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WHAT IS THE SPIN OF THE PION?

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ABSTRACT: Although it is generally believed that the pion is a spin-zero particle and therefore spherically symmetric, there is a group of experiments which strongly indicate that the distribution of muons from π-μ decay at rest is not isotropic. The spin-zero assignment for the charged pion is based on interpretation of another group of experiments. In this paper we have attempted to resolve this paradox by forming a model of the pion which can satisfy both groups of experiments. The model consists of a composite pion formed of two massless spin-½ particles. This composite pion is a vector particle and like the photon it does not exist in all three m_s states; this composite pion exists only in the $m_s = 0$ state. This crude model satisfies the results of both groups of experiments, but is deficient in that only a massless pion has been constructed so far. However, the model does predict experimental results which could prove conclusively that the pion has spin. The argument showing that the neutral pion has zero spin is re-examined, and it is shown that a different assumption regarding the statistics of the photon could allow the π^0 to be a vector particle that decays into two photons.

I. INTRODUCTION

As the title suggests, the question to be discussed in this paper is: What is the spin of the pion? There is a group of experiments[1-7] dating back to 1950 which indicate an asymmetry in the angular distribution of muons in the π-μ decay at rest. We will call these Group 1 experiments. There is a second group of experiments[8-12] (we call these Group 2) whose results are consistent with no asymmetry in the π-μ decay. Then there is a third group of experiments (which we designate as Group 3) which consist of: (1) the detailed-balance experiments[13-15] involving the reaction $p + p \rightarrow \pi^+ + d$ and its inverse, (2) an experiment[16] showing small or zero magnetic moment for the π^-, (3) the observed predominance of the π-μ decay mode over the π-e decay mode[17-18] and the polarization of the muon from π decay[19]. (One might include the observed decay of the π^0 into two photons in this third group, but we shall discuss the spin of the π^0 separately in Sec. III).

In some[1,4] of the experiments in Group 1 the chance of a statistical fluctuation causing the observed asymmetry is less than 1 in 100 and in others[2,3,5] it is less than 1 in 1000. With this same result occurring in several experiments there appears to be little chance of explaining the results of Group 1 by some wild statistical fluctuation. Systematic errors must be considered next and Hulubei and co-workers[2] have done a good job of this. Thus the experiments of Group 1 strongly indicate that the observed asymmetry in the π-μ decay is a genuine effect.

The experimental evidence of Group 2 (indicating no asymmetry in π-μ decay) is much weaker than the evidence of Group 1. If one assumes that the pion beams were only slightly polarized or unpolarized due to production conditions,[2,20] most of these experiments are not in contradiction with the results of Group 1. (One experiment[11] was done with emulsions from the same stack as one used by Hulubei *et al.*[2] and this argument does not apply for that case.) Some of the earlier negative results[8] were reanalized by Hulubei *et al.*[2] showing that "several authors, yielding to general opinion, have formulated negative conclusions in spite of their positive results." The electronic counter experiments of Crewe *et al.*[9] are not in disagreement with the results of Group 1 since they only looked for transverse polarization while the results of Group 1 indicate longitudinal polarization. The only other counter experiment[10] showed a slight effect, which the authors chose to eliminate in their final result by an interesting averaging technique.

Therefore, we shall disregard the results of Group 2. The experimental results of Group 3 appear solid and beyond reproach.

Next, the question arises: Can the results of Group 1 be reconciled

TABLE I

Summary of Group 1 and Group 3 experimental results

Group 1.

The spin of the charged pion is non-zero. (This follows from the observed asymmetry in π-μ decay if the spin is defined in terms of rotational invariance of the system.)

(a) The pion tends to be produced with longitudinal polarization in the direction of the proton beam from which it is produced.[1, 2, 7, 26]

(b) The magnetic moment of the pion vanishes to first order.[1, 26]

Group 3.

