Likelihood of origin of Paleolithic tools as viewed from their entropy

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Lithic tools are physical objects whose surface can be characterised by the scars left on them by the removal of stone flakes. In a recent article [1], we quantified the information contained in Paleolithic stone tools about their manufacturing process using Shannon's information theory [2], with the notions of amount of information and entropy. The approach permitted to assess the probability that such objects were made by our hominid ancestors, and also the amount of information they carry. Here, we dig deeper into the physico-mathematical aspects of the subject and show that the entropy of a lithic tool can be defined on a physical basis following Boltzmann's arguments [3]. Thus, the entropy of a stone tool acquires a physical meaning that considerably enlightens their interest. We also extend our previous treatment by considering the effects of curvature of the lithic surface on the probability density of strokes imparted randomly on it, and by taking into account the fact that there can be many tools effectively similar to the one being investigated. Although the number of tools equivalent to a given one is exceedingly large, the probability of observing any of them is still much smaller than that of observing a similar but roughly battered stone. In this work, however, we have not dealt with the archaeological implications of this work, which will be considered elsewhere.

Keywords: Stone tools; entropy; information; paleolithic; likelihood.

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1. Entropy and information of stone tools

The rise of stone-tool technology in the Pleistocene played a crucial rôle in the process of humanisation [4]. Recently [1] we calculated how extremely unlikely it is for a stone to be shaped as a tool solely by random natural forces. The approach relied on the concepts of Shannon's entropy and information [2]. Here we show that the entropy of a lithic tool, LT, has physical meaning, following arguments similar to those used by Boltzmann in a famous paper [3]. This new physical approach complements the informational treatment, reinforces our conclusions and opens new avenues of inquiry.

The statistical meaning of entropy was created by Boltzmann [3] and more modernly can be expressed in its general form as

$$S = -k \sum_j P_j \ln P_j, \tag{1}$$

where P_j is the probability for a physical system to be in its j-th complexion (as Boltzmann named a microstate). A crucial proposition of Boltzmann was to state that $P(Y)$, the probability for the system being in macro state Y with a certain distribution of energy among its elements is proportional to the number of complexions accessible in the state Y. The constant k in Eq. (1) is introduced to give S the same units as it has in Thermodynamics. Seventy years later, Shannon introduced the concept of the amount of information of a message j, an element of a set of messages $\mathcal{M}$, by

$$I_j^{\text{Shan}} = -\log_2 P_j^{\text{Shan}}, \tag{2}$$

where P_j^{Shan} is the probability of choosing the message from the set $\mathcal{M}$. It is customary in information science to use de $\log_2$ basis so that the unit of information quantity is the bit. Shannon also introduced the average information or uncertainty of $\mathcal{M}$ by

$$H = -\sum_{j \in \mathcal{M}} P_j^{\text{Shan}} \log_2 P_j^{\text{Shan}}. \tag{3}$$

Disregarding the difference in the meaning of P_j, the formal identity between S and H in Eqs. (1) and (3) is remarkable; hence, H is also known as the informational entropy. H has been used in Archaeology to quantify richness, heterogeneity and complexity in archaeological collections.

In our previous work we determined the probability that a given LT could have been produced by a natural random process, *i.e.*, without hominin intervention. We also determined Shannon's I and H, for 10 stone tools, between 3.3 My and 160 ky-40 ky of age and exhibited that with the passage of time and the evolution of technology, the entropy of a typical LT decreases and its quantity of information increases.

2. The stone tool as a physical system

But the stone tool is, after all, a physical system. Its surface shows the scars of flakes, thin slices of stone removed each by a blow imparted at a certain point $\mathbf{r}$ and oriented in a certain direction $\mathbf{k}$, see the illustration in Fig. 1. Hence the pattern of scars characterises the LT. For a given shape of the LT, $\mathbf{r}$ is determined by two polar angles (u, v) and if the strength of the blow is disregarded, the director $\mathbf{k}$ is also determined by two polar angles (θ, ϕ) giving its orientation with respect to the normal to the surface; see illustration in Fig. 2.

