. \quad (47)$$

The next step is to hadronize the quark bilinears appearing in each amplitude. They turn out to be

$$\mathcal{M}_\gamma^{P\overline{P}} = \frac{e^2}{Q^2} F_V^{P_1P_2}(s) \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} \overline{\mu}(p')[Q^2(\not{p}_q - \not{p}_{\overline{q}})F_L^\gamma(Q^2)P_L + 2im_\tau p_q^\mu \sigma_{\mu\nu} p_{\overline{q}}^\nu F_M^\gamma(Q^2)P_R]\tau(p),$$
$$\mathcal{M}_Z^{P\overline{P}} = g^2 \frac{2s_W^2 - 1}{2c_W M_Z^2} F_V^{P_1P_2}(s) \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} \overline{\mu}(p')(\not{p}_q - \not{p}_{\overline{q}})[\gamma^\mu(F_L^Z P_L + F_R^Z P_R)]\tau(p),$$
$$\mathcal{M}_{\text{Box}}^{P\overline{P}} = \frac{g^2}{2} F_V^{P_1P_2}(s) \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} (B_u^j - B_d^j) \overline{\mu}(p')(\not{p}_q - \not{p}_{\overline{q}}) P_L \tau(p). \quad (48)$$

After computing each amplitude, we get the following branching ratio

$$\text{Br}(\tau \to \mu PP) = \frac{\kappa_{PP}}{64\pi^3 m_\tau^2 \Gamma_\tau} \int_{s_-}^{s_+} ds \int_{t_-}^{t_+} dt \frac{1}{2} \sum_{i,f} |\mathcal{M}_{\tau\to\mu PP}|^2, \quad (49)$$

where κ_{PP} is 1 for $PP = \pi^+\pi^-, K^+K^-, K^0\bar{K}^0$ and $1/2$ for $PP = \pi^0\pi^0$. In terms of the pseudoscalars momenta, $s = (p_q + p_{\overline{q}})^2$ and $t = (p - p_{\overline{q}})^2$, so that

$$t_-^+ = \frac{1}{4s}\left[\left(m_\tau^2 - m_\mu^2\right)^2 - \left(\lambda^{1/2}(s, m_{P_1}^2, m_{P_2}^2) \mp \lambda^{1/2}(m_\tau^2, s, m_\mu^2) \right)^2 \right], \qquad s_- = 4m_P^2, \qquad s_+ = (m_\tau - m_\mu)^2. \quad (50)$$

7.4. $\tau \to \mu V$

When conducting an experiment that involves measuring a final state with a vector resonance, the experimenters reconstruct it via pseudoscalar mesons pairs with an invariant mass approaching m_V, where V can be either ρ or ϕ ($\omega \to \pi\pi$ decays are suppressed by isospin symmetry).

There, the branching ratio of $\tau \to \mu V$ is related to the one of $\tau \to \mu PP$ by implementing:

$$\text{Br}(\tau \to \mu V) = \sum_{P_1,P_2} \text{Br}(\mu P_1 P_2) \Big|_V . \quad (51)$$

Above, the s limits are now cut to

$$s_- = M_V^2 - \frac{1}{2}M_V\Gamma_V, \quad s_+ = M_V^2 + \frac{1}{2}M_V\Gamma_V. \quad (52)$$

Then, when $V = \rho, \phi$ the corresponding branching ratios are

$$\text{Br}(\tau \to \mu\rho) = \text{Br}(\tau \to \mu\pi^+\pi^-)\big|_\rho ,$$
$$\text{Br}(\tau \to \mu\phi) = \text{Br}(\tau \to \mu K^+K^-)\big|_\phi + \text{Br}(\tau \to \mu K^0\bar{K}^0)\big|_\phi . \quad (53)$$