If the pion is a simple (unstructured) particle, it must have spin zero. If the pion has non-zero spin it must be a composite particle which exists only in the $m_s = 0$ spin state.

(a) The detailed-balance experiments[13-15] involving studies of the reaction

$$p + p \rightarrow \pi^+ + d$$

and its inverse show that either:

(1) the spin of the pion is zero, or

(2) the pions are completely polarized in the production process which can only occur in the $m_s = 0$ state, or

(3) the pions only exist in the $m_s = 0$ spin state.

(b) The magnetic moment of the pion vanishes to first order.[16]

(c) The pion has spin zero or it exists only in the $m_s = 0$ spin state.[17, 18]

with the results of Group 3? A few attempts[21-23] have been made by assuming that a new particle with spin but mass degenerate with that of the pion is contaminating the pion beam and causing the observed effect. Simple models of this type[21] have been shown[17, 18] to fail since they would predict that this new particle should decay via the electron mode about as often as the muon mode. Also, as pointed out in Ref. 21 the muons from the decay of these pion-like particles (in the spin state $m_s = +1$ or -1) would be polarized in the opposite direction to those from pion decay. (This assumes that the neutrino has the same helicity as in ordinary π-μ decay.) If their argument is correct, this polarization should alter the electron distribution from μ-e decay, which it apparently does not.[24] It is possible that some complex model[23] for these pion-like particles may circumvent these problems.

In this paper we are going to take the point of view that the results of Group 1 and Group 3 are not in contradiction, but are caused by some unexpected property of the pion. If one took the results of Group 1 to imply that the pion has spin 1 and the results of Group 3 to imply that the pion has zero spin, then the results of the different experiments are obviously contradictory. However, there is some theoretical analysis involved in going from these experimental results to the conclusions about spin. By the definition of spin[25] concerning the transformation properties under rotation, one can conclude that the results of Group 1 imply that the pion has non-zero spin. We shall argue that the Group 3 experimental results do not prove that the pion has zero spin.

The experimental results of the first and third groups are listed in Table 1. In listing these results we have made some interpretations of the experiments. Combining the results in Table I, we are led to the conclusion that the charged pion has spin and exists only in the $m_s = 0$ spin state. The problem of how a particle can exist only in the $m_s = 0$ spin state is solved in Appendix A for a particle with zero mass. From our present results one can make definite predictions of experiments which (if that theory is correct) will establish beyond doubt that the pion has spin.

II. PROBLEM OF SPIN-1 PARTICLE EXISTING ONLY IN $m_s = 0$ STATE

We shall consider a model in which the pion has spin 1. (From its interaction with other particles it can be determined that its spin must be integral.) If the pion is a simple unstructured particle, it could not exist only in the $m_s = 0$ spin state. It could be formed only in the $m_s = 0$ state in the production process and this would explain the detailed-balance experiments[13-15] as noted by Durbin *et al.*[13] However, by superposition of states along different axes in the rest frame of the particle, one can form states of $m_s = \pm 1$. Therefore, complete polarization in the production process cannot explain the absence of a magnetic moment[16] and, more importantly, the observed ration of π-μ to π-e decay.[17,18]

The same type of calculation, which shows how an $m_s = \pm 1$ state can be formed by a superposition of $m_s = 0$ states, can be used to show that and $m_s = 0$ state can be formed from an $m_s = +1$ or $m_s = -1$ state. These arguments break down for a massless particle. An integral-spin massless particle can exist in the $m_s = \pm 1$ spin states only; it cannot be transformed to an $m_s = 0$ spin state.[27] Similarly, a massless spin-½ particle can exist only in the $m_s = +\frac{1}{2}$ or $m_s = -\frac{1}{2}$ state.[27] By combining two massless spin-½ particles one can form a massless particle which exists only in the $m_s = 0$ state. Just as a massless particle with $m_s = \frac{1}{2}$ or $-\frac{1}{2}$ along its direction of propagation is a relativistically invariant concept, a composite particle (formed of two massless spin-½ particles) with $m_s = 0$ along its direction of propagation is a relativistically invariant concept (see Appendix A).

