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Radiative corrections and new physics tests in semileptonic tau decays

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I review recent advances in the computation of radiative corrections in one- and two-meson tau decays and sketch their main applications in new physics tests: of lepton universality and CKM unitarity, as well as searching for non-standard interactions.

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1. Introduction

Exclusive hadronic tau decays are interesting for a bunch of reasons[i] (see Ref. [3] for a recent summary). On the purely Standard Model (SM) side, they are a clean laboratory to understand the hadronization of QCD currents at low and intermediate energies, and allow for a precise extraction of resonance pole parameters. Concerning beyond the SM (BSM) tests, they permit further (beyond those of the purely lepton decays) verifications of lepton universality, of the three-family CKM unitarity, and of the possible presence of non-standard interactions (NSI). Further BSM tests extend to discrete symmetries, such as CP and T. Even with all such good quality data on $\sigma(e^+e^- \to \mathrm{hadrons})$ produced by the B-factories, semileptonic tau decays should still be important in determining the leading contribution to the hadronic vacuum polarization part of the muon gyromagnetic factor [4], a particularly sensitive New Physics (NP) probe.

Effective field theory (EFT) techniques enable the combination of the tau decays bounds on NP with those coming from both low- (kaon, hyperon, nuclear and pion decays) and high-energy (electroweak precision observables and LHC) experiments. Perhaps counterintuitively, tau decay limits can increase the reach on NP of the other probes in some instances.

Here in Sec. 2, we will recall the essential aspects of one- and two-meson tau decays in the SM, as well as the main features (and consequences in NP searches) of their associated radiative corrections, topics that have interested me from my early research experiences. Conclusions and outlook in Sec. 3 close this contribution.

2. Semileptonic tau decays within and beyond the SM

Generally, semileptonic tau decays can be split into a lepton and a hadron current, describing the creation, from the hadronic vacuum, of some given final-state mesons, by the left-handed weak charged current. This hadron vector can be written in terms of a number of allowed Lorentz structures times a set of scalar functions depending on kinematical invariants, the relevant form factors. In the one-meson case, these reduce to just the pseudoscalar meson ($P = \pi, K$) decay constant, f_P. Within the SM, f_P can be extracted either from the P decay, $P^- \to \mu^-\bar{\nu}_\mu$, or from lattice QCD. NP effects can distinguish both determinations of f_P, so it is essential to use the latter value when studying possible BSM effects. At Born level, form factors appear for two or more mesons. Their determination is mainly guided by data, although QCD predicts their behaviour in the low (chiral) [5–7] and high (asymptotic) [8–10] energy limits. Fundamental properties of quantum field theory, like analyticity, crossing-symmetry and unitarity need also be respected. Dispersion relations are the most convenient way to enforce these properties and have been widely pursued in two-meson tau decays [11–21].

2.1. Radiative corrections to $\tau^- \to P^-\nu_\tau$ and their applications

These radiative corrections are essential, at the current level of precision, in several NP tests. In the case of lepton universality and CKM unitarity, there are some hints for their violation in semileptonic decays involving heavy flavors and in the first row test [22], respectively. Lepton universality is however precisely verified in the ratio of lepton tau decays to the muon decay [23] or comparing different W leptonic decays [24, 25]. Here we will test it through the ratio

$$R_{\tau/P} \equiv \frac{\Gamma(\tau^- \to P^-\nu_\tau[\gamma])}{\Gamma(P^- \to \mu^-\bar{\nu}_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|^2_P R^{(0)}_{\tau/P}(1+\delta R_{\tau/P})\,, \quad (1)$$

where $R^{(0)}_{\tau/P}$ is a known function of M_τ, m_P and m_μ, and $\delta R_{\tau/P}$ is the radiative correction to this ratio, needed to verify if $g_\tau = g_\mu$, as predicted by the SM electroweak symmetry, or not. We will describe briefly numerator and denominator of Eq. (1), emphasizing their respective radiative corrections, next.

