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LOW-ENERGY LEVEL-SPACING DISTRIBUTION IN THE NUCLEAR TABLE*

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ABSTRACT: The nuclei in the nuclear table are considered as a realization of an ensemble of Hamiltonians, and the distribution of spacings s_{01} between the ground and first excited state is analyzed. We show that, once nuclei with a systematic s_{01} behaviour are eliminated, s_{01}, measured in units of its average, follows a Poisson distribution.

I. INTRODUCTION

Statistical properties of nuclear spectra have always been studied considering for a given nucleus a stretch of highly excited energy levels[1,2]. The properties analyzed can be divided into two broad classes: local and global properties. The global properties, a typical example of which is the level density ρ, vary with the excitation energy E; on the other hand, the

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local properties, for example the spacing distributions, correspond to the fluctuations in the spectrum and if necessary are corrected for the energy variation

Fluctuations have been analyzed using random matrix theory, in which an ensemble of random Hamiltonians is introduced, as a mathematical device, to compute among other things the k-th neighbour spacing distributions $p(k;s)$, that is, the probability of finding two levels at a distance s with k levels in between. The predictions of random-matrix theory are well confirmed experimentally[3]. In particular, level sequences characterized by the same values of the only good quantum numbers (total spin J and parity π) show in regions of constant density a nearest-neighbour spacing distribution $p(0;s)$ which is closely approximated by Wigner's surmise[4]

$$p_W(0;s) = (\pi/2D)(s/D)\exp\left[-(\pi/4)(s/D)^2\right] , \qquad (1)$$

where $s = E_{i+1} - E_i$ and D is the mean spacing, related to the local level density ρ by $D = \rho^{-1}$. Furthermore, if level sequences of many values of J and π are mixed together[5], the nearest neighbour spacings are distributed according to a Poisson distribution

$$p_P(0;s) = D^{-1}\exp(-s/D) , \qquad (2)$$

which corresponds to a completely random sequence of levels.

As we mentioned above, the ensemble usually plays the role of a mathematical device, used to compute fluctuations of the spectrum of a given nucleus. For example, in order to calculate the nearest-neighbour spacing distribution, *a single* spacing is taken for each spectrum and its distribution throughout the ensemble is computed[2]; the assumption is then made that the statistical properties of the ensemble coincide with those of "almost all" spectra (ergodic property)[6,7]. In this paper we shall study a "realization" of a sample from the ensemble, by considering all the nuclei in the nuclear table. We shall thus take *a single* spacing for each nucleus and compute its distribution throughout the nuclear table. In other words, the experimental comparison will follow step by step what is done in the theoretical calculation and the ergodic property is not needed.

Specifically, we shall discuss the distribution of the energy difference s_{01} between the first excited and the ground state of all nuclei in the nuclear table, without taking into account their J^π values. This is to be