Thus from considerations of relativistic invariance we have been led to a model in which the pion is formed of *massless* fermions. Indications that one should think in terms of massless fermions as constituents of the pion come also from the analogy of the photon and pion. The pion was originally conceived[28] in analogy with the photon, and it is significant that the photon does not exist in all three m_s states, since we want a pion which exists in one instead of three m_s states. If either the photon or pion is a composite particle, we would expect the other to be a composite particle from the analogy. If we assume that the photon is a composite particle composed of a neutrino-antineutrino pair[29] as de Broglie suggested years ago, by analogy one might expect the pion to be a composite particle formed of a neutrino-antineutrino pair. (If the pion were composed of a nucleon-antinucleon pair, then by analogy one would be tempted to conclude that the photon should be composed of an electron-positron pair.)

In Appendix A a massless particle is formed by combining neutrino-antineutrino states. The method is identical to that used to formulate a neutrino theory of photons[29,30] except we require the component of spin along the direction of propagation to be zero instead of ± 1. The composite particle so formed is described by a four-vector and is longitudinally polarized whereas the photon is transversely polarized. The crude theory has a glaring deficiency in that it does not account for the pion's rest mass. There have been a number of papers on massless pions (see, for example, Refs. 31-33), and it seems to be a useful concept for doing calculations. Our massless pion model *does* predict a vector particle existing only in the $m_s = 0$ spin state, and such a concept is relativistically invariant for a massless particle. If this model were extended by starting with two massive fermions,[34-37] the resulting composite particle could not exist only in the $m_s = 0$ spin state. Thus, we must consider that the pion is composed of particles with zero mass in order to maintain relativistic invariance of the $m_s = 0$ spin state. We do not go into the interaction that gives rise to the pion mass in this paper. How mass might arise from the interaction of massless particles has been summarized elsewhere.[38]

III. SPIN OF THE NEUTRAL PION

The experimental evidence[1-7] indicating that the pion has spin involves only the charged pions. Thus one could consider the π^+ and π^- to have spin 1 with the π^0 as an unrelated particle with spin zero.

However, in any consistent theory it seems essential that the neutral and charged pions have the same spin. Therefore, in this section we shall consider the question: Can a spin-1 particle decay into *two* photons? It will be shown that the standard proof that a spin-1 particle cannot decay into two photons is based on an assumption which we think is questionable. If this assumption is wrong then a spin-1 particle can decay into two photons.

In the decay process the transformation properties of the final state must be the same as the initial state, and it has been argued[39-41] that there is no state of two photons which transforms under rotation as a spin-1 particle. Further, it might be argued that a fermion-antifermion system in an $m_s = 0$ state could not decay into two photons without violating C-invariance.[41-43] This follows from the argument that a state of two photons cannot change sign under charge conjugation while the state of such a composite vector particle does change sign [see Eq. (A41)].

For our particular model the initial particle (see Sec. II) has the magnetic quantum number $m_s = 0$ with respect to, say, the z axis and transforms like a vector along the z axis. Therefore, the question reduces to: Is there a state of two photons of the same helicity (both right-handed or both left-handed) emitted in opposite directions which transforms as a vector pointing along the axis of emission?

The state of two photons can be described in terms of three vectors: the complex polarization vectors of the two photons ϵ_1 and ϵ_2, and the relative momentum vector $\boldsymbol{p}\boldsymbol{n} = \boldsymbol{q}_1 - \boldsymbol{q}_2 = 2\boldsymbol{q}_1 = -2\boldsymbol{q}_2$, as was pointed out by Wolfenstein and Ravenhall.[41] Also, since the polarization vectors are directly connected with the photon creation operators which act on the vacuum state and since each creation operator acts once, the expression for the state must be bilinear in ϵ_1 and ϵ_2.

