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Electromagnetic form factors in a contact interaction: scalar and pseudoscalar mesons

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The determination of fundamental properties of hadrons is important to clearly understand the experimental measurements such as the anomalous magnetic moment of the muon. However, since a first principle approach is hard to find, several groups among the world are approaching phenomenologically to such description of hadrons. In this poster, we present a comprehensive survey of electromagnetic form factors of all light, heavy and heavy-light ground-state pseudoscalar and scalar mesons. Specifically, in this work we are interested on the determination of Elastic Meson Form Factors (EFF) for Scalar (S) and Pseudoscalar (PS) mesons. However, the calculation of all EFF requires the computation of the quark propagator, the Bethe-Salpeter (BS) equation and the Bethe-Salpeter Amplitudes (BSA) of mesons, their masses as well as the knowledge of the quark-photon interaction at different probing momenta.

Keywords: Hadron physics; form factors; pseudoscalar mesons; scalar mesons.

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1. Introduction

The description of hadrons is an interesting and complex problem at the same time, since dynamics of elementary degrees of freedom of quantum chromodynamics (QCD), namely, quarks and gluons are in a confined state. The task is basically the connection between the Green functions of dressed quarks through relativistic bound state equations with the properties of hadrons measured in experiments. The simplest bound states of QCD are the two-particle systems (mesons) composed of a quark and an antiquark ($q\bar{q}'$), so-called mesons. In this document we analyse the elements to determine the Electromagnetic Form Factors (EFFs) of Pseudoscalar and Scalar mesons in the framework of a Contact Interaction (CI) model.[i]

2. GAP equation

It is known that the masses of hadrons are not simply the addition of the masses of constituent quarks but they acquire their mass from interactions studied via the Bethe-Salpeter Equation (BSE) [2]. Furthermore, dressed masses of quarks are obtained by use of the equation,

$$S(p)^{-1} = i\,\gamma\cdot p + m_f + \Sigma(p)\,, \tag{1}$$

where the one-loop contribution to the quark propagator, $S(p)^{-1} = p^2 + M_f^2$, is given by [3, 4],

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\,\gamma_\mu\, S(q)\,\Gamma_\nu(p,q)\,, \tag{2}$$

with m_f and M_f the current and dressed masses of the f-quark, respectively, $D_{\mu\nu}$ the vector boson propagator, g the coupling of the interaction and Γ_ν the quark-vector boson vertex. By means of the CI model we consider,

$$g^2 D_{\mu\nu}(k) = 4\pi\hat{\alpha}_{\rm IR}\delta_{\mu\nu}, \tag{3}$$

where $\hat{\alpha}_{\rm IR} = \alpha_{\rm IR}/m_g^2$, with the scale m_g interpreted as the infrared gluon mass scale generated dynamically within QCD [5–7]. We take currently accepted value $m_g = 500\,$MeV [8–11]. It is clear that in the CI gap equation, the effective coupling which appears is $\hat{\alpha}_{\rm IR}$ instead of $\alpha_{\rm IR}$. We choose

TABLE I. Ultraviolet regulator and coupling constant for different combinations of quarks in PS mesons. $\hat{\alpha}_{\rm IR} = \hat{\alpha}_{\rm IRL}/Z_H$, where $\hat{\alpha}_{\rm IRL} = 4.57$ is extracted from the best-fit to data as explained in Ref. [14]. $\Lambda_{\rm IR} = 0.24$ GeV is a fixed parameter.

quarks	Z_H	$\Lambda_{\rm UV}$ [GeV]	$\hat{\alpha}_{\rm IR}$
u, d, s	1	0.905	4.57
c, u, s	3.034	1.322	1.50
c	13.122	2.305	0.35
b, u	11.273	3.222	0.41
b, s	17.537	3.574	0.26
b, c	30.537	3.886	0.15
b	129.513	7.159	0.035

TABLE II. Current (m_f) and dressed masses (M_f) for quarks in GeV, required as an input for the BSE and the EFFs.