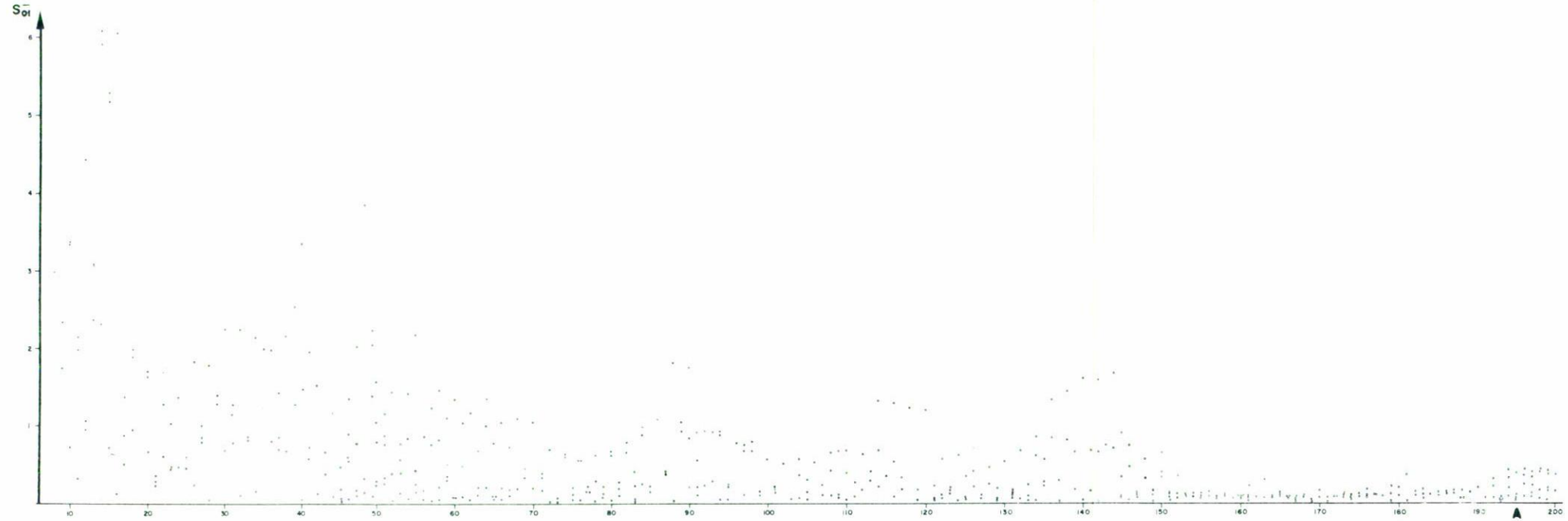

Fig. 1. Diagram of the first-excited state energy s_{01} for all nuclei with $8 \leq A \leq 200$. Notice the erratic behaviour, except for nuclei in the deformed region.

contrasted with the usual applications of statistics in nuclear physics, where only the highly excited region of the spectrum is analyzed.

In our problem, the equivalent of global properties would be those which show a systematic variation with the mass number A. This will be studied in Section II for the energy distance s_{01}. In order to study local properties, i.e. fluctuations with respect to this mean value, the systematic behaviour of s_{01} must be eliminated; this is done in Section III, where the probability distribution of s_{01} is analyzed.

II. SYSTEMATIC BEHAVIOUR OF THE FIRST EXCITED-STATE ENERGY

The variation of s_{01}, the first excited-state excitation energy, throughout the nuclear table was first studied soon after the birth of nuclear-shell theory. The values of s_{01} for even-even nuclei plotted versus the neutron number N lie on a rather smooth curve[8]. This is not true, however, if all the nuclei are taken into account, as is shown in Fig. 1, for nuclei with mass number A, with $8 < A < 200$. Here, s_{01} is plotted versus A and the erratic behaviour from one nucleus to the next is apparent, except for nuclei in the deformed region.

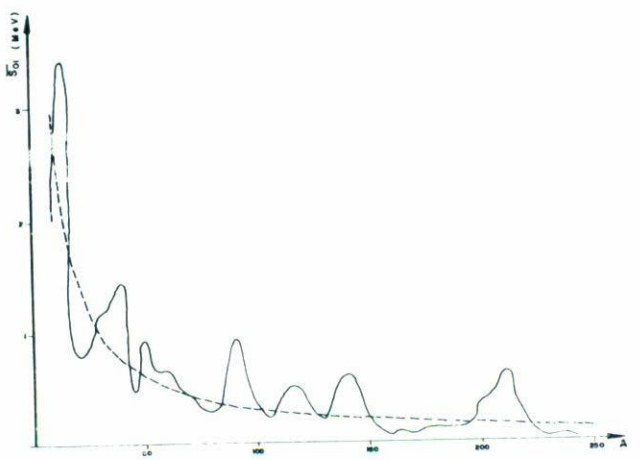

Fig. 2. Diagram showing the average of s_{01} as a function of A. Both the "intermediate structure" (solid line) as well as the "gross structure" curve (dotted line) $s_{01} = 30/A$ are given.

On the average, however, s_{01} clearly decreases with A. This is shown (solid line) in Fig. 2, where for each A the average value of s_{01} for ten neighbouring nuclei is given. The peaks due to magic and semi-magic nuclei show up clearly, as well as the valley corresponding to nuclei in the deformed region.

We therefore have in Fig. 1 examples of the "fine structure" behaviour of s_{01}, whereas in Fig. 2 the average of s_{01} still shows evidence of an "intermediate structure". One could go a step further and obtain the "gross structure" by eliminating the peaks and valleys appearing in Fig. 2. For this gross structure we propose a function of the type

$$\overline{s}_{01} = \alpha/A \tag{3}$$

and determine α by a least-squares fit to 610 spacings s_{01}, taken directly from the nuclear tables[9]. The value obtained was $\alpha = 30$ MeV and the resulting curve is plotted as a dashed line in Fig. 2.*

We are now prepared to tackle the problem of the fluctuations of s_{01}, which is the main object of this paper.

