Suplemento de la Revista Mexicana de Física **4** 021122 (2023) 1–5

The top quark chromomagnetic dipole moment in the SM

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Received 20 May 2023; accepted 20 April 2023

We present our results on the top quark chromomagnetic dipole moment, which is based on the dimension-5 effective Lagrangian operator that characterizes the chromodipolar vertex functions $gt\bar{t}$ and $ggt\bar{t}$. The chromomagnetic dipole $\hat{\mu}_t$ is derived via quantum fluctuation at the 1-loop level. We evaluate $\hat{\mu}_t(s)$ as a function of the energy scale $s = \pm E^2$, for $E = [10, 1000]$ GeV. In particular, we focus on the conventional high-energy scale $E = m_Z$, analogously as with $\alpha_s(m_Z^2)$ and $s_W(m_Z^2)$. The spacelike evaluation matches quite well with the experimental central value, while the timelike case deviates from it.

Keywords: Quantum chromodynamics; electric and magnetic moments; top quark; gluons.

DOI: https://doi.org/10.31349/SuplRevMexFis.4.021122

1. Introduction

In this proceeding, we present our results on the top quark chromomagnetic dipole moment in the Standard Model (SM). The anomalous chromomagnetic dipole moment (CMDM), $\hat{\mu}_q$, in the SM is induced at the 1-loop level and receives quantum-loop contributions from quantum chromodynamics (QCD), electroweak (EW) and Yukawa (YK) sectors [1–4]. On the experimental side, the CMS Collaboration reported for the top quark CMDM $\hat{\mu}_t^{\rm Exp} = -0.024^{+0.013}_{-0.009}({\rm stat})\ ^{+0.016}_{-0.011}({\rm syst})$ [5], using pp collisions at the centre-of-mass energy of 13 TeV with an integrated luminosity of 35.9 fb^{-1}.

Theoretically, in the literature these results of the CMDM, $\hat{\mu}_t$, has been exhaustively studied from the 3-body vertex $gt\bar{t}$ at the 1-loop level [1–4]. The most promising prediction yields $\mathrm{Re}\,\hat{\mu}_t(-m_Z^2) = -0.0224$ [2, 3], this result was evaluated in the spacelike domain ($s = -m_Z^2$), just as occurs for $\alpha_s(m_Z^2)$, $s_W(m_Z^2)$ and $\alpha(m_Z^2)$ which are conventionally fixed at the Z gauge boson mass energy scale for high-energy physics processes [6–11]. In addition, it was found an absorptive contribution, $\mathrm{Im}\,\hat{\mu}_t(-m_Z^2) = -0.000925i$, which is strictly induced by virtual charged currents. In contrast, the timelike prediction was $\hat{\mu}_t(m_Z^2) = -0.0133\ -0.0267i$, whose real part deviates from the experimental measurement.

Nevertheless, the dimension-5 effective Lagrangian operator that characterizes $\hat{\mu}_q$ and the chromoelectric dipole moment (CEDM) $\hat{d}_q$ states that both quantities are proportional to both the 3-body $gq\bar{q}$ and to the 4-body $ggq\bar{q}$ vertices [12]. Thus, in this talk we present the CMDM result from both vertex functions. Our calculation of the CMDM from $ggq\bar{q}$ is given in the Ref. [13].

2. The chromomagnetic dipole moment

The dimension-5 effective Lagrangian operator that characterizes the quantum fluctuation that induces the chromoelectromagnetic dipole moments (CEMDM) [12, 14, 15], has the form