8. Numerical analysis

We first recall the masses of LHT particles:

$$M_W = \frac{\upsilon}{2s_W}\left(1-\frac{\upsilon^2}{12f^2}\right),$$
$$M_Z = M_W/c_W, \quad \upsilon \simeq 246 \text{ GeV}, \tag{54}$$
$$M_{W_H} = M_{Z_H} = \frac{f}{s_W}\left(1-\frac{\upsilon^2}{8f^2}\right) \approx 0.65f, \tag{55}$$
$$M_{A_H} = \frac{f}{\sqrt{5}c_W}\left(1-\frac{5\upsilon^2}{8f^2}\right) \approx 0.16f,$$
$$M_\Phi = \sqrt{2}M_h\frac{f}{\upsilon}, \tag{56}$$
$$m_{\ell_H^i} = \sqrt{2}\kappa_{ii}f \equiv m_{Hi},$$
$$m_{\nu_H^i} = m_{Hi}\left(1-\frac{\upsilon^2}{8f^2}\right), \quad m_{\ell^c,\nu^c} = \kappa_2. \tag{57}$$

We can approximate

$$M_\Phi = \sqrt{2}M_h\frac{f}{\upsilon} \approx \frac{\sqrt{2}}{2}f. \tag{58}$$

Considering just mixing between two lepton families, the mixing matrix of T-odd leptons $(V_{H\ell}^{i\mu *}V_{H\ell}^{i\tau})$ and the mixing matrix among partner leptons $(W_{ij}^{\dagger}W_{jk})$ can be parameterized as follows [36]

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_V & \sin\theta_V \\ 0 & -\sin\theta_V & \cos\theta_V \end{pmatrix},$$
$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_W & \sin\theta_W \\ 0 & -\sin\theta_W & \cos\theta_W \end{pmatrix}, \tag{59}$$

where $\theta_V, \theta_W \in [0, \pi/2)$ is the physical range for the mixing angles and θ_W must not be confused with the weak-mixing angle. We similarly assumed $\mu-\tau$ mixing, for the evaluation $\tau - e$ transitions.

No extra quark mixing is assumed and we take degenerate heavy quarks, so $V_{H\ell}^q$ and W^q will be the identity. Thus, the other free parameter are: θ_V, θ_W and neutral couplings of heavy Majorana neutrinos: $(\theta S\theta^\dagger)_{\mu\tau}$.

Two analyses are presented. The initial one assumes negligible contributions from heavy Majorana neutrinos (and was not done before), while the second case takes them into account. Our simulation combines these processes and computes them concurrently, in a single Monte Carlo simulation.

TABLE IV. Mean values obtained by Monte Carlo simulation of $\tau \to \ell P$ ($\ell = e, \mu$) processes where Majorana neutrinos contribution is not considered.

$\tau \to \ell P$ ($\ell = e, \mu$) (C.L. $= 90\%$) without Majorana neutrinos contribution.			
New physics (NP) scale (TeV)		Mixing angles	
f	1.49	θ_V	42.78°
Branching ratio		θ_W	42.69°
Br($\tau \to e\pi^0$)	5.24×10^{-9}	Masses of partner leptons ($m_{\nu^c} = m_{\ell^c}$)(TeV)	
Br($\tau \to \mu\pi^0$)	3.42×10^{-9}	$m_{\nu_1^c}$	3.12
Br($\tau \to e\eta$)	2.32×10^{-9}	$m_{\nu_2^c}$	3.15
Br($\tau \to \mu\eta$)	1.91×10^{-9}	$m_{\nu_3^c}$	3.37
Br($\tau \to e\eta'$)	2.20×10^{-8}	Masses of partner quarks ($m_{u^c} = m_{d^c}$) (TeV)	
Br($\tau \to \mu\eta'$)	1.79×10^{-8}	$m_{u_i^c}$	3.55
Masses of T-odd leptons (TeV)			
$m_{\ell_H^1}$	2.11		
$m_{\ell_H^2}$	2.11		
$m_{\ell_H^3}$	2.12		
$m_{\nu_H^1}$	2.10		
$m_{\nu_H^2}$	2.11		
$m_{\nu_H^3}$	2.11		
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	2.71		
$m_{u_H^i}$	2.70		

TABLE V. Mean values for $\tau \to \ell P$ $(\ell = e, \mu)$ processes.