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*One could use the slightly more general form α/A^{β} to represent the gross structure of s_{01}. However, if one performs a least-squares fit with this function, there are several sets of values of α and β which are equally good and nothing is gained. In fact, using the x_1-statistic[10], B. Chen has shown[11] that the errors for α and β are of the order of 50%. One should then be guided by a simple model: consider the problem of computing the level density of a noninteracting Fermi gas contained in a spherical cavity of radius $r_0 A^{1/3}$. To first order in the wave number κ, the single-particle level density is proportional to κV, V being the volume of the cavity[12]. Evaluating the quantity at the Fermi momentum κ_F, and taking into account spin and isospin degeneracy factors, one obtains

$$\overline{s}_{01} = (3\pi^2/8)^{1/3} (\hbar^2/2mr_0^2)\, A^{-1} , \tag{4}$$

leading to a value of $\beta = 1$. Here m is the effective nucleon mass. Using $\alpha = 30$ MeV and m equal to the free nucleon mass, a value of $r_0 = 1.0f$ is obtained. One should recall that for even-even nuclei the gross-structure behaviour of s_{01} was analyzed theoretically a long time ago using the collective model[13]. The result was $\alpha = 100$ MeV and $f = 5/6$; this, however, does not fit the experimental data.

III. FLUCTUATIONS OF THE FIRST EXCITED-STATE ENERGY

To analyze the fluctuations of s_{01}, each of these energies is measured in units of $\bar{s}_{01}$ as given by Eq. (3), thus eliminating the systematic behavior as a function of A.

We have plotted in Fig. 3 the histogram of $s_{01}/\bar{s}_{01}$, for all nuclei corresponding to $8 < A \leq 200$. The corresponding Poisson distribution, which

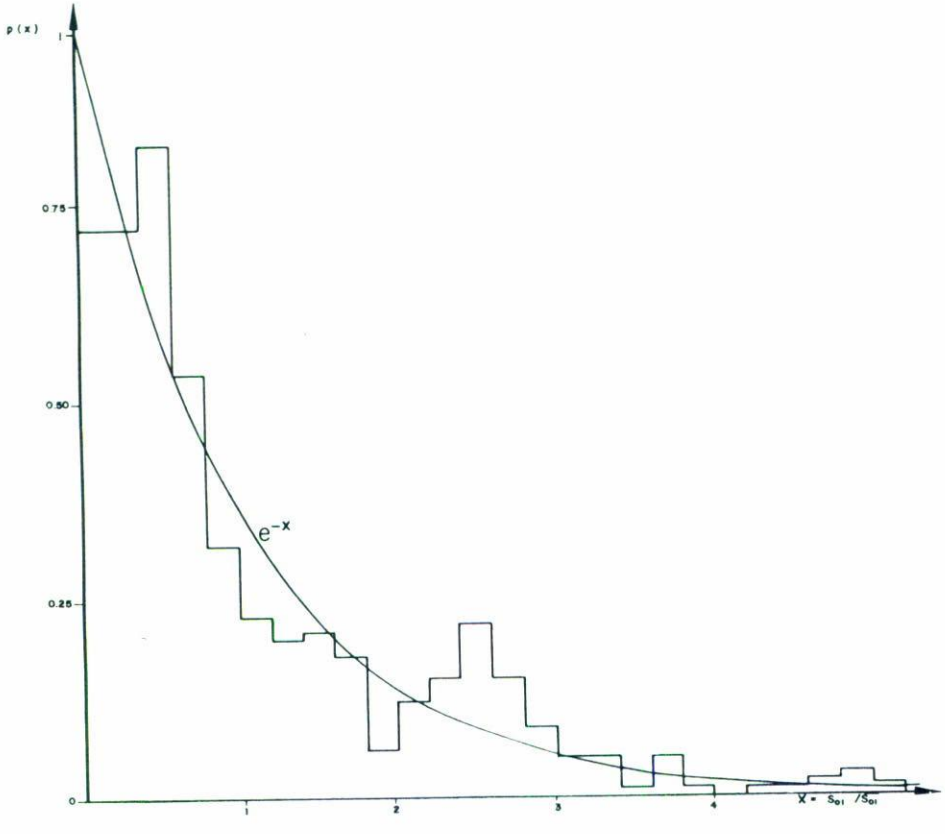

Fig. 3. Histogram for $x = s_{01}/\bar{s}_{01}$ for all nuclei with $8 < A < 200$, compared to the corresponding Poisson distribution. The histogram contains 505 points.

would be valid for a completely random sequence of distances, is also shown in Fig. 3. As is clear from this figure, the histogram does not follow a Poisson distribution: one notices the accumulation of values around $s_{01} \sim 0.4\,\bar{s}_{01}$ and $s_{01} \sim 2.5\,\bar{s}_{01}$. The bump around $0.4\,\bar{s}_{01}$, is due to distances belonging to deformed nuclei. As was mentioned in the previous section, and as is shown in Fig. 4, for these nuclei the s_{01} lie too close to their mean value, which in the units used in the histogram is approximately 0.4.