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\bar{q}_A\sigma^{\mu\nu}\left(\mu_q + id_q\gamma^5\right)q_B G^a_{\mu\nu}T^a_{AB}, \qquad (1)$$

where

$$G^a_{\mu\nu} = \partial_\mu g^a_\nu - \partial_\nu g^a_\mu - g_s f_{abc} g^b_\mu g^c_\nu, \qquad (2)$$

is the gluon strength field, the CMDM μ_q conserves CP and the CEDM d_q violates CP, $\sigma^{\mu\nu} \equiv (i/2)[\gamma^\mu, \gamma^\nu]$, $\bar{q}_A$ and q_B are the spinor fields with A and B the quark color indices, g^a_μ is the gluon field with $a = 1, ..., 8$, T^a_{AB} are the color generators and f_{abc} are the structure constants of the SU(3)$_C$ group; $g_s = \sqrt{4\pi\alpha_s}$ is the group coupling constant of QCD, with the strong running coupling constant $\alpha_s(m_Z^2) = 0.1179$ [6–11] obtained in the spacelike domain at the m_Z gauge boson mass scale for high-energy physics.

The CMDM that arises from the Abelian 3-body vertex $gq\bar{q}$ is analogous to the anomalous magnetic dipole moment of quantum electrodynamics (QED), it is also described by an effective operator of dimension-5 [17, 18], this was published by Schwinger in 1948 from a radiative correction at the 1-loop level yielding the famous result $a_e = \alpha/(2\pi)$ [19, 20] (see the modern reviews [17, 18]).

On the other hand, unlike QED, QCD is a more sophisticated case due to its non-Abelian nature, which predicts that the chromodipoles can originate from two separate vertices: the Lagrangian (1) states that μ_q and d_q are proportional to both the Abelian term $\partial_\mu g^a_\nu - \partial_\nu g^a_\mu$, responsible for the 3-body vertex $gq\bar{q}$, and the non-Abelian term $g_s f_{abc} g^b_\mu g^c_\nu$, responsible for the 4-body vertex $ggq\bar{q}$. Therefore, the chromodipoles can be derived from both $gq\bar{q}$ and $ggq\bar{q}$ vertex

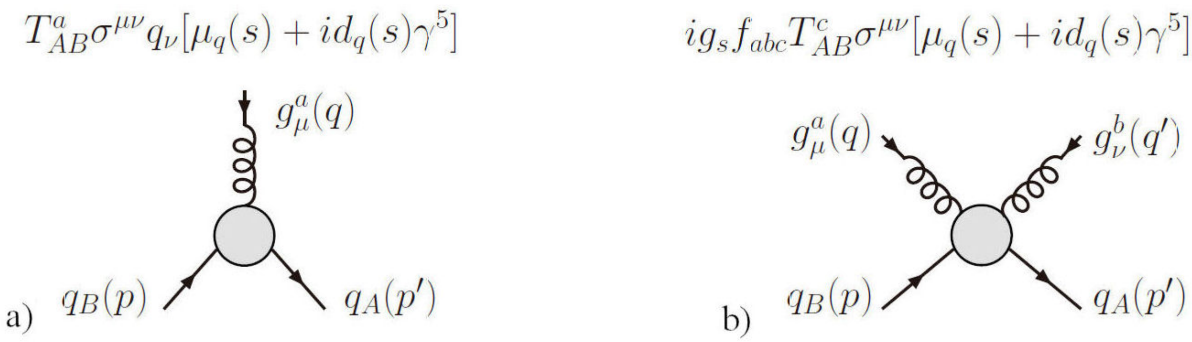

FIGURE 1. Chromodipolar quantum-loop induced Feynman rules as function of the Lorentz-invariant scale energy s: a) Abelian 3-body vertex function $gq\bar{q}$, with kinematics $q+p=p'$, and b) non-Abelian 4-body vertex function $ggq\bar{q}$, with $q+q'+p=p'$.