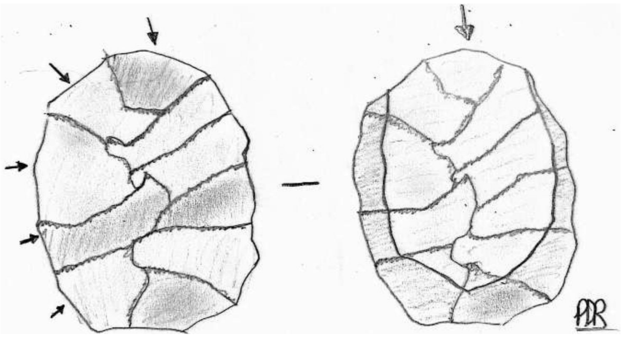

FIGURE 1. Classic Levallois lithic core from the Mousterian tradition. The inner irregular lines mark the boundaries between neighbouring scars. The arrows show the points of impact of 22 strokes and their direction. Drawing by PdRM based on Fig. 4 of [5].

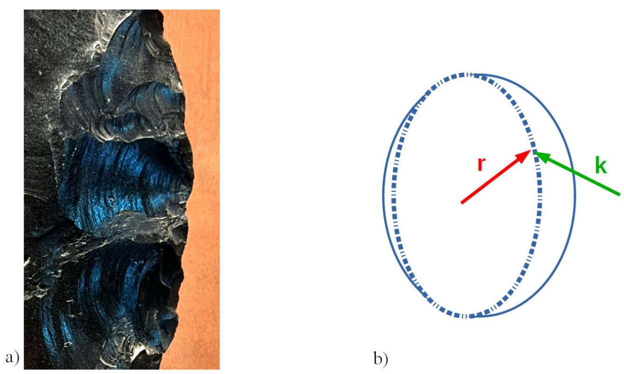

FIGURE 2. The state of a lithic tool is described by the scars left by the removal of flakes by strokes. a) The characteristic conchoidal scars left on an obsidian piece. [Photograph courtesy of Prof. G. Jackson.] b) Each stroke is described by a point of impact vector **r** and a director of the blow **k**.

2.1. Configuration of a stone tool

The blow that produces a flake scar on the stone core is then represented by four angles $(u, v), (\theta, \phi)$, which are in turn represented by a point in a 4-dimensional space Γ_c, which is the product of two subspaces,: $\Gamma_c = \Gamma_{pos} \otimes \Gamma_{dir}$. See Fig. 3. It is convenient, however, to discretise these spaces by defining elemental cells of size δa partitioning Γ_{pos} and $\delta\omega$ partitioning Γ_{dir}. See Fig. 4. The configuration of a LT with m strokes, with respect to its reduction process, is given by m cells in Γ_c. Hence an elemental configuration is denoted:

$$\Phi_{elem} = \{\delta\gamma_j = (\delta a_j, \delta\omega_j); j = 1, 2, ..., m\}, \qquad (4)$$

where j indicates a cell. If A is the surface area of the LT and Ω the available solid angle for a blow, there are $\mathcal{N}_a = A/\delta a$ cells in Γ_{pos} and $\mathcal{N}_\omega = \Omega/\delta\omega$ in Γ_{dir} for a total of $\mathcal{N}_\gamma = \mathcal{N}_a \times \mathcal{N}_\omega$ cells. The elemental parameters are $\delta a = 0.33$ cm^2 and $\delta\omega = \pi/1000$ sr, and are set from the estimated precision of the stone knappers. We propose that the cells $\delta\gamma_j$ play the rôle of Boltzmann's complexions.

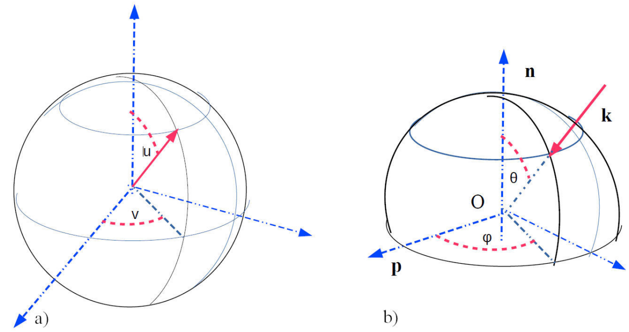

FIGURE 3. a) The configuration of a lithic tool is represented by a point γ_c in the four-dimensional space Γ_c formed by a two-dimensional position subspace, $\Gamma_{\rm pos}$, and b) a two-dimensional director subspace, $\Gamma_{\rm dir}$.