We have thus eliminated distances belonging to deformed nuclei; we are left with the histogram shown in Fig. 5, which now resembles a Poisson distribution, except for the bump around $s_{01} = 2.5\,\bar{s}_{01}$. A χ^2 test shows that this bump is statistically significant: $P(\chi^2)$, the probability of χ^2, is of the order of 10^{-5}. One should, therefore, seek for other sets of nuclei that, like those belonging to the deformed region, show a systematic behavior of s_{01}.

At this point we have followed two different citeria: the first, we have removed all singly and doubly closed-shell nuclei and in the second all even-even nuclei. The resulting histograms are shown for $8 \leq A \leq 150$ in Figs. 6 and 7, respectively. For the case of Fig. 6 a value of $P(\chi^2) \sim 0.04$ is ob-

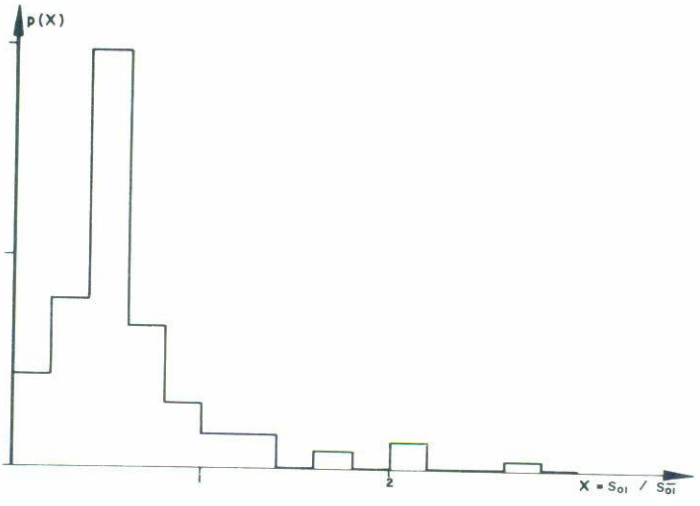

Fig. 4. Distribution of s_{01} for 108 nuclei in the deformed region, $150 < A < 190$.

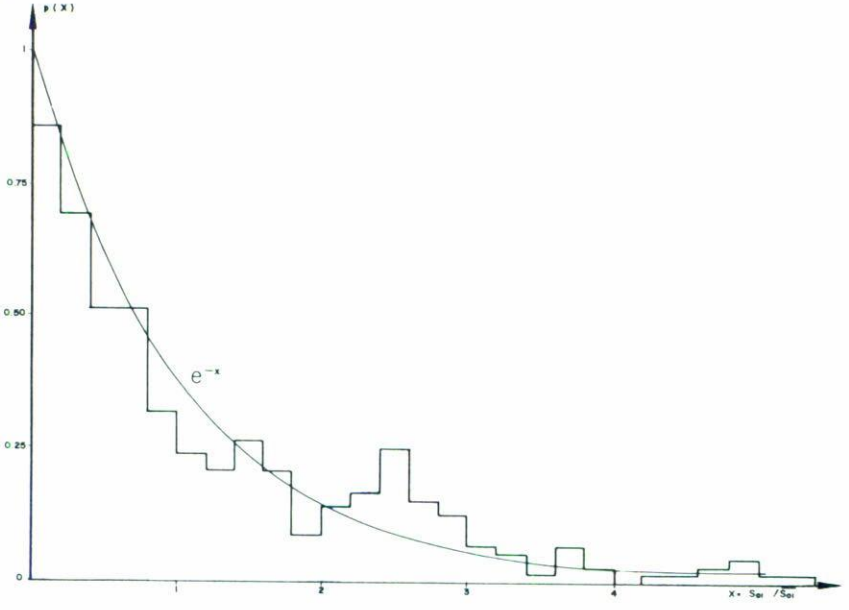

Fig. 5. Comparison of the Poisson distribution with the resulting histogram of $s_{01}/\bar{s}_{01}$, once the nuclei in the deformed region have been excluded; 369 nuclei with $8 \leqslant A \leqslant 150$ are taken into account.