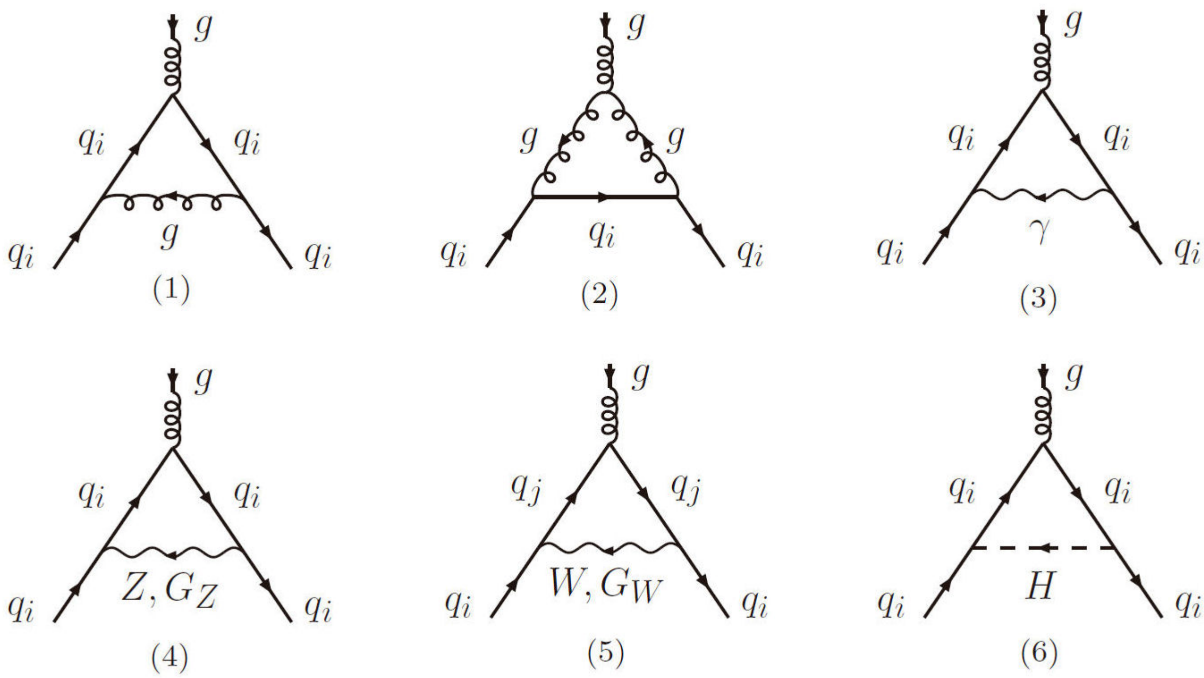

FIGURE 2. 1-Loop diagrams that induce the CMDM of the Abelian 3-body vertex in the $\xi=1$ Feynman-'t Hooft gauge, where $q_i=t$ and $q_j=d,s,b$; 12 diagrams in total.

functions, that in quantum field theory arise perturbatively due to effects of virtual loops.

From the Lagrangian, the Feynman rules of the chromodipolar interactions, the 3-body vertex $gq\bar{q}$ and the 4-body vertex $ggq\bar{q}$, are

$$\Gamma^\mu_{3\mathrm{b}} = T^a_{AB}\sigma^{\mu\nu}q_\nu\left(\mu_q+id_q\gamma^5\right), \tag{3}$$

and

$$\Gamma^{\mu\nu}_{4\mathrm{b}} = ig_sf_{abc}T^c_{AB}\sigma^{\mu\nu}\left(\mu_q+id_q\gamma^5\right), \tag{4}$$

respectively, being q_ν the gluon transfer momentum. In Fig. 1 are presented the loop induced Feynman rules, with the quark momenta defined as incoming for q_B and outgoing for q_A, the gluon momenta are incoming, A and B are color indices.

For comparison with the literature and the experimental reports, the dimensionless chromodipoles are defined as [6, 12, 14, 15]

$$\hat{\mu}_q \equiv \frac{m_q}{g_s}\mu_q \quad \text{and} \quad \hat{d}_q \equiv \frac{m_q}{g_s}d_q. \tag{5}$$

In general, the chromodipoles are complex quantities that can develop absorptive imaginary parts [14, 15].