Two spherically symmetric states are:

$$\epsilon_1(\boldsymbol{n}) \cdot \epsilon_2(-\boldsymbol{n})\, f(p) \tag{1}$$

and

$$[\,\epsilon_1(\boldsymbol{n}) \times \epsilon_2(-\boldsymbol{n})] \cdot \boldsymbol{n} f(p)\,. \tag{2}$$

A vector state is

$$[\,\epsilon_1(\boldsymbol{n}) \times \epsilon_2(-\boldsymbol{n})]\, f(p) = i\boldsymbol{n} f(p)\,, \tag{3}$$

where we have fixed the relative phases so that $\epsilon_2(-\boldsymbol{n}) = \epsilon_1^*(\boldsymbol{n})$.

Thus, we can form a vector state of two photons which transforms like a spin-1 particle in the $m_s = 0$ state. However, this vector state is antisymmetric under an interchange of the two photons ($p\boldsymbol{n} \to -p\boldsymbol{n}$). Up to this point we are in agreement with the standard argument. On the assumption that the photon is an exact Bose particle, it is argued that a state of two photons must be symmetric under interchange.

Planck's distribution is usually cited as direct evidence for exact Bose statistics for the photon. However, a recent paper has shown[29] that a composite photon formed of a neutrino-antineutrino pair could satisfy the experimental results with regard to Planck's law. These composite photons are approximate bosons in the same sense that a deuteron is an approximate boson. Unlike true bosons or fermions the states of these composite particles contain both terms which are symmetric and asymmetric under interchange.

We shall *not* use the approximate "principle of persistence of statistics"[44] which assumes that the state of two composite particles can be represented by two composite particle creation operators acting on the vacuum. The wavefunction for such a state would not have the correct symmetry properties for the constituent fermions.

Considering the state of two photons to be a state of two neutrinos and two antineutrinos,

$$|\gamma_1\gamma_2\rangle = \int d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{q}_1 d\boldsymbol{q}_2\, a^\dagger(\tfrac{1}{2}\boldsymbol{q}_1 - \boldsymbol{k}_1)\, c^\dagger(\tfrac{1}{2}\boldsymbol{q}_1 + \boldsymbol{k}_1)\, a^\dagger(\tfrac{1}{2}\boldsymbol{q}_2 - \boldsymbol{k}_2)\, c^\dagger(\tfrac{1}{2}\boldsymbol{q}_2 + \boldsymbol{k}_2) \times$$

$$\times\, f(\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{k}_1, \boldsymbol{k}_2)|\,0\rangle\ , \tag{4}$$

where $a^\dagger$ and $c^\dagger$ are creation operators for a neutrino and antineutrino respectively. We shall neglect spin for the moment. We take

$$f(-\mathbf{q}_1, -\mathbf{q}_2, -\mathbf{k}_1, -\mathbf{k}_2) = f(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_1). \tag{5}$$

The wavefunction for this state is

$$F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) = <0 \mid c(\tfrac{1}{2}\mathbf{q}_2 + \mathbf{k}_2)\, a(\tfrac{1}{2}\mathbf{q}_2 - \mathbf{k}_2)\, c(\tfrac{1}{2}\mathbf{q}_1 + \mathbf{k}_1)\, a(\tfrac{1}{2}\mathbf{q}_1 - \mathbf{k}_1) \mid \gamma_1 \gamma_2 >. \tag{6}$$