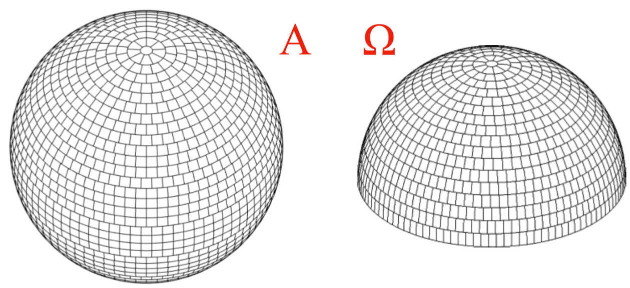

FIGURE 4. Discrete versions of the subspaces $\Gamma_{\rm pos}$ and $\Gamma_{\rm dir}$ partitioned respectively in elemental cells of sizes δa and $\delta\omega$. $\Gamma_{\rm pos}$ has an area A characteristic of each lithic tool, and the solid angle $\Gamma_{\rm dir}$ measures 2π sr for all convex tools.

2.2. A thought experiment: the rolling stones

Imagine an experiment in which $\mathcal{N} \gg 1$ identical stone cores are subjected each to m random blows –random both in space and direction– such as stones rolling down on a riverbed or being carried by an avalanche. At the end we examine the stones for blow scars. The blows on a stone are denoted $\{g_\alpha; \alpha = 1, 2, ..., m]$. How to tell whether one of these battered stones looks like a tool?

A roughly battered stone shows scars scattered all over its external surface A without any resemblance of order – actually some blows do not remove any flake because their direction $\mathbf{k}_\alpha$ is not well oriented. In contrast, a stone tool has strokes arranged in patterns that imply a correlation between both their positions and directions. The ordering is not absolutely precise, however: in the real world $(\mathbf{r}_\alpha, \mathbf{k}_\alpha)$ may appear slightly perturbed while keeping the effective use of the tool. For the LT this means that there are small regions $(\Delta a, \Delta\omega)$ on which $(\mathbf{r}_\alpha, \mathbf{k}_\alpha)$ can be focalised while maintaining the practical use of the tool, but when the perturbation is large and $(\Delta a, \Delta\omega)$ are too large, the stone has no practical use. These least focalised strokes correspond to $(\Delta a = A/2, \Delta\omega = \Omega/2)$. In summary there are two relevant features of the arrays (configurations): their focalisation and correlation. We have shown that the degree of focalisation of the strokes is enough to distinguish between a stone tool and a roughly battered stone.

This means that we shall not be interested in whether the blow is observed in a certain elemental cell $\delta\gamma$, but rather on whether it is found within a given subregion $\Delta\gamma$ of Γ_c.

A stone tool is the result of strokes $\{g_\alpha; \alpha = 1, 2, ..., m]$ focalised in small regions forming certain patterns. In contrast, a roughly battered stone corresponds to observe scattered strokes; the observation with the most scattered results has $\Delta a = A/2$ and $\Delta\omega = \Omega/2$.

2.3. Example of a lithic tool

In order to illustrate our discussion, we shall refer to a specific LT: a unifacial tool from Gona, Ethiopia, with $m = 9$ blows, and area $A = 124.75$ cm^2. This stone tool has $756,700$ elemental cells and $34,550$ effective subregions $(\Delta\gamma)$.