Radiative corrections to the $P^- \to \mu^- \bar{\nu}_\mu$ decays can be computed unambiguously in the SM, using Chiral Perturbation Theory techniques [5–7]. Following Refs. [26–28], one has

$$\Gamma(P^- \to \mu^- \bar{\nu}_\mu[\gamma]) = \Gamma(P^- \to \mu^- \bar{\nu}_\mu[\gamma])^0 S_{EW}^P \times \left\{1 + \frac{\alpha}{\pi} F(m_\mu^2/M_P^2)\right\} \times \left\{1 - \frac{\alpha}{\pi}\left(\frac{3}{2}\log\frac{M_\rho}{m_P} + M(m_\rho^2, m_P^2, m_\mu^2)\right)\right\}, \quad (2)$$

where $\Gamma(P^- \to \mu^- \bar{\nu}_\mu[\gamma])^0$ is the tree-level contribution in the Fermi theory, S_{EW} is the short-distance electroweak radiative correction factor ($S_{EW} = 1.0232, 1.0201$ at the π^- [26] and τ^- [29] mass scales, respectively[ii]), $F(m_\mu^2/M_P^2)$ encodes the structure-independent radiative corrections (which can be computed in the point-like approximation, first obtained by Kinoshita [30]) and $M(m_\rho^2, m_P^2, m_\mu^2)$ includes the structure-dependent radiative corrections, reported in Refs. [27, 28]. The only model dependence concerns the determination of the counterterms entering the function M. This is done using the same framework which describes the $\tau^- \to P^- \nu_\tau[\gamma]$ decays, namely Resonance Chiral Theory [31, 32], $R\chi T$. Within this setting, requiring that relevant Green functions fulfil QCD asymptotic behaviour restricts these counterterms.

Similarly, for the $\tau^- \to P^- \nu_\tau[\gamma]$ decays, we write [33, 34]

$$\Gamma(\tau^- \to P^- \nu_\tau[\gamma]) = \Gamma(\tau^- \to P^- \nu_\tau[\gamma])^0 S_{EW}^\tau \times \left\{1 + \frac{\alpha}{\pi} G(m_\tau^2/M_P^2)\right\} \times \left\{1 - \frac{3\alpha}{2\pi}\log\frac{M_\rho}{M_\tau} + \delta_{\tau P}|_{rSD} + \delta_{\tau P}|_{vSD}\right\}, \quad (3)$$

where the structure-independent radiative corrections are subsumed into the function G [30, 38], and now model-dependent corrections ($\delta_{\tau P}$) were split into their real ($_{rSD}$) and virtual ($_{vSD}$) parts (the former is negligible for $P^- \to \mu^- \bar{\nu}_\nu$ decays). The first one was first computed within $R\chi T$ in Ref. [35] (see also Refs. [36,37]) and the latter was studied only recently, in Refs. [33, 34].

We will not dwell here into the details of the computation of $\delta_{\tau P}|_{vSD}$ [33, 34]. Suffice it to say that they depend on three (one vector- and two axial-current) form factors. We have obtained them within schemes that ensure well-behaved two- and three-point Green functions in the chiral and $U(3)$ flavor limits. In this way our results depend only on the spin-one meson masses. We have checked that both chiral and flavor symmetry breaking corrections induce subdominant uncertainties to our final results. We have as well estimated our model-dependent error by restricting to simpler form factors, which are only suitable for two-point Green functions. The corresponding systematic error is again subdominant with respect to the one coming from the μ dependence of the loop integrations, that completely saturates our model-dependent uncertainty, shown below (μ was varied in a conservative interval enclosing the relevant hadron scale [26], *i.e.* resonance mass, $[0.5, 1]$ GeV).

Our numerical results for these corrections are [33, 34]

$$\begin{aligned} \delta R_{\tau/\pi}|_{SI} &= 1.05\%, \\ \delta R_{\tau/K}|_{SI} &= 1.67\%, \\ \delta R_{\tau/\pi}|_{rSD} &= 0.15\%, \\ \delta R_{\tau/K}|_{rSD} &= (0.18 \pm 0.15)\%, \\ \delta R_{\tau/\pi}|_{vSD} &= -(1.02 \pm 0.57)\%, \\ \delta R_{\tau/K}|_{vSD} &= -(0.88 \pm 0.58)\%, \\ \delta R_{\tau/\pi}| &= (0.18 \pm 0.57)\%, \\ \delta R_{\tau/K}| &= (0.97 \pm 0.58)\%. \end{aligned} \quad (4)$$

We will proceed now to discuss their applications.