$m_u = 0.007$	$m_s = 0.17$	$m_c = 1.08$	$m_b = 3.92$
$M_u = 0.367$	$M_s = 0.53$	$M_c = 1.52$	$M_b = 4.75$

$\alpha_{\rm IR}/\pi$ to be 0.36 so that $\hat{\alpha}_{\rm IR}$ has exactly the same value as in all related previous works [6,7,12,13]. The interaction vertex is bare, *i.e.*, $\Gamma_\nu(q,p) = \gamma_\nu$. Furthermore, it is possible to find dressed masses of mesons M_f by,

$$M_f = m_f + M_f \, \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \, \mathcal{C}(M_f^2) \,, \tag{4}$$

where $\mathcal{C}(z)/z = \Gamma(-1, z\,\tau_{\rm UV}^2) - \Gamma(-1, z\,\tau_{\rm IR}^2)$, with $\Gamma(\alpha, z)$ the incomplete gamma-function, $\tau_{\rm IR}$ and $\tau_{\rm UV}$ the Infrared and Ultraviolet regulators, respectively [6]. In order to find a phenomenological agreement, we set the parameters of Eq. (4) according to Table I. Therefore, in Table II, we present the current quark masses m_f used herein and the dynamically generated dressed masses M_f of quarks using the GAP equation.

3. Bethe-Salpeter equation

The bound-state problem for mesons isa studied using the homogeneous Bethe-Salpeter (BS) equation [2],

$$[\Gamma(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4} [\chi(q;P)]_{sr} \mathcal{K}_{tu}^{rs}(q,k;P) \,, \tag{5}$$

where $[\Gamma(k;P)]_{tu}$ represents the bound-state's BS amplitude and $\chi(q;P) = S(q+P)\Gamma S(q)$ is the BS wave-function; r, s, t, u represent colour, flavor and spinor indices; and $\mathcal{K}$ is the relevant quark-antiquark scattering kernel. This equation possesses solutions on that discrete set of P^2-values for which bound-states exist.

Decomposition of the BSA for the PS and the S mesons ($f_1 \bar{f}_2$) in the CI has the following form [15],

$$\Gamma_{ps}(P) = i\gamma_5 \, E_{ps}(P) + \frac{1}{2M_R} \gamma_5 \gamma \cdot P \, F_{ps}(P) \,,$$
$$\Gamma_s(P) = I_D \, E_s(P) \,. \tag{6}$$

Note that $E_i(P)$ and $F_i(P)$ with $i \in \{PS, S\}$ are known as the BS amplitudes of the meson under consideration, P is its total momentum, I_D is the identity matrix and $M_R = M_{f_1} M_{\bar{f}_2}/[M_{f_1} + M_{\bar{f}_2}]$ is the reduced mass of the system. Eq. (5) has a solution when $P^2 = -M_M^2$ with M_M being the meson mass.

4. Electromagnetic form factors

EFF can be extracted by studying the process $M\gamma M$, where M is a general meson. The kinematics of the process is given by the Feynman diagram depicted in Fig. 1.

And the standard momentum parametrization, $k_i = K + Q/2$ and $k_f = K - Q/2$. Using Feynman rules, we find that for a general meson, this process can be written as [1],

$$\Lambda^{M,f_1} = N_c \int \frac{d^4\ell}{(2\pi)^4} {\rm Tr} \, \mathcal{G}^{M,f_1} \,, \tag{7}$$

where we have omitted the Dirac indices, and,

$$\mathcal{G}^{M,f_1} = i\Gamma_M(k_i) \, S(\ell + k_i, M_{f_1}) \, i\Gamma_\lambda(Q, M_{f_1})$$
$$\times \; S(\ell + k_f, M_{f_1}) i\bar{\Gamma}_M(-k_f) \, S(\ell, M_{\bar{f}_2}) \,. \tag{8}$$