$\tau \to \ell P$ $(\ell = e, \mu)$ (C.L. $= 90\%$) with Majorana neutrinos contribution.			
New physics (NP) scale (TeV)		Mixing angles	
f	1.51	θ_V	43.07°
Branching ratio		θ_W	42.82°
Br$(\tau \to e\pi^0)$	8.69×10^{-9}	Masses of partner leptons $(m_{\nu^c} = m_{\ell^c})$(TeV)	
Br$(\tau \to \mu\pi^0)$	6.96×10^{-9}	$m_{\nu_1^c}$	3.26
Br$(\tau \to e\eta)$	6.19×10^{-9}	$m_{\nu_2^c}$	3.26
Br$(\tau \to \mu\eta)$	5.19×10^{-9}	$m_{\nu_3^c}$	3.30
Br$(\tau \to e\eta')$	2.19×10^{-8}	Masses of partner quarks $(m_{u^c} = m_{d^c})$ (TeV)	
Br$(\tau \to \mu\eta')$	1.94×10^{-8}	$m_{u_i^c}$	3.31
Masses of T-odd leptons (TeV)		Masses of heavy Majorana neutrinos (TeV)	
$m_{\ell_H^1}$	3.06	M_1	19.18
$m_{\ell_H^2}$	3.03	M_2	19.07
$m_{\ell_H^3}$	3.03	M_3	19.25
$m_{\nu_H^1}$	3.05	Neutral couplings of heavy Majorana neutrinos	
$m_{\nu_H^2}$	3.02	$\lvert(\theta S \theta^\dagger)_{e\tau}\rvert$	3.32×10^{-7}
$m_{\nu_H^3}$	3.02	$\lvert(\theta S \theta^\dagger)_{\mu\tau}\rvert$	3.90×10^{-7}
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	2.78		
$m_{u_H^i}$	2.78		

TABLE VI. Mean values for $\tau \to \ell PP,\ \ell V$ $(\ell = e, \mu)$ processes without Majorana neutrinos.

$\tau \to \ell PP,\ \ell V$ $(\ell = e, \mu)$ (C.L. $= 90\%$) without Majorana neutrinos contribution			
New physics (NP) scale (TeV)		Mixing angles	
f	1.50	θ_V	43.36°
Branching ratio		θ_W	41.50°
Br$(\tau \to e\pi^+\pi^-)$	3.92×10^{-9}	Masses of partner leptons $(m_{\nu^c} = m_{\ell^c})$(TeV)	
Br$(\tau \to \mu\pi^+\pi^-)$	3.96×10^{-9}	$m_{\nu_1^c}$	3.20
Br$(\tau \to eK^+K^-)$	2.38×10^{-9}	$m_{\nu_2^c}$	3.15
Br$(\tau \to \mu K^+K^-)$	2.85×10^{-9}	$m_{\nu_3^c}$	3.31
Br$(\tau \to eK^0\overline{K^0})$	1.15×10^{-9}	Masses of partner quarks $(m_{u^c} = m_{d^c})$ (TeV)	
Br$(\tau \to \mu K^0\overline{K^0})$	1.33×10^{-9}	$m_{u_i^c}$	3.32
Br$(\tau \to e\rho)$	1.10×10^{-9}		
Br$(\tau \to \mu\rho)$	1.12×10^{-9}		
Br$(\tau \to e\phi)$	1.77×10^{-9}		
Br$(\tau \to \mu\phi)$	1.87×10^{-9}		
Masses of T-odd leptons (TeV)			
$m_{\ell_H^1}$	3.13		
$m_{\ell_H^2}$	2.99		
$m_{\ell_H^3}$	3.10		
$m_{\nu_H^1}$	3.12		
$m_{\nu_H^2}$	2.98		
$m_{\nu_H^3}$	3.09		
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	2.92		
$m_{u_H^i}$	2.91		

TABLE VII. Mean values for $\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ processes.