From Eqs. (4) and (6) and the use of the fermion anticommutation relations we obtain

$$\begin{aligned} F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) &= f(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) + f(\mathbf{q}_2, \mathbf{q}_1, \mathbf{k}_2, \mathbf{k}_1) - f\,[\tfrac{1}{2}(\mathbf{q}_1 + \mathbf{q}_2) + \\ &+ \mathbf{k}_1 - \mathbf{k}_2, \tfrac{1}{2}(\mathbf{q}_1 + \mathbf{q}_2) + \mathbf{k}_2 - \mathbf{k}_1, \tfrac{1}{4}(\mathbf{q}_1 - \mathbf{q}_2) + \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2), \tfrac{1}{4}(\mathbf{q}_2 - \mathbf{q}_1) + \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)] - \\ &- f\,[\tfrac{1}{2}(\mathbf{q}_1 + \mathbf{q}_2) + \mathbf{k}_2 - \mathbf{k}_1, \tfrac{1}{2}(\mathbf{q}_1 + \mathbf{q}_2) + \mathbf{k}_1 - \mathbf{k}_2, \tfrac{1}{4}(\mathbf{q}_2 - \mathbf{q}_1) + \\ &+ \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2), \tfrac{1}{4}(\mathbf{q}_1 - \mathbf{q}_2) + \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)]. \end{aligned} \tag{7}$$

From Eq. (4) we can see that $F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2)$ is properly antisymmetrized with respect to exchange of identical fermions. However, it contains both symmetric and antisymmetric terms with respect to exchange of the composite photons. (These considerations are similar to those of Erhenfest and Oppenheimer[44] for composite electron-proton systems.) We have

$$F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) = F_S(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) + F_A(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2), \tag{8}$$

where

$$F_{S,A}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) = \tfrac{1}{2}[F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) \pm F(\mathbf{q}_2, \mathbf{q}_1, \mathbf{k}_1, \mathbf{k}_2)]. \tag{9}$$

Note that the antisymmetric wavefunction does not vanish for two non-overlapping composite particles. Wavefunction overlap at the time of formation is important in determining the probability of the composite particles being in the antisymmetric state, but not for the existence of an antisymmetric state.

From Eqs. (5) and (7),

$$F(-\mathbf{q}_1, -\mathbf{q}_2, -\mathbf{k}_1, -\mathbf{k}_2) = F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2) \ . \tag{10}$$

Now we shall let γ_R and γ_L represent right and left circularly polarized photons respectively. The creation operators $a_1^\dagger$ and $a_2^\dagger$ refer to positive energy neutrino states with spin parallel and antiparallel to their momentum respectively. The creation operators $c_1^\dagger$ and $c_2^\dagger$ refer to the antiparticles with spin antiparallel and parallel to their momentum respectively.

Now consider the two-photon state:

$$\begin{aligned}
&|\gamma_R\gamma_R - \gamma_L\gamma_L> \\
&= \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{q}_1 d\mathbf{q}_2 \, [\, a_1^\dagger(\tfrac{1}{2}\mathbf{q}_1 - \mathbf{k}_1)\, c_2^\dagger(\tfrac{1}{2}\mathbf{q}_1 + \mathbf{k}_1)\, a_1^\dagger(\tfrac{1}{2}\mathbf{q}_2 - \mathbf{k}_2)\, c_2^\dagger(\tfrac{1}{2}\mathbf{q}_2 + \mathbf{k}_2) - \\
&- a_2^\dagger(\tfrac{1}{2}\mathbf{q}_1 - \mathbf{k}_1)\, c_1^\dagger(\tfrac{1}{2}\mathbf{q}_1 + \mathbf{k}_1)\, a_2^\dagger(\tfrac{1}{2}\mathbf{q}_2 - \mathbf{k}_2)\, c_1^\dagger(\tfrac{1}{2}\mathbf{q}_2 + \mathbf{k}_2)] \times \\
&\times F(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}_1, \mathbf{k}_2)\,|\,0> \ .
\end{aligned} \tag{11}$$

Using Eqs. (A27)-(A30) and Eq. (10), we obtain the transformation properties of this state under parity:

$$P\,|\gamma_R\gamma_R - \gamma_L\gamma_L> \; = - \,|\gamma_R\gamma_R - \gamma_L\gamma_L> \ . \tag{12}$$