3. The two sources of the CMDM in the SM

As indicated above, the Lagrangian operator predicts that $\hat{\mu}_q$ will arise from both the Abelian 3-body $gt\bar{t}$ and the non-Abelian 4-body $ggt\bar{t}$ vertex functions, thus, in order to distinguish the origin of the CMDM, we will label them as $\hat{\mu}^{3\mathrm{b}}_q$ and $\hat{\mu}^{4\mathrm{b}}_q$, respectively. With respect to the CEDM, it will result

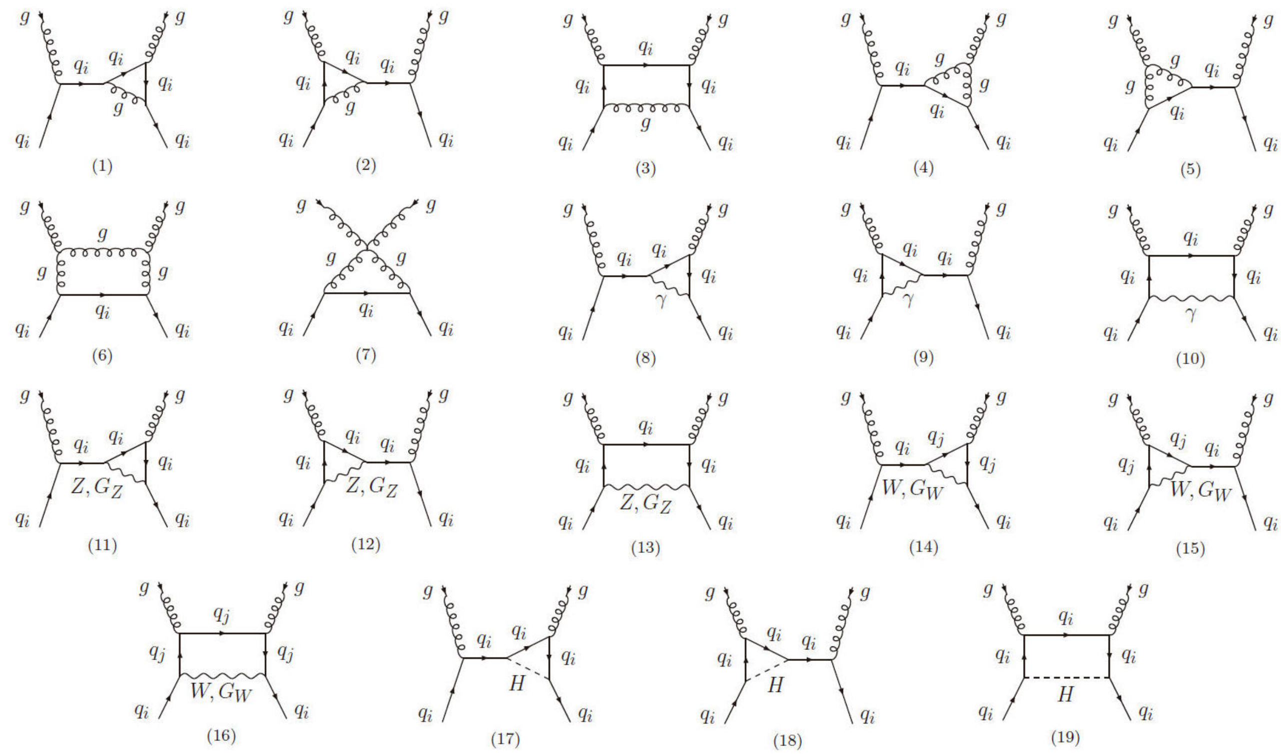

FIGURE 3. 1-loop diagrams that induce the CMDM of the non-Abelian 4-body vertex in the $\xi = 1$ Feynman-'t Hooft gauge, where $q_i = t$ and $q_j = d, s, b$; in total there are 73 diagrams due to the Bose symmetry of external gluons.

$\hat{d}_q^{3\text{b}} = \hat{d}_q^{4\text{b}} = 0$ in the SM at the 1-loop level. On the other hand, it is known that the electric dipole moment (EDM) of quarks arises at the 3-loop level [16], and despite that the calculation has not been done for the CEDM case, this suggests that it could also occur for it.