2.4. Observations of strokes as events

Keeping in mind the rolling-stones experiment, we now consider the probability that each of the $\{g_\alpha\}$ strokes is found within given subregions Δa_α located at $\mathbf{r}_\alpha^0$ and $\Delta\omega_\alpha$ with its axis at $\mathbf{k}_\alpha^0$ and call this an effective observation, and denote:

$$\Lambda_{\text{eff}} = \{\mathbf{r}_\alpha \in (\Delta a_\alpha; \mathbf{r}_\alpha^0), \mathbf{k}_\alpha \in (\Delta\omega_\alpha; \mathbf{k}_\alpha^0)\}. \tag{5}$$

Then the fraction of stones found with strokes in those effective subregions determines its probability

$$\mathcal{N}(\Lambda_{\text{eff}})/\mathcal{N} \;\rightarrow\; P(\Lambda_{\text{eff}}). \tag{6}$$

For simplicity we keep $\Delta a_\alpha = \Delta a$ and $\Delta\omega_\alpha = \Delta\omega$ constant for a given stone tool. The effective configuration of focused blows corresponds to the stone tool:

$$\Lambda_{\text{tool}} = \Lambda_{\text{eff}}(\Delta a \ll A,\, \Delta\omega \ll \Omega). \tag{7}$$

3. Basic assumptions

We assume the rolling-stones experiment to be completely random, as natural elements tend to be. Hence any stroke hits randomly on the surface S of the stone tool, and so $\mathbf{k}$ and $\mathbf{r}$ are independent events. For a continuous Γ_c with probability density $p(u,v)$ with $\mathbf{r} = (u,v)$ on the surface S, we have

$$P(\mathbf{r} \in \Delta a) = \int_{\Delta a} du dv\, p(u,v). \tag{8}$$

The collision frequency $f(u,v)$ at a point (u,v) of S of a stone, hit randomly by other stones, has been investigated by several authors and found to depend linearly on the Gaussian and mean curvatures of S at that point, $\rho_G(u,v)$ and $\rho_m(u,v)$ [7, 8]. Hence we make the plausible assumption that the probability density

$$p(u,v) \propto f(\rho_m(u,v), \rho_G(u,v)). \tag{9}$$

Using the result from the literature and normalising the density, we find

$$p(u,v) = \frac{1 + 2R_{\text{eff}}\rho_m(u,v) + R_{\text{eff}}^2\rho_G(u,v)}{16\pi R_m^2}. \tag{10}$$

Here, R_{eff} is an effective radius of mean curvature and the shape of S is a parametric relation $(x,y,z) = \psi(u,v)$. We have also assumed, for simplicity, that the stone core has approximately a tri-axial ellipsoidal shape with semi-axes (a,b,c). All the necessary quantities have been calculated for a wide range of eccentricities and expressed in terms of the semi-axes (a,b,c).

A final simplifying assumption is to neglect the effect of curvature and assume a uniform density, which normalised reads

$$p(u,v) = p(u)p(v) = \frac{1}{A}. \tag{11}$$

The probability for joint positive outcomes $(\sigma_a, \sigma_\omega) = (+,+)$ in position and direction in finite effective subregions $\Delta\gamma = (\Delta a, \Delta\omega)$ is

$$P_{\Delta\gamma}(+,+) = P(\mathbf{r} \in \Delta a)P(\mathbf{k} \in \Delta\omega). \tag{12}$$

The uniform probability densities in Eq. (11) are $p(\mathbf{r}) = p(u,v) = 1/A$ and $p(\mathbf{k}) = p(\theta,\phi) = 1/\Omega$, therefore

$$P_{\Delta\gamma}(+,+) = (\Delta a/A)(\Delta\omega/\Omega). \tag{13}$$

In order to reach an important result, we multiply and divide by the elemental cells $\delta a, \delta\omega$ to write

$$\Delta a/A = (\Delta a/\delta a)/(A/\delta a) = \mathcal{N}_{\Delta a}/\mathcal{N}_A, \tag{14}$$

and

$$\Delta\omega/\Omega = (\Delta\omega/\delta\omega)/(\Omega/\delta\omega) = \mathcal{N}_{\Delta\omega}/\mathcal{N}_\Omega, \tag{15}$$

with $\mathcal{N}_{\Delta a}, \mathcal{N}_{\Delta\omega}, \mathcal{N}_A$ and $\mathcal{N}_\Omega$ respectively equal to the number of elemental cells in Δa, $\Delta\omega$, A and Ω. Then, of course,

$$\mathcal{N}_{\Delta\gamma} = \mathcal{N}_{\Delta a}\mathcal{N}_{\Delta\omega}. \tag{16}$$

The probability to observe stroke g_α in the subregion $\Delta\gamma$ is, in terms of elemental cells,

$$P_\alpha(+,+) = P_{\Delta\gamma}^{(\alpha)}(+,+) = \mathcal{N}_{\Delta\gamma}^{(\alpha)}/\mathcal{N}_{\Gamma_c}, \tag{17}$$

where $\mathcal{N}_{\Gamma_c}$ is the number of elemental cells in Γ_c.