In addition, we shall consider that the photon interacts with the quark with flavor f_1, and the fermion $\bar{f}_2$ is a spectator. Λ^{M,f_1} can be written as a function of the EFF as,

$$\Lambda^{S,f_1} = -2k_\lambda F^{S,f_1} \,, \; \Lambda^{PS,f_1} = -2k_\lambda F^{PS,f_1} \,. \tag{9}$$

In order to consider the total elastic form factor of mesons, we use[ii],

$$F^M(Q^2) = e_{f_1} F^{M,f_1}(Q^2) + e_{\bar{f}_2} F^{M,\bar{f}_2}(Q^2) \,, \tag{10}$$

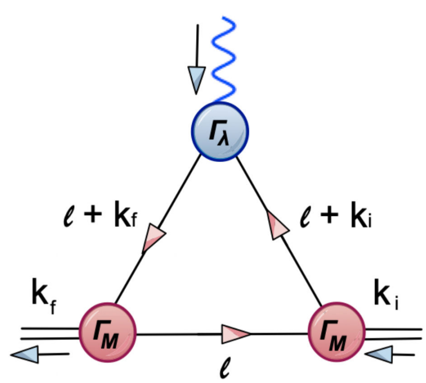

FIGURE 1. The triangle diagram for the impulse approximation to the $M\gamma M$ vertex.

TABLE III. Ultraviolet regulator and the coupling constant for different combinations of quarks in S mesons. As before, $\hat{\alpha}_{\mathrm{IR}} = \hat{\alpha}_{\mathrm{IRL}}/Z_H$, where $\hat{\alpha}_{\mathrm{IRL}} = 4.57$ is extracted from a best-fit to data as explained in Ref. [14]. $\Lambda_{\mathrm{IR}} = 0.24$ GeV.

quarks	Z_H	Λ_{UV} [GeV]	$\hat{\alpha}_{\mathrm{IR}}$
u, d, s	1	0.905	4.57
c, u	3.034	1.322	1.50
c, s	3.034	2.222	1.50
c	13.122	2.305	0.35
b, u	18.473	10.670	0.25
b, s	29.537	11.064	0.15
b, c	34.216	14.328	0.13
b	127.013	26.873	0.036

5. Results

In the following lines, we present our numerical results for the computation of the EFFs for S and PS mesons in the CI approach. It is important to mention that for PS mesons we used the values in Table I and for S mesons we use the values in Table III to predict the values of the masses of mesons by solving the BS equation.

5.1. Meson masses

In the following tables, we present the solution of the Bethe-Salpeter equation for scalar and pseudoscalar mesons used for the calculations of EFF [12] and the BS amplitudes which are needed for the calculation of the EFFs. In addition, we present predictions for masses of some S and a PS mesons which has not been computed yet experimentally.

Before closing this subsection, we point out that the values obtained of meson masses are in perfect agreement with the values reported in Ref. [7].

TABLE IV. Computed values of the S mesons masses and BSAs in the CI model, see [7] for comparison, using the parameters listed in Tables II and III.

	Mass [GeV]	E_S	m_S^{exp} [GeV]
$u\bar{d}$	1.22	0.66	—
$u\bar{s}$	1.38	0.65	—
$s\bar{s}$	1.46	0.64	—
$c\bar{u}$	2.31	0.39	2.30
$c\bar{s}$	2.42	0.42	2.32
$u\bar{b}$	5.30	1.53	—
$s\bar{b}$	5.64	0.26	—
$c\bar{b}$	6.36	1.23	6.71
$c\bar{c}$	3.33	0.16	3.42
$b\bar{b}$	9.57	0.69	9.86