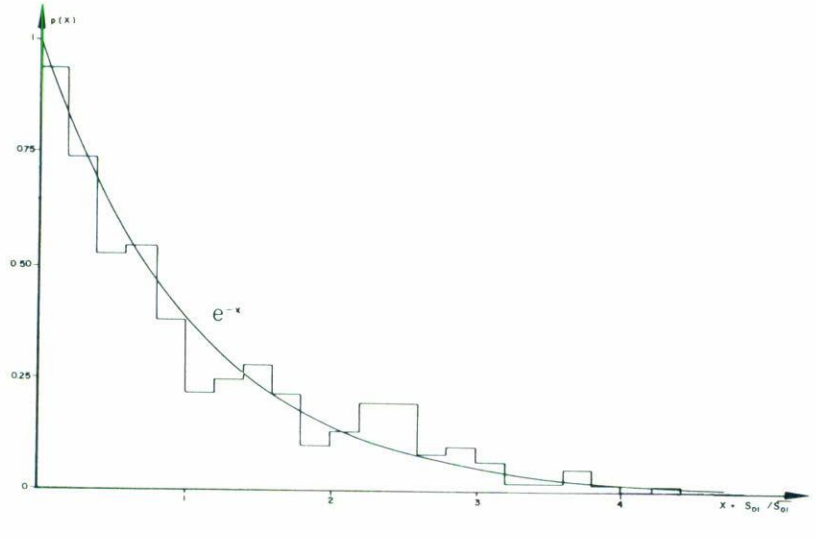

Fig. 6. Histogram for the spacing s_{01} obtained after the rotational as well as magic and semi-magic nuclei have been excluded. The corresponding Poisson distribution is also shown. The mass number A ranges from $A = 8$ to $A = 150$ and 308 nuclei were taken into account.

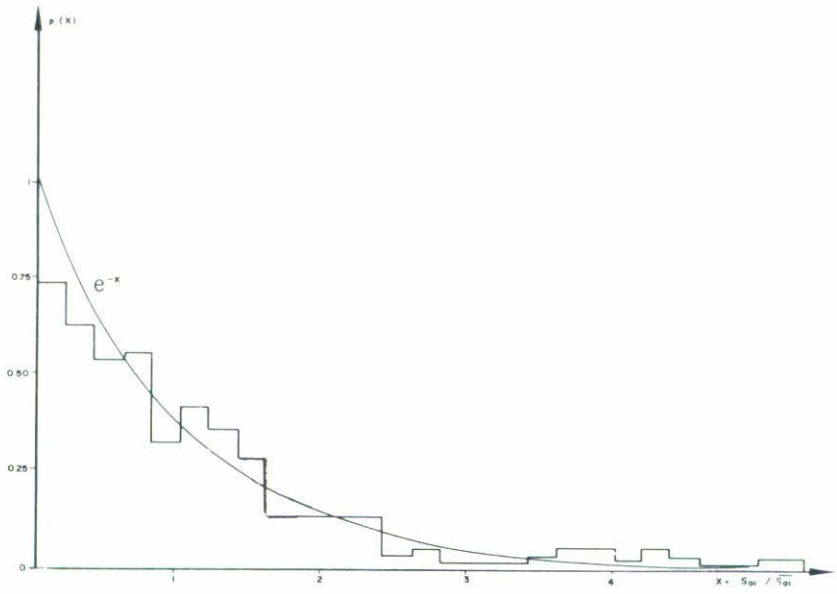

Fig. 7. Histogram for $s_{01}/\bar{s}_{01}$ after spacings from even-even nuclei as well as those from nuclei in the deformed region are eliminated; 264 distances are left. The average $\bar{s}_{01}$ is now represented by the form $\bar{s}_{01} = 18.7/A$, obtained from the data by least-squares fit.