In the Fig. 2 are displayed the set of diagrams that give rise to $\hat{\mu}_q^{3\text{b}}$ in the SM in the $\xi = 1$ Feynman-'t Hooft gauge, this can be consulted in detail in the Ref. [3]. In the Fig. 3 are shown the set of diagrams that produce $\hat{\mu}_q^{4\text{b}}$, it has been calculated in Ref. [13].

Let us first discuss on the evaluation of $\hat{\mu}_q(s)$ in the spacelike ($s < 0$) and timelike ($s > 0$) domains. Perturbative QCD and the renormalization group method are directly applicable to the study of the strong interaction processes only in the spacelike (Euclidean) domain in a consistent way (α_s is conceived in this way). The Lorentz-invariant transfer momentum must respect $s \neq 0$, or an IR divergence arise, instead, $s < 0$ in the spacelike domain is the correct way to evaluate the chromodipole, being of special interest the typical high-energy scale convention $s = -m_Z^2$. On the other hand, we also evaluate the timelike domain, $s > 0$, just to appreciate and understand how it behaves, despite this could be physically inconsistent respect to perturbative QCD.

In regard to the timelike (Minkowskian) domain, some studies establish that perturbative QCD is not directly applicable to the study of strong interactions processes, *i.e.* $e\bar{e} \to hadrons$, and that the proper and consistent description of such processes should be carried out only through the appropriate dispersion relations. Thus, this requires the specific study of the hadronic vacuum polarization and the Adler functions in the timelike context [10, 11].

On the other hand, the CMDM from both sources $gt\bar{t}$ and $ggt\bar{t}$ is IR divergent when $\hat{\mu}_q(0)$; in Ref. [3] it is demonstrated via dimensional regularization, the explicit appearance of the IR pole $1/\epsilon_{\text{IR}}$ for $\hat{\mu}_q^{3\text{b}}(0)$. Additionally, the CMDM $\hat{\mu}_t$ is entirely free of UV divergences, all the poles $1/\epsilon_{\text{UV}}$ contained in the 1-point (A_0) and 2-point (B_0) Passarino-Veltman scalar functions (PaVes), that arise in all the diagrams, cancel each other out in each sub contribution $\hat{\mu}_t(g)$, $\hat{\mu}_t(\gamma)$, $\hat{\mu}_t(Z)$, $\hat{\mu}_t(W)$, and $\hat{\mu}_t(H)$.

4. Results

Let us focus on the top quark, we evaluate $\hat{\mu}_t(s)$ at both spacelike $s = -E^2$ and timelike $s = E^2$ domain within the energy scale range $E = [10, 1000]$ GeV. The results of $\hat{\mu}_t(s)$ derived from the two different vertex function sources are displayed in the Fig. 4, where the CMDM and its PaVes A_0, B_0, C_0 and D_0 are evaluated with `Package-X` [21] in `Mathematica`. $\hat{\mu}_t^{3\text{b}}(s)$ and $\hat{\mu}_t^{4\text{b}}(s)$ are pretty close to each other for the spacelike evaluation, in fact, when evaluated at the high-energy convention Z mass gauge boson scale $s = -m_Z^2$, the resulting values of $\hat{\mu}_q^{3\text{b}}(-m_Z^2)$ and $\hat{\mu}_q^{4\text{b}}(-m_Z^2)$ encloses the experimental central value $\hat{\mu}_t^{\text{Exp}} = -0.024$. It is noticeable that the time-like domain is quite irregular for $\hat{\mu}_q^{4\text{b}}(s > 0)$. The Table I shows the explicit values