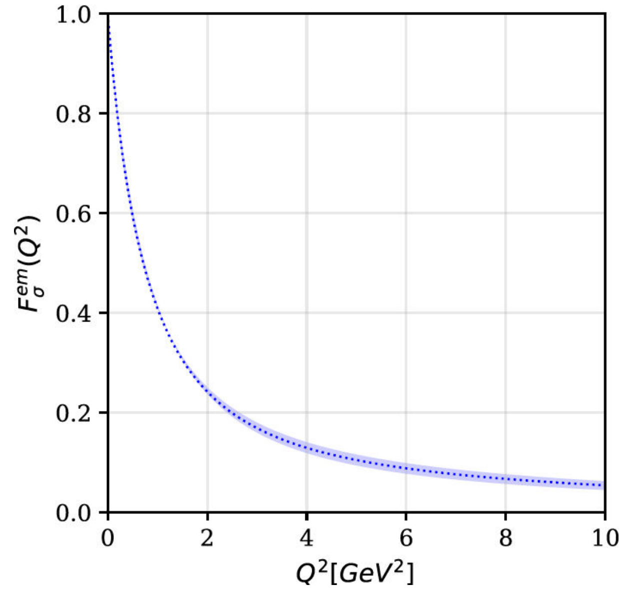

FIGURE 2. EFF for σ-meson. The central curve is obtained using the Λ_{UV} value from Table V while the band represents a 5% variation in the charge radius.

TABLE V. Calculated values for the BSAs and masses for PS mesons in CI model using the parameters in Tables II and I (compare the parameters with the ones in Ref. [7]).

	Mass[GeV]	E_{ps}	F_{PS}	m_{PS}^{exp}[GeV]
$u\bar{d}$	0.139	3.59	0.47	0.139
$u\bar{s}$	0.499	3.81	0.59	0.493
$s\bar{s}$	0.701	4.04	0.75	—
$c\bar{u}$	1.855	3.03	0.37	1.864
$c\bar{s}$	1.945	3.24	0.51	1.986
$u\bar{b}$	5.082	3.72	0.21	5.279
$s\bar{b}$	5.281	2.85	0.21	5.366
$c\bar{b}$	6.138	2.58	0.39	6.274
$c\bar{c}$	2.952	2.15	0.40	2.983
$b\bar{b}$	9.280	2.04	0.39	9.398

5.2. Electromagnetic form factors

We are now with all ingredients needed for the computation of EFFs of S and PS mesons. For the sake of simplicity, we depicted in the following figures EFF for the lightest scalar and pseudoscalar mesons, sigma and pion meson. In particular, it is of great interest the determination of the charge radii defined as,

$$r_M^2 = -6 \left. \frac{dF_M(Q^2)}{dQ^2} \right|_{Q^2=0} . \tag{11}$$

In Figs. 2 and 3 we present our results of the EFFs for the σ and π mesons. The error bands correspond to the 5% variation of the charge radii. In addition, for π mesons, we present comparison with experimental results.

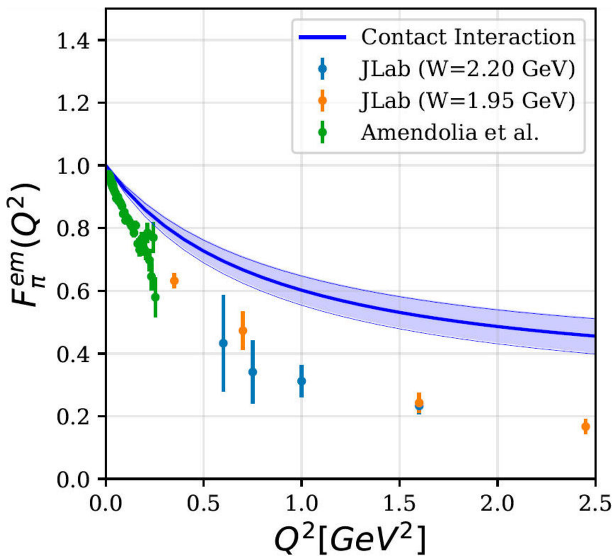

FIGURE 3. EFF for π-meson. The central curve is obtained using the $\tau_{\rm UV}$ value from the Table IV. The filled band allows for a 5% variation in the charge radius. Dots represent the experimental data from Refs. [16–18].