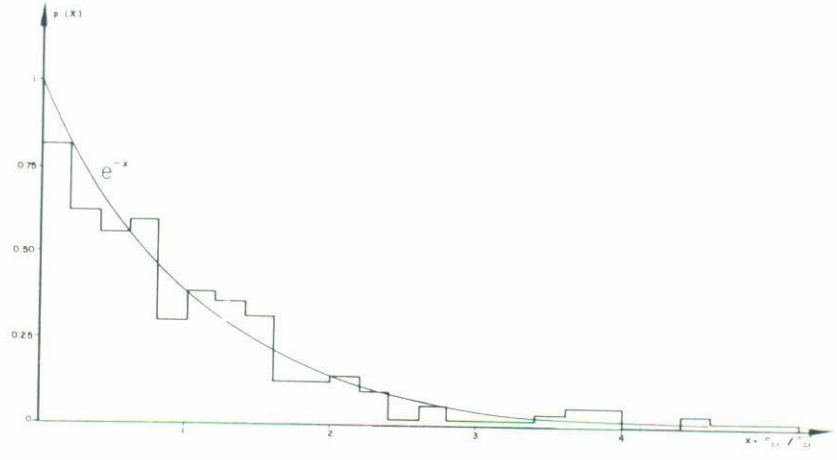

Fig. 8. Histogram corresponding to the first nearest-neighbour distance $s_{01}/\bar{s}_{01}$, obtained after the rotational, the magic and semi-magic, as well as the even-even nuclei have been eliminated. The unit $\bar{s}_{01}$ used is the same as in Fig. 7. The mass number A varies from $A = 8$ to $A = 150$ and 232 nuclei are considered.

tained, while $P(\chi^2)$ rises to 0.08 for the histogram shown in Fig. 7. Finally, when both magic, semi-magic and even-even nuclei are taken out, the histogram of Fig. 8 is obtained, which follows a Poisson distribution with a $P(X^2) \sim 0.50$.

IV. FINAL REMARKS

We have shown that, once nuclei with a systematic s_{01} behavior are eliminated, the energy differences s_{01} between the first excited and ground states, measured in units of the average $\bar{s}_{01}$, follow a Poisson distribution. This statistical regularity show up only after three different sets of nuclei are not taken into account: even-even, magic and semi-magic and rotational nuclei.

This is only the first step in the analysis of nuclear spectrum fluctuations in the ground state region, using the nuclear table as a realization of an ensemble of hamiltonians. For instance, one should consider the next few nearest-neighbour distances s_{12}, s_{23} ..., as well as the next-to-nearest neighbour energy differences s_{13}, s_{24}, ..., again without regarding their J^π values.

The aim of this analysis will be to gain greater understanding of the relations among nuclei from the standpoint of random matrix theory. An important question arises from the fact that we have found a Poisson distribution for the s_{01}; this arises from the superposition of independent ensembles[5], each of which possesses internal correlations that lead to quite different spacing distributions, and we must ask whether a classification of the nuclei can be found that reveals such structure. For instance, distances between nuclear levels belonging to the same J and π should be analyzed, and the possible principles of classification should be studied.

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REFERENCES

1. C. E. Porter (ed.) *Statistical Theories of Spectra*, (Academic Press, N. Y., 1965)
2. M. L. Mehta, *Random Matrices*, (Academic Press, N. Y., 1967)
3. H. I. Liou, H. Camarda, S. Wynchank, M. Slagowitz, G. Hacken, F. Rahn, and J. Rainwater, Phys. Rev. C5 (1972) 974.
4. E. P. Wigner, *Gatlinburg Conference on Neutron Physics*, ORNL 2309 (1957) 59 (reprinted in ref. 1).
5. I. I. Gurevich and M. I. Pevsner, Nucl. Phys. 2 (1956) 575; N. Rosenzweig and C. E. Porter, Phys. Rev. 120 (1960) 1698.
6. W. H. Olson and V. R. Uppuluri, *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability*, (U. of California Press, 1970) Vol. III, p. 615.
7. T. A. Brody and P. A. Mello, Phys. Letters 37A (1971) 429.
8. G. Sharff-Goldhaber, Phys. Rev. 90 (1953) 587.
9. Nuclear Data Group, Section B, Current Sheets.
10. C. Marmasse, Rev. Mex. Fís., 20 (1971) 239.
11. B. Chen, private communication.
12. R. Balian and C. Bloch, Ann. Phys. (N.Y.) 60 (1970) 401.
13. K. W. Ford, Phys. Rev. 90 (1953) 29.

RESUMEN

Se considera a los núcleos de la tabla nuclear como una realización de un ensemble de Hamiltonianos y se analiza la distribución del espaciamiento s_{01} entre el primer estado excitado y el estado base. Se demuestra que, una vez que se han eliminado núcleos con un comportamiento sistemático, el espaciamiento s_{01}, medido en unidades de su promedio, sigue una distribución de Poisson.