Assuming the strokes to be non-overlapping

$$P^{(m)}(+,+) = \prod_{\alpha=1}^{m} P_\alpha(+,+) = \frac{1}{\mathcal{N}_{\Gamma_c}^{\alpha}} \frac{\mathcal{N}_{\Delta\gamma}^{(m)}!}{m!(\mathcal{N}_{\Delta\gamma}^{(m)} - m)!}. \tag{18}$$

In conclusion, the probability to have Λ_{eff} equals the fraction of combinations of accessible elemental cells. Although, of course, this differs in many ways from the thorough and ample work of Boltzmann, the fact is that the probability of making an effective observation of the set of strokes is proportional to the combinatorial expression in Eq. (18), and this is in the direction of his great work. This assertion completes the analogy with Boltzmann's treatment.

3.1. Observing the outcomes of a stroke

The single process of observing an effective blow on the stone core has four outcomes with probability distribution

$$P_\alpha(\sigma_a, \sigma_\omega) = \{P_\alpha(+,+), P_\alpha(-,+), P_\alpha(+,-), P_\alpha(-,-)\}. \quad (19)$$

The first outcome has a probability determined by Eq.(12), the rest are

$$P_{\Delta\gamma}(-,+) = (1 - \Delta a/A)(\Delta\omega/\Omega), \quad (20)$$

$$P_{\Delta\gamma}(+,-) = (\Delta a/A)(1 - \Delta\omega/\Omega), \quad (21)$$

and

$$P_{\Delta\gamma}(-,-) = (1 - \Delta a/A)(1 - \Delta\omega/\Omega). \quad (22)$$

Since for a focalised stroke $\Delta a/A \ll 1$ and $\Delta\omega/\Omega \ll 1$ the distribution $P_\alpha^{\text{foc}}(\sigma_a, \sigma_\omega)$ is very skewed. In our LT example from Gona the focalised distribution is

$$P_{foc}^{(\alpha)-ex} = \{1.06 \times 10^{-5}, 9.89 \times 10^{-4}, 1.06 \times 10^{-2}, 0.975\}.$$

But in sharp contrast, the scattered distribution is always uniform

$$P_{scat}^{(\alpha)} = 1/4, 1/4, 1/4, 1/4. \quad (23)$$

4. Probability, information and entropy of a stone tool

The probability $P(\Lambda_{\text{eff}})$ of observing in the random rolling-stones experiment a stone tool characterised by a certain focalised configuration, is that of observing each of its m blows within a corresponding focalised region $\Delta\gamma_\alpha$. We associate this probability with the stone tool, explicitly

$$P_{\text{tool}} = \prod_{\alpha=1}^{m} P_{\text{foc}}^{(\alpha)}(+,+). \quad (24)$$

As a result, the Gona LT example has

$$P_{\text{tool}}^{ex} = 4.66 \times 10^{-43}.$$

The rough stone has probability

$$P_{\text{rough}} = \prod_{\alpha=1}^{m} P_{\text{scat}}^{(\alpha)}(+,+), \quad (25)$$

which gives

$$P_{\text{rough}}^{ex} = 3.82 \times 10^{-6}.$$

So the lithic tool is 2.4×10^{-93} less probable than the natural battered stone.