Under charge conjugation [see Eqs. (A31)-(A34)] this two-photon state transforms so that

$$C\,|\gamma_R\gamma_R - \gamma_L\gamma_L>$$

$$= \int dk_1 dk_2 dq_1 dq_2 \,[a_1^\dagger(\tfrac{1}{2}q_1 - k_1)\, c_2^\dagger(\tfrac{1}{2}q_1 + k_1)\, a_1^\dagger(\tfrac{1}{2}q_2 - k_2)\, c_2^\dagger(\tfrac{1}{2}q_2 + k_2) -$$

$$- a_2^\dagger(\tfrac{1}{2}q_1 - k_1)\, c_1^\dagger(\tfrac{1}{2}q_1 + k_1)\, a_2^\dagger(\tfrac{1}{2}q_2 - k_2)\, c_1^\dagger(\tfrac{1}{2}q_2 + k_2)] \times$$

$$\times F(q_2, q_1, k_1, k_2)\,|\,0> , \tag{13}$$

where we have used the fact that

$$F(q_1, q_2, -k_1, -k_2) = F(q_1, -q_2, k_1, k_2) = F(q_2, q_1, k_1, k_2), \tag{14}$$

which in turn follows from Eq. (10) and $q_1 = -q_2$. Thus from Eqs. (9) and (13) we see that the symmetric and antisymmetric terms transform differently under C,

$$C\,|\gamma_R\gamma_R - \gamma_L\gamma_L>_S = |\gamma_R\gamma_R - \gamma_L\gamma_L>_S , \tag{15}$$

and

$$C\,|\gamma_R\gamma_R - \gamma_L\gamma_L>_A = -\,|\gamma_R\gamma_R - \gamma_L\gamma_L>_A . \tag{16}$$

Summarizing, we have shown that if the photon is a composite particle (composed of a fermion and an antifermion), then the state of two photons contains an antisymmetric term as well as a symmetric term [see Eqs. (8) and (11)]. Further, this antisymmetric part changes sign under charge conjugation [see Eq. (16)]. Thus, if our assumption that the photon is a composite particle is correct, then there is a state of two photons which transforms as a vector and this two-photon state has the same transformation properties as our vector particle under parity and charge conjugation. [Compare Eqs. (12) and (16) with (A40) and (A41).]

The antisymmetric decay matrix element is comparable with the symmetric one since there is a significant overlap of the photon wavefunctions because $\lambda_\gamma (= 2\hbar/m_\pi c) > \lambda_\pi (= \hbar/m_\pi c)$. Thus the ratio of the antisymmetric two-photon decay to the symmetric three-photon decay is governed mainly by phase-space considerations (see Sec. IV).

The experimental observation that the spin-1 state of positronium decays predominately by the three photon decay mode is not in contradiction with the above theory. For positronium the antisymmetric decay matrix element is small compared with the symmetric one since the overlap of the photon wavefunctions is very small. Here $(\lambda_\gamma/\lambda_p)^3 \approx [(h/m_e c)/(2a_{\text{Bohr}})]^3 \approx 10^{-5}$

IV. EXPERIMENTAL IMPLICATIONS

Even in its crude formulation this theory gives predictions of relatively simple experiments which can conclusively test this model. The previous experiments[1-7] have lacked conclusive proof that the pion has spin, because they did not involve variation of some parameter which caused the observed polarization to change in a prescribed manner. For example, the one experiment of Garwin *et al.*[19] was sufficient to prove an asymmetry in the μ-e decay, because they could vary one parameter (the strength of the magnetic field) and show a predicted systematic variation in the observed effects. There are such parameters in the π-μ decay, as we shall now discuss.

