The top quark chromomagnetic dipole moment in the SM

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We present our results on the top quark chromomagnetic dipole moment, which is based on the dimension-5 effective Lagrangian operator that characterizes the chromodipolar vertex functions $gt\ell$ and $ggt\ell$. The chromomagnetic dipole $\mu_t$ is derived via quantum fluctuation at the 1-loop level. We evaluate $\mu_t(s)$ as a function of the energy scale $s = \pm E^2$, for $E = [10, 1000]$ GeV. In particular, we focus on the conventional high-energy scale $E = m_Z$, analogously as with $\alpha_s(m_Z^2)$ and $s_W(m_Z^2)$. The spacelike evaluation matches quite well with the experimental central value, while the timelike case deviates from it.

Keywords: Quantum chromodynamics; electric and magnetic moments; top quark; gluons.

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1. Introduction

In this proceeding, we present our results on the top quark chromomagnetic dipole moment in the Standard Model (SM). The anomalous chromomagnetic dipole moment (CMDM), $\mu_t$, in the SM is induced at the 1-loop level and receives quantum-loop contributions from quantum chromodynamics (QCD), electroweak (EW) and Yukawa (YK) sectors [1–4]. On the experimental side, the CMS Collaboration reported for the top quark CMDM $\hat{\mu}_t$ at the 1-loop level yielding the famous result $\hat{\mu}_t(s) = -0.024^{+0.010}_{-0.009} \text{(stat)}^{+0.011}_{-0.011} \text{(syst)}[5]$, using $pp$ collisions at the centre-of-mass energy of $13$ TeV with an integrated luminosity of $35.9$ fb$^{-1}$.

Theoretically, in the literature these results of the CMDM, $\hat{\mu}_t$, has been exhaustively studied from the 3-body vertex $gt\ell$ at the 1-loop level [1–4]. The most promising prediction yields $\text{Re} \hat{\mu}_t(-m_Z^2) = -0.0224$ [2, 3], this result was evaluated in the spacelike domain ($s = -m_Z^2$), just as occurs for $\alpha_s(m_Z^2)$, $s_W(m_Z^2)$ and $\alpha(m_Z^2)$ which are conventionally fixed at the $Z$ gauge boson mass energy scale for high-energy physics processes [6–11]. In addition, it was found an absorptive contribution, $\text{Im} \hat{\mu}_t(-m_Z^2) = -0.000925i$, which is strictly induced by virtual charged currents. In contrast, the timelike prediction was $\hat{\mu}_t(m_Z^2) = -0.0133 - 0.0267i$, whose real part deviates from the experimental measurement.

Nevertheless, the dimension-5 effective Lagrangian operator that characterizes $\mu_t$ and the chromoelectric dipole moment (CEDM) $d_q$ states that both quantities are proportional to both the 3-body $ggq$ and to the 4-body $ggqq$ vertices [12]. Thus, in this talk we present the CMDM result from both vertex functions. Our calculation of the CMDM from $ggqq$ is given in the Ref. [13].

2. The chromomagnetic dipole moment

The dimension-5 effective Lagrangian operator that characterizes the quantum fluctuation that induces the chromoelectromagnetic dipole moments (CEMDM) [12, 14, 15], has the form

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{2} \bar{q} A^{\mu\nu} \left( \mu_q + igd_q \gamma_5 \right) q B_{\mu\nu} T_{AB},
\]

where

\[
G_{\mu\nu} = \partial_{[\mu} q_{\nu]} - \partial_{[\mu} g_{\nu]} - g_s f_{ABC} g_{[A} g_{B]C},
\]

is the gluon strength field, the CMDM $\mu_q$ conserves CP and the CEDM $d_q$ violates CP, $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, $\bar{q}_A$ and $q_B$ are the spinor fields with $A$ and $B$ the quark color indices, $g_s^a$, is the gluon field with $a = 1, \ldots, 8$, $T_{AB}$ are the color generators and $f_{ABC}$ are the structure constants of the SU(3)$_C$ group; $g_s = \sqrt{4\pi\alpha_s}$ is the group coupling constant of QCD, with the strong running coupling constant $\alpha_s(m_Z^2) = 0.1179$ [6–11] obtained in the spacelike domain at the $m_Z$ gauge boson mass scale for high-energy physics.

The CMDM that arises from the Abelian 3-body vertex $ggq$ is analogous to the anomalous magnetic dipole moment of quantum electromodynamics (QED), it is also described by an effective operator of dimension-5 [17, 18], this was published by Schwinger in 1948 from a radiative correction at the 1-loop level yielding the famous result $\alpha_e = \alpha/(2\pi)$ [19, 20] (see the modern reviews [17, 18]).

On the other hand, unlike QED, QCD is a more sophisticated case due to its non-Abelian nature, which predicts that the chromodipoles can originate from two separate vertices: the Lagrangian (1) states that $\mu_q$ and $d_q$ are proportional to both the Abelian term $\partial_\mu g^a_{\mu} - \partial_\mu g^a_{\mu}$, responsible for the 3-body vertex $ggq$, and the non-Abelian term $g_s f_{ABC} g_{[A} g_{B]C}$, responsible for the 4-body vertex $ggqq$. Therefore, the chromodipoles can be derived from both $ggq$ and $ggqq$ vertex.
functions, that in quantum field theory arise perturbatively due to effects of virtual loops. From the Lagrangian, the Feynman rules of the chromodipolar interactions, the 3-body vertex $gq\bar{q}$ and the 4-body vertex $ggq\bar{q}$, are

$$\Gamma^\mu_{3b} = T^a_{AB} \sigma^{\mu\nu} q_\nu [\mu_q(s) + id_q(s)\gamma^5], \quad (3)$$

and

$$\Gamma^{\mu\nu}_{4b} = ig_s f_{abc} T^c_{AB} \sigma^{\mu\nu} (\mu_q + id_q\gamma^5), \quad (4)$$

respectively, being $q_\nu$ the gluon transfer momentum. In Fig. 1 are presented the loop induced Feynman rules, with the quark momenta defined as incoming for $q_B$ and outgoing for $q_A$, the gluon momenta are incoming, $A$ and $B$ are color indices.