We can first write the photon-inclusive one-meson tau decays at one-loop, as

$$\Gamma(\tau^- \to P^- \nu_\tau[\gamma]) = \Gamma(\tau^- \to P^- \nu_\tau[\gamma])^0 S_{EW}^\tau (1 + \delta_{\tau P}), \quad (5)$$

finding [33,34] $\delta_{\tau\pi} = -(0.24 \pm 0.56)\%$ and $\delta_{\tau K} = -(0.15 \pm 0.57)\%$.

Now, application of the radiative corrections highlighted in red in Eqs. (4) to Eq. (1) yields [33, 34]

$$\begin{aligned} \left|\frac{g_\tau}{g_\mu}\right|_\pi &= 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} \\ &= 0.9964 \pm 0.0038\,, \\ \left|\frac{g_\tau}{g_\mu}\right|_K &= 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} \\ &= 0.9857 \pm 0.0078\,. \end{aligned} \quad (6)$$

For the π case, this verifies LU at 0.9σ and it is at 1.8σ in the K case. Similar results are obtained by the last HFLAV review [23], which uses our radiative corrections [33,34]. Since the K case is limited by the experimental (exp) accuracy, improved measurements from B-factories would be extremely helpful to check finely LU in these processes.

Next application concerns CKM unitarity tests. First, by means of the ratio $\Gamma(\tau^- \to K^- \nu_\tau[\gamma])/\Gamma(\tau^- \to \pi^- \nu_\tau[\gamma])$, for which our radiative correction is $\delta = \delta_{\tau K} - \delta_{\tau\pi} = (0.10 \pm 0.80)\%$. PDG [22] values and the FLAG result for the ratio of meson decay constants F_K/F_π [39] give [33, 34]

$$\begin{aligned} \left|\frac{V_{us}}{V_{ud}}\right| &= 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} \\ &= 0.2288 \pm 0.0020\,, \end{aligned} \quad (7)$$

which is 2.1σ away from the unitarity constraint [22]. Tau-based results are not competitive (again because of experimental uncertainties) with the Kaon semileptonic decays, which reach $|V_{us}/V_{ud}| = 0.2291 \pm 0.0009$ [22].

A second unitarity test can be performed directly from one-Kaon tau decays, borrowing V_{ud} from the Fermi beta decays [22]. In this case, our $\delta_{\tau K}$ is applied together with the FLAG F_K [39] and the PDG branching ratio [22], resulting in Ref. [33, 34]

$$V_{us} = 0.2220 \pm 0.0008_{\rm th} \pm 0.0016_{\rm exp} \\ = 0.2220 \pm 0.0018\,, \qquad (8)$$

that is at 2.6σ from CKM unitarity. Our precision is similar to the HFLAV result [23], both of them not being competitive with the Kaon semileptonic decays value $V_{us} = 0.2231 \pm 0.0006$ [40], because of lack of statistics in tau decays.

Our final application concerns constraining non-standard interactions. These, for the one-meson tau decays have been discussed in Refs. [41–43]. Accounting for both radiative corrections $\delta_{\tau P}$ and possible NP effects $\Delta^{\tau P}$, they can be written[iii]

$$\Gamma(\tau^- \to P^-\nu_\tau[\gamma]) = \Gamma(\tau^- \to P^-\nu_\tau[\gamma])^0 S_{EW}^\tau \\ \times (1 + \delta_{\tau P} + 2\Delta^{\tau P}), \qquad (9)$$

with[iv] $(D = d, s)$

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)}\epsilon_P^\tau\,. \qquad (10)$$

From the last expression we find

$$\Delta^{\tau\pi} \times 10^2 = -(0.15 \pm 0.72), \\ \Delta^{\tau K} \times 10^2 = -(0.36 \pm 1.18)\,, \qquad (11)$$

which are compatible with the results in Refs. [41–43] and push NP affecting $\Delta^{\tau P}$ above $\sim 2, 3$ TeV.