According to the model, pions would exist only in the $m_s = 0$ spin state and thus they would be longitudinally polarized. The longitudinal polarization would lead to forward-backward asymmetries only and indeed this agrees with the observed asymmetries of muons in π-μ decay from pions produced in nucleon-nucleon collisions[2,7] and in τ^+ decay.[4,5] Actually, the longitudinal polarization applies only to the center-of-mass system in which the pion is emitted. Although all pions would be in the $m_s = 0$ state, the beams may not be completely polarized in the sense that the axis along which $m_s = 0$ may not be the same for all the pions. (In visualizing this $m_s = 0$ axis one can think of the axis along which the neutrino and antineutrino spins are oriented in the pion rest frame.) A magnetic field changes the direction of the pion's momentum but not the direction of its polarization, since the pion has no magnetic moment. Therefore, in the laboratory system it should be possible by use of a magnetic field to form a beam of pions that are polarized at any desired angle with respect to their momentum.

Thus, in producing polarized pions, the pions should undergo small or nearly equal deflections by the magnetic field. Therefore, production outside the cyclotron or synchrotron field in a hydrogen or hydrogen-rich target with forward emission seems preferable. (This was noted as being important by Hulubei *et al.*[2, 20]). With forward emission the polarization vector will be parallel to the pion momentum.

To be specific, we shall list three experiments – any of which can prove that the charged pion has spin – and a fourth experiment to test the spin of the neutral pion.

1) *Emulsion experiment.* Take the π^+ beam produced in the manner described above and stop it in an emulsion. From previous experimental results, one should observe a muon angular distribution of the form

$$I_\mu = 1 - \xi_0 \cos \theta \ , \tag{17}$$

where θ is the angle between the pion and muon momentum and $\xi_0 \approx 0.15$ from experiment. In the coordinate system in which pion momentum and polarization is along the z_0 axis, the muon distribution [Eq. (17)] has the form

$$I_\mu = 1 - \xi_0 z_0 (x_0^2 + y_0^2 + z_0^2)^{-\frac{1}{2}} \ . \tag{18}$$

If one bends the pion beam by an angle φ in a magnetic field before it enters the emulsion, this has the same effect as observing the distribution in a coordinate system that is rotated by φ from the $(x_0 y_0 z_0)$ system. If we rotate about the x_0 axis (i.e., put the x_0 axis along the direction of the magnetic field), then the new coordinate system is related to the old system by

$$\begin{aligned} x_1 &= x_0 \ , \\ y_1 &= y_0 \cos \varphi + z_0 \sin \varphi \ , \\ z_1 &= - y_0 \sin \varphi + z_0 \cos \varphi \ , \end{aligned} \tag{19}$$

and the muon distribution in the new coordinate system becomes [from Eqs. (18) and (19)],

$$I_\mu = 1 - \xi_0 (z_1 \cos \varphi + y_1 \sin \varphi)(x_1^2 + y_1^2 + z_1^2)^{-\frac{1}{2}} \ . \tag{20}$$

By varying φ one can check Eq. (20). This will thus give systematic variation of the direction of polarization versus the direction of the pion's momentum.

2) *Experiment using a μ-e analyzer.* Using the production conditions noted above, repeat the counter experiment of Ref. 10. Let ω be the angle through which the muon precesses in Δt seconds $= (\mu \Delta t / s\hbar) H$. If the magnetic field is aligned with the direction of polarization (z_0 axis), then the magnetic field will cause the coordinates of the muon direction (x_0, y_0, z_0) to change to (x_1, y_1, z_1) in Δt seconds, where

$$x_1 = r_0 \cos(\theta_0 + \omega) ,$$

$$y_1 = r_0 \sin(\theta_0 + \omega) , \qquad (21)$$

$$z_1 = z_0 \quad .$$

Since Eq. (18) is invariant under the transformation of Eq. (21), the angular distribution of muon spins is not changed by precession with the magnetic field along the axis of the pion polarization. Thereby the electron distribution from the muon decay will not be affected for this direction of the magnetic field.