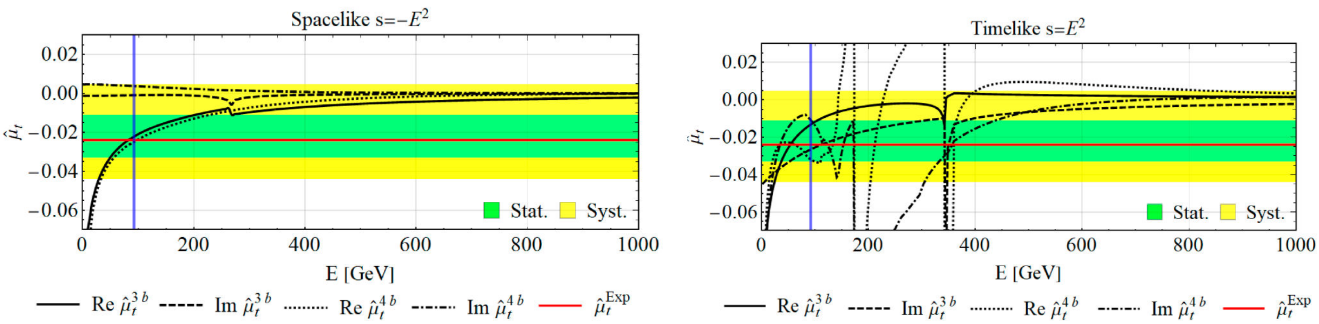

FIGURE 4. CMDM of the top quark from the 3- and 4-body vertices. The blue vertical line indicates $E = m_Z$, the experimental value is $\hat{\mu}_t^{\rm Exp} = -0.024^{+0.013}_{-0.009}(\rm stat)^{+0.016}_{-0.011}(\rm syst)$.

TABLE I. Top quark anomalous CMDM evaluated at the high-energy convention scale of the Z gauge boson mass. The total absolute values are $\|\hat{\mu}_t^{\rm 3b}\| = 0.0224$ and $\|\hat{\mu}_t^{\rm 4b}\| = 0.0253$. The experimental central value is $\hat{\mu}_t^{\rm Exp} = -0.024$ [5].

	3-body	4-body
$\hat{\mu}_t\left(-m_Z^2\right)$	$-0.0224 - 0.000923i$	$-0.025 + 0.00384i$
$\hat{\mu}_t(m_Z^2)$	$-0.0133 - 0.0267i$	$-0.0318 - 0.0106i$

at $s = \pm m_Z^2$. We can conclude that the spacelike evaluation at the Z gauge boson mass scale is the favored one respect to the experimental central value $\|\hat{\mu}_t^{\rm 3b}(-m_Z^2)\| \lessapprox \|\hat{\mu}_t^{\rm EXP}\| \lessapprox \|\hat{\mu}_t^{\rm 4b}(-m_Z^2)\|$.

In regard to the CMDM of small quarks (u, d, s, c, b), they will be published elsewhere. The CMDM of top quark is the only one that has experimental measurement available, so it deserved an individual and in-depth analysis.

5. Conclusions

We have presented the results of $\hat{\mu}_t$ from the 3- and 4-body vertex functions predicted by the Lagrangian, the calculation has been made in a process independent way. $\hat{\mu}_t^{\rm 3b}(s)$ and $\hat{\mu}_t^{\rm 4b}(s)$ are IR divergent at $s = 0$. The spacelike evaluation, $s = -E^2$, is the well-behaved one, and its imaginary part is induced by the W contribution. Despite theoretically is expected $\hat{\mu}_t^{\rm 3b} = \hat{\mu}_t^{\rm 4b}$, our numerical evaluations show that $\|\,\hat{\mu}_t^{\rm 3b}\| \simeq \|\,\hat{\mu}_t^{\rm 4b}\|$. The spacelike evaluation of the two sources at the energy scale of $s = -m_Z^2$, as $\alpha_s(m_Z^2)$ and $s_W(m_Z^2)$ are conventionally evaluated, enclose the experimental value: $\|\hat{\mu}_t^{\rm 3b}(-m_Z^2)\| \lessapprox \|\hat{\mu}_t^{\rm EXP}\| \lessapprox \|\hat{\mu}_t^{\rm 4b}(-m_Z^2)\|$.

Acknowledgments

This work has been partially supported by SNI-CONHACYT and CIC-UMSNH. JMD thanks to the CONAHCYT program Investigadoras e Investigadores por México, project 1753. JMD thanks to Mario Rodriguez-Cahuantzi for the invitation to the XVIII Mexican Workshop on Particles and Fields 2022.

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