2.2. Radiative corrections to $\tau^- \to (PP^{(\prime)})^-\nu_\tau$ and their consequences

Need for these corrections was put forward [44,45] within the study of the $\pi^+\pi^-$ contribution to the leading order hadronic vacuum polarization piece of the muon g-2 ($a_\mu^{HVP,LO}$) [46], which could benefit from using tau decay data on the $\pi^-\pi^0$ mode [47], once corrected by isospin symmetry violation [44,45]. These analyses were updated over the years [48–50], employing the computations (within $R\chi T$) of Refs. [44, 45] and the vector meson dominance results of Refs. [51,52]. The essential ingredient here is the G_{EM} function, defined by (t in the invariant mass of the di-pion system)

$$\frac{d\Gamma(\tau^-\to\pi^-\pi^0\nu_\tau[\gamma])}{dt} = \frac{d\Gamma(\tau^-\to\pi^-\pi^0\nu_\tau)}{dt}^0 \\ \times S_{EW}^\tau G_{EM}(t)\,, \qquad (12)$$

so that long-distance (electromagnetic) radiative corrections are encoded in $G_{EM}(t) - 1$. We revisited recently [53] the original $R\chi T$ calculations [44, 45] aiming at estimating their error induced by uncertainties on the resonance Lagrangian couplings as well as from missing higher-order corrections (see discussion in Ref. [4]). We have now [54] applied these results to find the correction to this mode's branching ratio, via

$$\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau[\gamma]) = \Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)^0 \\ \times S_{EW}^\tau(1 + \delta_{EM}^{\pi\pi})^2\,, \qquad (13)$$

and extend it to all $P^-P^{(\prime)0}$ ($P = \pi,\, K$) modes[v,vi]. We will be summarizing the main results of this work [54] (see also Refs. [45,53]) and outlining their possible implications in the remainder of this section.

We split the $G_{EM}(t)$ function in two pieces: $G_{\rm EM}^{(0)}(t)$, standing for the leading Low approximation plus non-radiative contributions, and the rest, $\delta G_{\rm EM}(t)$, which includes the SD contributions to the amplitude. The predictions for both are shown in Fig. 1.

On the left-hand side of Fig. 1, the curves labeled '1' and '2' stand for two different prescriptions for including the radiative corrections in the form factors. We favor case '1' because it warrants smooth corrections, as physically expected. The difference of the result '2' with respect to '1' is taken as an asymmetric theory uncertainty (which turns out to be the dominant error) of our results. On the right-hand side of Fig. 1, the curve 'SI' stands for the structure independent effects, while '2F' and '3F' include model-dependent corrections. These are only functions of three resonance couplings, which can be determined so as to fulfill QCD asymptotics in two-point Green functions (case '2F', standing for $F_V^2 = 2F^2$ [31, 32], where F is the pion decay constant and F_V parametrizes the coupling of the vector resonance to the vector current). On the contrary, the consistent set of constraints (named '3F') on the relevant two- and three-point Green functions [57–59] includes the relation $F_V^2 = 3F^2$. The difference between the '2F' and '3F' results estimates the associated model-dependence [33, 34, 37, 53, 60], which is much smaller than the difference between the '1' and '2' cases.

The main results of our analysis are

$$\delta^{K^-\pi^0} = -\left(0.009^{+0.008}_{-0.118}\right)\%\,, \\ \delta^{\bar{K}^0\pi^-} = -\left(0.166^{+0.010}_{-0.122}\right)\%\,, \\ \delta^{K^-K^0} = -\left(0.030^{+0.026}_{-0.179}\right)\%\,, \\ \delta^{\pi^-\pi^0} = -\left(0.186^{+0.024}_{-0.169}\right)\%\,. \qquad (14)$$

As expected, these radiative corrections are larger for modes with a π^- than for those with a K^-, as the inner bremsstrahlung part depends on $1/m_P$. Relations between these modes also depend on the corresponding flavor Clebsch-Gordan coefficients. Our results for the $(K\pi)^-$