The quantity of information conveyed by a message occurring with probability P_α has been introduced as [2, 6]

$$I_\alpha = -\log_2 P^{(\alpha)}, \quad (26)$$

where we have opted to keep the unit of bit for I. We thus define the amounts of information obtained by finding the tool, that is, by observing the m blows within the focalised regions $\Delta\gamma_\alpha^{\text{foc}}$ as

$$I_{\text{tool}} = -\sum_{\alpha=1}^{m} \log_2 P_{\text{foc}}^{(\alpha)}(+,+), \quad (27)$$

and by observing the fully scattered blows on a rough stone

$$I_{\text{rough}} = -\sum_{\alpha=1}^{m} \log_2 P_{\text{scat}}^{(\alpha)}(+,+). \quad (28)$$

For the stone of our example, we obtain

$$I_{\text{tool}}^{ex} = 140.6\,\text{bits}; \; I_{\text{rough}}^{ex} = 18\,\text{bits}. \quad (29)$$

Originally, the entropy of a message was introduced by Shannon as the mean information carried by it. For an isolated event with probabilities $p(+)$ and $p(-) = 1 - p(+)$, the mean information carried by both outcomes of the distribution $P = (p(+), p(-))$ is

$$\langle I \rangle(p) = -p(+) \; \log_2 p(+) - p(-) \; \log_2 p(-), \quad (30)$$

a quantity that is zero for both certain outcomes $p(+) = 0$ and $p(+) = 1$, but reaches its maximum $\langle I \rangle = \log_2 2 = 41$ when the two outcomes are equally likely: $p(+) = p(-) = 1/2$. [Information quantities are given in units of bits.]

The probability distribution corresponding to a stone tool involves all its outcomes. The distribution for stroke g_α its distribution is given by Eq, (19). Thus the event of giving one blow has entropy (also meaning information or uncertainty) given by

$$S_{foc}^{(\alpha)} = -\sum_{\sigma_a, \sigma_\omega} P_{foc}^{(\alpha)}(\sigma_a, \sigma_\omega) \ln P_{\text{foc}}^{(\alpha)}(\sigma_a, \sigma_\omega). \quad (31)$$

We have omitted the Boltzmann constant factor of k in this definition but kept the natural logarithm, then the units of S are nats. For the stone tool, in similarity with Eq. (27), we define S_{tool} on the positive outcomes that gave rise to the LT, that is,

$$S_{\text{tool}} = -\sum_{\alpha=1}^{m} S_{foc}^{(\alpha)}(+,+). \quad (32)$$

We note that this differs from the entropy reported in our previous publication [1]. Now, for the rough stone

$$S_{\text{rough}} = -\sum_{\alpha=1}^{m} S_{scat}^{(\alpha)}(+,+). \quad (33)$$

For the lithic tool of the example: $S_{\text{tool}} = 9.90 \times 10^{-4}$ nats, and $S_{\text{rough}} = 1.355$ nats. The entropy of a stone tool is really a property of the reduction process. The scattered process that leads to the rough stone is 1400 times more entropic or uncertain than the process that leads to the tool of the example.

4.1. Uniqueness of lithic tool

Up to this point we have analised a lithic tool characterised by blows in certain subregions of its surface and within certain solid-angle cones. We must consider, however, the fact that there are many other equivalent arrangements of strokes but located on different parts of the surface. By equivalent we mean that the elements Δa_α and $\Delta\omega_\alpha$ are the same in number m and size, but located in different places of the area A and directions within the solid angle Ω.

Since there are $\mathcal{N}_a = \Delta a/A$ subareas in A and $\mathcal{N}_\omega = \Delta\omega/\Omega$ sub-cones in Ω, and there are m strokes, the number of equivalent LT with the same effective observation is increased by the double combinatorial factor

$$M_m = \binom{\mathcal{N}_a}{m}\binom{\mathcal{N}_\omega}{m}, \tag{34}$$

and the probability of finding any equivalent tool is

$$P_{\rm equi} = M_m \times P_{\rm tool}. \tag{35}$$

We find that, even though M_m can be a very large number, the probability for an equivalent LT $P_{\rm equi}$ remains small. Thus, for the example at hand, we find that $P_{\rm equi} = 6.50 \times 10^{-14}$.

5. Selected results

There is scarce information in the literature about the parameters δa and $\delta\omega$. There are experimental studies with modern knappers. [13, 14]. On this basis, our choice of parameters was $\delta a = 0.33$ cm^2 and $\delta_\omega = \pi/1000$ sr; the latter corresponds to an apex angle of 3.5^o.