For comparison with the literature and the experimental reports, the dimensionless chromodipoles are defined as

$$\hat{\mu}_q \equiv \frac{m_q}{gs} \mu_q \quad \text{and} \quad \hat{d}_q \equiv \frac{m_q}{gs} d_q. \quad (5)$$

In general, the chromodipoles are complex quantities that can develop absorptive imaginary parts [14, 15].

### 3. The two sources of the CMDM in the SM

As indicated above, the Lagrangian operator predicts that $\hat{\mu}_q$ will arise from both the Abelian 3-body $gt\bar{t}$ and the non-Abelian 4-body $ggt\bar{t}$ vertex functions, thus, in order to distinguish the origin of the CMDM, we will label them as $\hat{\mu}^{3b}_q$ and $\hat{\mu}^{4b}_q$, respectively. With respect to the CEDM, it will result
In the Fig. 2 are displayed the set of diagrams that give rise to $\hat{\mu}_q^{3b}$ in the SM in the $\xi = 1$ Feynman-'t Hooft gauge, this can be consulted in detail in the Ref. [3]. In the Fig. 3 are shown the set of diagrams that produce $\hat{\mu}_q^{4b}$, it has been calculated in Ref. [13].

Let us first discuss on the evaluation of $\hat{\mu}_q(s)$ in the spacelike ($s < 0$) and timelike ($s > 0$) domains. Perturbative QCD and the renormalization group method are directly applicable to the study of the strong interaction processes only in the spacelike (Euclidean) domain in a consistent way ($\alpha_s$ is conceived in this way). The Lorentz-invariant transfer momentum must respect $s \neq 0$, or an IR divergence arise, instead, $s < 0$ in the spacelike domain is the correct way to evaluate the chromodipole, being of special interest the typical high-energy scale convention $s = -m_Z^2$. On the other hand, we also evaluate the timelike domain, $s > 0$, just to appreciate and understand how it behaves, despite this could be physically inconsistent respect to perturbative QCD.

In regard to the timelike (Minkowskian) domain, some studies establish that perturbative QCD is not directly applicable to the study of strong interactions processes, i.e. $e\bar{e} \rightarrow \text{hadrons}$, and that the proper and consistent description of such processes should be carried out only through the appropriate dispersion relations. Thus, this requires the specific study of the hadronic vacuum polarization and the Adler functions in the timelike context [10, 11].

On the other hand, the CMDM from both sources $g t \bar{t}$ and $g g t \bar{t}$ is IR divergent when $\hat{\mu}_q(0)$; in Ref. [3] it is demonstrated via dimensional regularization, the explicit appearance of the IR pole $1/\epsilon_{IR}$ for $\hat{\mu}_q^{3b}(0)$. Additionally, the CMDM $\hat{\mu}_t$ is entirely free of UV divergences, all the poles $1/\epsilon_{UV}$ contained in the 1-point ($A_0$) and 2-point ($B_0$) Passarino-Veltman scalar functions (PaVes), that arise in all the diagrams, cancel each other in each sub contribution $\hat{\mu}_t(g)$, $\hat{\mu}_t(Z)$, $\hat{\mu}_t(W)$, and $\hat{\mu}_t(H)$.

## 4. Results

Let us focus on the top quark, we evaluate $\hat{\mu}_t(s)$ at both spacelike $s = -E^2$ and timelike $s = E^2$ domain within the energy scale range $E = [10, 1000]$ GeV. The results of $\hat{\mu}_t(s)$ derived from the two different vertex function sources are displayed in the Fig. 4, where the CMDM and its PaVes $A_0, B_0, C_0$ and $D_0$ are evaluated with Package-X [21] in Mathematica. $\hat{\mu}_t^{3b}(s)$ and $\hat{\mu}_t^{4b}(s)$ are pretty close to each other for the spacelike evaluation, in fact, when evaluated at the high-energy convention $Z$ mass gauge boson scale $s = -m_Z^2$, the resulting values of $\hat{\mu}_t^{3b}(-m_Z^2)$ and $\hat{\mu}_t^{4b}(-m_Z^2)$ encloses the experimental central value $\hat{\mu}_t^{\text{Exp}} = -0.024$. It is noticeable that the time-like domain is quite irregular for $\hat{\mu}_q^{4b}(s > 0)$. The Table I shows the explicit values

![Figure 3](image-url)
they will be published elsewhere. The CMDM of top quark vertex functions predicted by the Lagrangian, the calculation

\[ \hat{\mu}_t = \frac{1}{\alpha_s(m_Z^2)} \left( \frac{g_2^2}{16\pi^2} \right) \sum \text{sources} \]

The blue vertical line indicates \( E = m_Z \), the experimental value is \( \hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009} \text{(stat)}^{+0.016}_{-0.011} \text{(syst)} \).


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