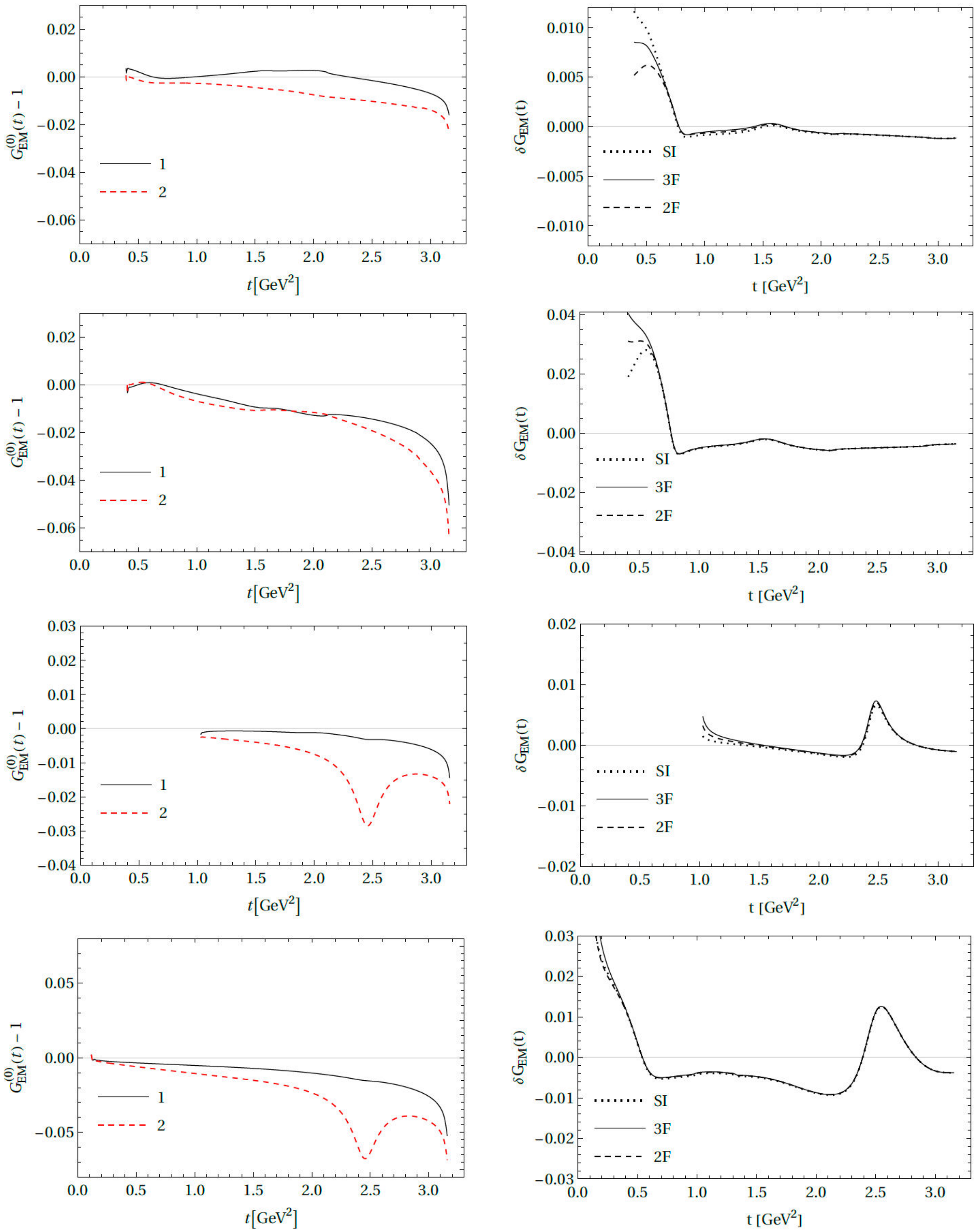

FIGURE 1. Correction factors $G^{(0)}_{\rm EM}(t)-1$ (left) and $\delta G_{\rm EM}(t)$ (right) to the differential decay rates of the $K^-\pi^0$, $\bar{K}^0\pi^-$, K^-K^0, and $\pi^-\pi^0$ modes from top to bottom (from Ref. [54]). See the main text for details.

modes agree with those in Ref. [16] and improve their precision by a factor ~ 2, as a result of computing the model-dependent part of δ, which was only estimated before. Our findings for the $\pi^-\pi^0$ case agree, for the G_{EM} function, with earlier evaluations within $R\chi T$ [45, 53] and VMD [51]. This is the first computation of these corrections for the K^-K^0 channel, as well.