$\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ (C.L. $= 90\%$) with Majorana neutrinos contribution			
New physics (NP) scale (TeV)		Mixing angles	
f	1.54	θ_V	$43.61°$
Branching ratio		θ_W	$42.21°$
Br($\tau \to e\pi^+\pi^-$)	4.75×10^{-9}	Masses of partner leptons ($m_{\nu^c} = m_{\ell^c}$)(TeV)	
Br($\tau \to \mu\pi^+\pi^-$)	4.90×10^{-9}	$m_{\nu_1^c}$	2.91
Br($\tau \to eK^+K^-$)	2.53×10^{-9}	$m_{\nu_2^c}$	2.99
Br($\tau \to \mu K^+K^-$)	3.38×10^{-9}	$m_{\nu_3^c}$	2.95
Br($\tau \to eK^0\overline{K^0}$)	1.16×10^{-9}	Masses of partner quarks ($m_{u^c} = m_{d^c}$) (TeV)	
Br($\tau \to \mu K^0\overline{K^0}$)	1.50×10^{-9}	$m_{u_i^c}$	3.08
Br($\tau \to e\rho$)	1.33×10^{-9}	Masses of heavy Majorana neutrinos (TeV)	
Br($\tau \to \mu\rho$)	1.39×10^{-9}	M_1	18.82
Br($\tau \to e\phi$)	1.78×10^{-9}	M_2	19.33
Br($\tau \to \mu\phi$)	2.08×10^{-9}	M_3	18.92
Masses of T-odd leptons (TeV)		Neutral couplings of heavy Majorana neutrinos	
$m_{\ell_H^1}$	3.49	$\lvert(\theta S\theta^\dagger)_{e\tau}\rvert$	2.20×10^{-7}
$m_{\ell_H^2}$	3.47	$\lvert(\theta S\theta^\dagger)_{\mu\tau}\rvert$	3.14×10^{-7}
$m_{\ell_H^3}$	3.21		
$m_{\nu_H^1}$	3.48		
$m_{\nu_H^2}$	3.46		
$m_{\nu_H^3}$	3.20		
Masses of T-odd quarks (TeV)			
$m_{d_H^i}$	3.73		
$m_{u_H^i}$	3.71		

8.1. $\tau \to \ell P\ (\ell = e, \mu)$ without Majorana neutrinos

The mean values for mixing angles θ_V and θ_W are $42.78°$ and $42.69°$, respectively, both $\approx \pi/4.2$, close to maximize the LFV effects ($\pi/4$). Table IV summarizes our results.

8.2. $\tau \to \ell P\ (\ell = e, \mu)$ with Majorana neutrinos

In contrast to the previous analysis, here the heavy Majorana neutrinos are barely correlated among them. Recalling the results obtained in previous sections, the mean value for heavy Majorana masses is around 17.2 TeV, differing slightly ($\sim 0.12\%$) in all cases. In this analysis, the mean value for heavy Majorana neutrinos is ~ 19.16 TeV. Thus, the difference between both is just $\sim 10.23\%$. The two neutral couplings $|(\theta S\theta^\dagger)_{\ell\tau}|$ $(\ell = e, \mu)$ have the same order of magnitude, $\mathcal{O}(10^{-7})$. Our results are summarized in Table V.

8.3. $\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ without Majorana neutrinos

Our results are collected in Table VI.

8.4. $\tau \to \ell PP,\ \ell V\ (\ell = e, \mu)$ with Majorana neutrinos

Our results are collected in Table VII.

The branching ratios for all processes, when Majorana neutrinos contribution is considered, are greater than when this contribution is absent. All our results for $\tau \to \ell P, PP, V\ (\ell = e, \mu)$ are very promising, as they are only, at most, 2 orders of magnitude smaller than current bounds [41].

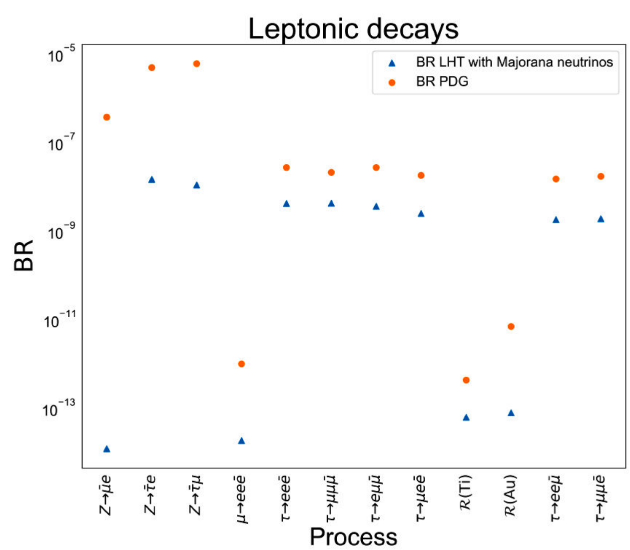

FIGURE 3. Leptonic decays considering Majorana neutrinos.