5.1. Stone tools analised

In the previous paper [1], we treated 10 lithic tools from different sites and dates. Here we illustrate the method with information about 5 lithic tools of the same sites. The tools denoted by the name of the site. Their site (abbreviation), age and reference are the following: 1) Lomekwi, Kenya, (Lom) 3.3 My [17]; 2) Gona, Ethiopia, (Gona) 2.3 / 2.5 My [16]; 3) Omo, Ethiopia, (Omo) 2.3 My [15]; 4) Melka Kunture, Ethiopia, (MK) 1.5 / 0.83 My [18]; 5) Corbehem, France, (Cor);150 / 40 ky [19]. Drawings of the first four of these LT are shown in Figs. 5, 6 and 7.

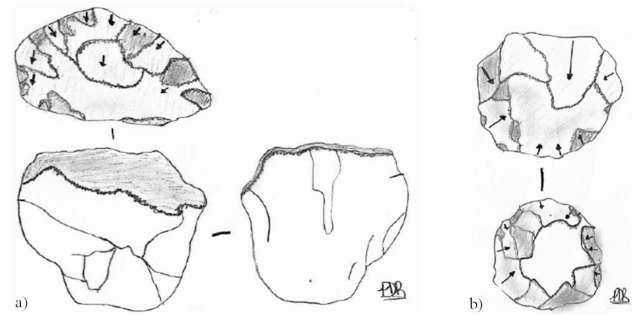

FIGURE 6. a) Depiction of lithic tools from Lomekwi, Kenya [17] and b) from Omo, Ethiopia [15]. Drawing by PdR; the tools are not to scale.

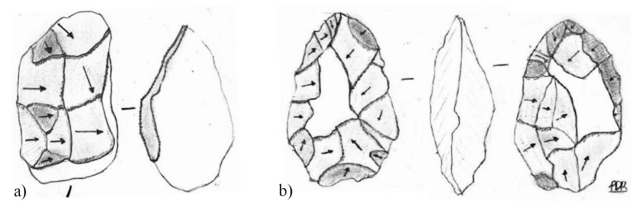

FIGURE 7. a) Depiction of lithic tools from Gona, Ethiopia [16] and b) from Melka Kundera, also in Ethiopia [18]. Drawing by PdR; the tools are not to scale.

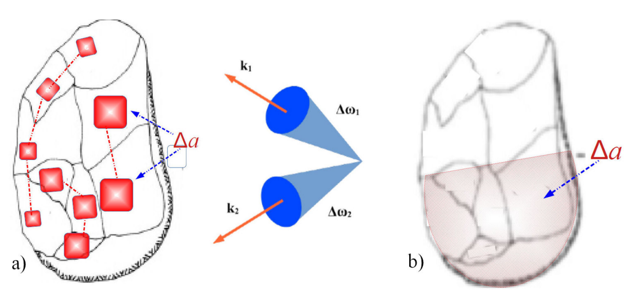

FIGURE 5. Observations as events. The blows are within subregions $(\Delta a_\alpha, \Delta\omega_\alpha)$ a) focalized effective observation of 9 strokes on a lithic tool from Gona, Ethiopia. b) Scattered effective observation of the same 9 blows on the same lithic tool. The available solid angle is half a hemisphere, $\Delta\omega_{scat} = \pi$ sr.

The relevant probability measures $P_{\rm tool}$ and $P_{\rm rough}$ about the five LT analysed here are given in Table I. The same table includes the (larger) probability of observing an equivalent flaked stone P_{equi} form in Eq. (35), which even if it is much larger than $P_{\rm tool}$ is still much smaller than P_{rough}. Observing a focalised set of strokes on a stone is much, much less likely than observing a naturally battered stone. This unlikeness becomes accentuated as time changes from the oldest LT with 3.3 My of age to the most recent one with only between 150 ky and 40 ky.

Table II shows, for each LT treated here, the number of strokes m, the entropies $S_{\rm tool}$ and $S_{\rm rough}$. In all cases $S_{\rm tool}$ is consistently smaller than $S_{\rm rough}$, meaning that the probability distribution for the LT is more skewed than for the scattered distribution, which in all cases is uniform and reaches its maximum value; this last situation, in the realm of

TABLE I. Probability of observation of lithic tools vs rough stones.