For completeness, we have also evaluated these corrections for the $K^-\eta^{(\prime)}$ modes. In the $G_{\rm EM}^{(0)}$ approximation and using the respective dominance of the vector (scalar) form factor [17], we obtain

$$\delta^{K^-\eta} = -\left(0.026^{+0.024}_{-0.162}\right)\% \,,$$
$$\delta^{K^-\eta'} = -\left(0.304^{+0.380}_{-0.030}\right)\% \,. \quad (15)$$

The $\tau^- \to \pi\eta^{(\prime)}\nu_\tau$ decays are forbidden in the G-parity symmetry limit [20, 69]. Therefore, some electromagnetic corrections are not suppressed with respect to the tree level contribution. One needs to get rid of these by appropriate experimental cuts [70]. The remaining radiative corrections are negligible [71] until these decays are discovered and percent accuracy is reached in their measurements.

The above radiative corrections already are –and will certainly be– important in several NP tests. If the main $\pi\pi$ contribution to $a_\mu^{HVP,LO}$ is taken from tau data instead of from e^+e^- measurements, results have always been at $\sim 2\sigma$ of the muon g-2 measurements, versus $3-4\sigma$ [46], for the e^+e^--based results. In particular, we found [53] a 2.0σ difference for our reference value. This would be in line with the BMW lattice QCD evaluation [72], or with the recent CMD-3 [73] measurement (see also [83]).

CKM Unitarity tests can be performed using tau decays into strangeness states, either inclusively of exclusively [16, 22, 23]. Our results reviewed here shall be included to improve the accuracy of such determinations.

NSI have been studied either in two-meson tau decays or using the inclusive (non-)strange tau decay width [41–43, 61–68]. In particular, Ref. [41] found that the difference between the isospin-rotated $\tau^- \to \pi^-\pi^0\nu_\tau$ spectral function and $\sigma(e^+e^- \to \pi^+\pi^-)$ complements very nicely electroweak precision observables and LHC bounds on the Wilson coefficients plane for the participant left- and right-handed currents, assuming LU, and using the electroweak invariance of the EFT above the weak scale. This example highlights neatly the relevance of our results (14).

3. Conclusions and outlook

We have reviewed in this contribution the main recent improvements on the radiative corrections for one- and two-meson tau decay modes, which increase the precision of several NP tests, like LU, CKM unitarity or searches for NSI.

Ongoing efforts include using dispersive methods to compute these radiative corrections or simulate them on the lattice.

The understanding of the three-meson tau decays within $R\chi T$ [74–82] is not yet mature enough to tackle the corresponding radiative corrections. NSI effects in them were only sketched in Ref. [84] recently. This constitutes an interesting challenging area for future development, provided the corresponding measurements are improved at B- or charm-tau factories.

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i. Inclusive tau lepton decays are also extremely useful, although we will not review them here. See these topics in *e.g.* Refs. [1, 2].

ii. This running is immaterial with current precision.

iii. We have absorbed possible NP affecting the extraction of V_{ud} from nuclear beta decays $\propto \epsilon^e_{L+R}$ into this CKM matrix element entering the Born ^{0} decay width.

iv. Wilson coefficients $\left(\epsilon^{\tau/e}_{L/R}\right)$ and quark masses are given in the $\overline{MS}$ scheme and at $\mu = 2$ GeV.

v. Aside the di-pion mode, only the model-independent part of these corrections for the $K\pi$ cases was computed before [16, 55].

vi. $\tau^- \to \pi^-\pi^0\nu_\tau\ell^+\ell^-$ ($\ell = e, \mu$) was first calculated in Ref. [56].

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