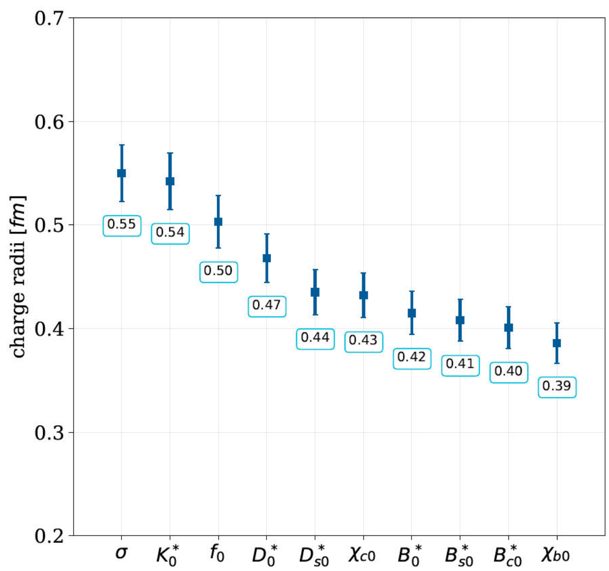

FIGURE 4. Charge radii of S mesons.

5.3. Charge radii

Following the definition of the charge radii of mesons, we proceed to the computation of the charge radii of S and PS mesons. We show in Figs. 4 and 5 the result of our numerical analysis of the charge radii.

Furthermore, we distinguish that our model predicts that mesons formed of light constituent quarks manifest a larger charge radii and, on the contrary, mesons formed of heavier constituent quarks have smaller charge radii.

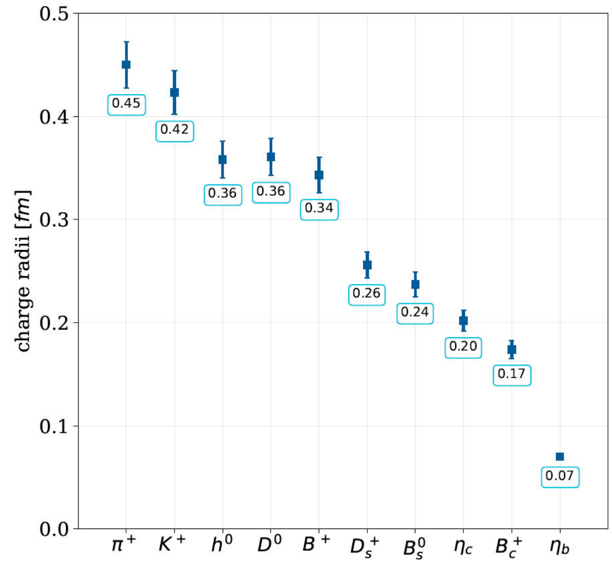

FIGURE 5. Charge radii of ground state PS mesons in the CI.

6. Conclusions

In this document we present a computation of EFFs employing the CI model for twenty ground state PS and S mesons. Note that the CI findings for light mesons and heavy quarkonia are already found in the literature as mentioned before [6,14,19,20]. We conclude that, to the best of our knowledge, there are no experimental or lattice results available for $c\bar{u}$, $c\bar{s}$, $u\bar{b}$, $s\bar{b}$ and $c\bar{b}$ mesons for comparison. However, the general trend of decreasing charge radii with increasing constituent quark mass seems reassuring, *e.g.*, the following hierarchies are noticeable:

$$r_{u\bar{d}} > r_{u\bar{s}} > r_{c\bar{u}} > r_{u\bar{b}}\,,$$
$$r_{u\bar{s}} > r_{s\bar{s}} > r_{c\bar{s}} > r_{s\bar{b}}\,,$$
$$r_{c\bar{u}} > r_{c\bar{s}} > r_{c\bar{c}} > r_{c\bar{b}}\,,$$
$$r_{u\bar{u}} > r_{s\bar{s}} > r_{c\bar{c}} > r_{b\bar{b}}\,.$$

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i. More details can be found in Ref. [1].

ii. For neutral mesons composed of same flavored quarks, the total EFF is simply $F^M = F^{M,f_1}$.

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