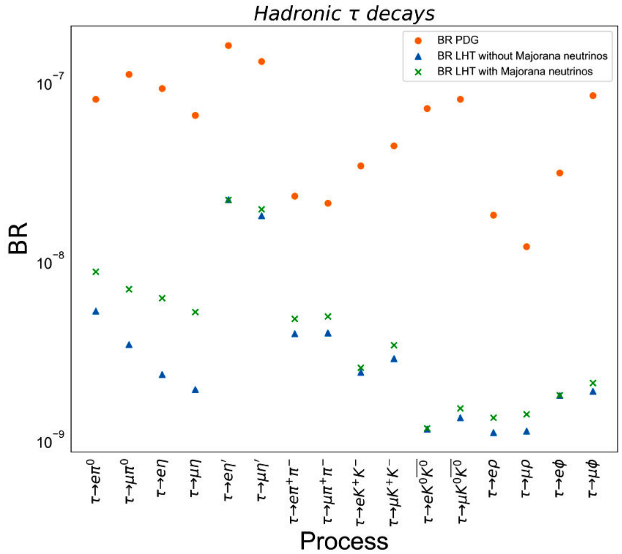

FIGURE 4. Hadronic τ decays considering Majorana neutrinos.

Our main findings (for mean branching fractions) are plotted in Figs. 3 and 4.

9. Conclusions

We have developed a comprehensive set of tools that enable us to numerically predict various physical properties such as branching ratios, particle masses, and couplings within the framework of our model. Our primary objective was to investigate purely leptonic decays. Having achieved that, we proceeded to study the lepton flavor violating (LFV) hadronic decays of the tau lepton. Our key findings are outlined below:

- The new physics (NP) energy scale is around $f \sim 1.36(1.50)$ TeV in our analyses. The difference between these two values is $\sim 9.34\%$, which is reasonable at this stage. We are studying light lepton to tau conversions in nuclei (as in Ref. [47]) and adding the contributions to the muon g-2 of the Majorana neutrinos, in order to perform a global analysis [48] that will complete the picture summarized here.
- LHT with Majorana neutrinos extension enables to bind the LNV couplings shown in Table III. This is a novel result since they were not restricted in [46].
- Neutral couplings of heavy Majorana neutrinos, denoted as $(\theta S\theta^\dagger)_{\ell\ell'}$ were yet unbound and agree in both sets of analyses.
- Masses of particles coming from LHT, T-odd and partner fermions, are below 4 TeV, almost 5 times lighter than heavy Majorana neutrinos. This is consistent with $f \sim 1.4$ TeV.
- Heavy Majorana neutrinos masses in the hadronic τ decays analysis are $\mathrm{M}_i \sim 19$ TeV (we recall that $\mathrm{M}_i \sim 4\pi f$). Compared to the leptonic analyses, these are heavier by ~ 2 TeV ($\sim 10\%$ of difference), with a fully consistent f.
- In all $\tau \to \ell\ell'\bar{\ell}''$ decays and in $\mu \to e$ conversion in Ti, the mean values of our simulated events satisfying all present bounds are only one order of magnitude smaller than current limits. In $\mu \to ee\bar{e}$, $Z \to \bar{\tau}\ell$ and conversion in Au, our predictions are around two orders of magnitude smaller than current bounds (only $Z \to \bar{\mu}e$ is much smaller than present limits).

These results have been published in Refs. [1,2] and offer bright prospects in the near future.

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i. See *e.g*. Refs. [43–45] for earlier studies.

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