Site	P_{tool}	P_{equi}	P_{rough}
Lom-2	6.77×10^{-44}	1.02×10^{-11}	3.81×10^{-6}
Omo-1	2.15×10^{-38}	3.30×10^{-10}	1.53×10^{-5}
Gona-1	4.66×10^{-43}	6.50×10^{-11}	3.82×10^{-6}
MK -1	7.03×10^{-230}	1.61×10^{-75}	2.07×10^{-25}
Cor-1	8.84×10^{-105}	8.83×10^{-20}	3.64×10^{-12}

TABLE II. Number of strokes, entropies and quantities of information of lithic tools vs rough stones; the unit of entropy is nat and the unit of quantities of information is bit.

Site	m	S_{tool}	S_{rough}	I_{tool}	I_{rough}
Lom-2	9	6.99×10^{-4}	1.355	143.4	18
Omo-1	8	7.37×10^{-4}	4.816	125.1	16
Gona-1	9	9.90×10^{-4}	1.355	140.6	18
MK -1	41	8.70×10^{-4}	6.171	761.3	82
Cor-1	19	3.47×10^{-4}	2.860	361.4	38

TABLE III. Effect of curvature: Effective radius $R_{\rm eff}$ and probability densities on ellipsoid: uniform p_0 and at vertices $p(a)$, $p(b)$ and $p(c)$.

Site	$R_{\rm eff}$/cm	p_0	$p(a)$	$p(b)$	$p(c)$
Lom-2	6.25	0.00145	0.00356	0.00288	0.00123
Omo-1	2.45	0.01481	0.0492	0.0275	0.0062
Gona-1	3.30	0.00802	0.0188	0.0069	0.0053
MK -1	5.40	0.00330	0.0191	0.0076	0.0011
Cor-1	2.97	0.01011	0.0290	0.0192	0.0043

thermodynamics, would correspond to the equilibrium states. This difference also means that the effective observation of the LT is a process with relatively small uncertainty.

The quantities of information for $I_{\rm tool}$ and $I_{\rm rough}$ also appear in Table II. The LT, in all cases, contains much more information than the roughly battered stone, and the ratio $I_{\rm tool}/I_{\rm rough}$ increases as time passes.

Finally, Table III contains the results about the influence of the curvature on the probability density $p(u, v)$ for each of the LT considered here. The shape of each LT was assumed to be ellipsoidal, with semi-axes (a, b, c) evaluated from previous publication [1]. The densities $p(u, v)$ were calculated from Eq. (10). For a given LT the points of largest curvature are the two vertices at the end of the largest semi-axis a, where the probability density $p(a)$ is also largest. The density $p(c)$ is the smallest at the vertices at the end of the shortest axes c, where the ellipsoids are flatter. Of course, $p(c) < p(b) < p(a)$. For reference, the table includes the uniform density p_0 and the effective radius obtained from the integrated mean curvature.

6. Conclusions

The entropy of a lithic tool can be defined by physical arguments following the arguments of Boltzmann in his 1877 paper. This fact establishes a bridge with the equivalent treatment based on Shannon's Theory of Information. The probability that a stone tool is produced by random natural strokes is extremely small. Much less than that of observing an equivalent rough stone with scattered scars. The entropy associated with the stone tool is much smaller than that of the roughly battered stone, which is the maximum entropy, as occurs to a thermodynamic system when reaching equilibrium. The lithic tool and its process of manufacture contain a considerable amount of information. A first analysis of 10 stone tools showed that the entropy of the lithic tools decreases and their information content increases with the course of prehistoric time. Nevertheless, the sample analysed is rather small to allow for archaeological conclusions. This proposal contributes a new quantitative tool in the Archaeology.

Work in course has provided with the practical basis to carry out the ellipsoidal model for the probability density. Here we have presented results to judge the effect of curvature. The focalised observation of strokes is being extended to account for correlation between strokes. The determination of elemental parameters δa and δ_ω is being improved by a Bayesian analysis. Most importantly, several distinguished archaeologists from the Universidad de Valencia, led by Prof. Valentín Villaverde Bonilla are joining the project, therefore enriching its scope.

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