We next consider the case with the magnetic field perpendicular to the axis of pion polarization. The coordinate system (x_1, y_1, z_1) is now taken such that the muon momentum is along the z_1 axis:

$$x_1 = x_0 \cos\alpha + y_0 \sin\alpha ,$$

$$y_1 = -x_0 \sin\alpha \cos\beta + y_0 \cos\alpha \cos\beta + z_0 \sin\beta , \qquad (22)$$

$$z_1 = x_0 \sin\alpha \sin\beta - y_0 \cos\alpha \sin\beta + z_0 \cos\beta ,$$

where α and β are the Euler angles. If we put the magnetic field along the y_0 axis, this will cause a precession of the muon's spin by an angle ω around the y_0 axis. This aligns the muon spin along the z_2 axis, where

$$z_2 = x_0(\sin\beta \cos\omega \sin\alpha + \cos\beta \sin\omega) - y_0 \sin\beta \cos\alpha +$$

$$+ z_0(\cos\beta \cos\omega - \sin\beta \sin\omega \sin\alpha) . \qquad (23)$$

The distribution of electrons from the decay of muons with spin along

z_2 has the form[19]

$$I_e = 1 - \xi_1 \cos \theta_2 \ , \tag{24}$$

where $\xi_1 = ⅓$ and

$$\cos \theta_2 = z_2 (x_2^2 + y_2^2 + z_2^2)^{-\frac{1}{2}} \ . \tag{25}$$

Weighing this electron distribution with the muon angular distribution, we obtain

$$I_e = (1/2\pi^2) \int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \, (1 - \xi_0 \cos \beta)(1 - \xi_1 \cos \theta_2) \ . \tag{26}$$

Substitution of Eqs. (23) and (25) into (26) results in

$$I_e = 1 + \tfrac{1}{2} \xi_0 \xi_1 (z_0 \cos \omega + x_0 \sin \omega)(x_0^2 + y_0^2 + z_0^2)^{-\frac{1}{2}} . \tag{27}$$

Thus, this experiment with the magnetic field perpendicular to the axis of pion polarization and the detectors in $x_0 - z_0$ plane should show a systematic variation with magnetic field strength as in the experiments of Ref. 19.

3) *Pion scattering experiment.* This is an experiment to look for a left-right asymmetry in pion scattering (see Ref. 9). The important point is that the pion must be bent through 90° by a magnetic field so that the axis of polarization is perpendicular to the pion's momentum. By bending the pions in the other direction by 90°, one can reverse the direction of the pion's polarization and the sign of the left-right scattering asymmetry. With the pion's polarization axis perpendicular to its momentum one can also look for a left-right asymmetry in the π-μ decay in flight (see Ref. 9). The muon distribution should have the form of Eq. (20) with $\varphi = 90°$, z_1 along the pion momentum after bending, and y_1 along the pion momentum before bending.

4) *Decay modes of neutral pion.* An experimental test of the spin of the neutral pion can be achieved by looking for the three photon decay mode. If we take the antisymmetric and symmetric decay matrix elements to be

equal (see Sec. III) then the ratio of decay rates is given[45] by the phase-space factor times α for one order higher in electromagnetic interactions

$$R = \frac{\Gamma(\pi^0 \to 3\gamma)}{\Gamma(\pi^0 \to 2\gamma)} \approx \frac{1/[8(4\pi)^3]}{1/16\pi}(1/137) \approx 2 \times 10^{-5}. \qquad (28)$$

The bubble chamber experiment of Cline and Dowd[46] shows that $R < 4 \times 10^{-4}$. The counter experiments of Kutin *et al.*[47] and Duclos *et al.*[48] showing $R < 5 \times 10^{-6}$ are not applicable since they used a special counter distribution and assumed Bose statistics in the determination of R.

Thus one can test this theory by looking for the three photon decay mode with a detection arrangement that is not so sensitive to the angular distribution of the three photons as used previously[47, 48] and which can test for $R \approx 2 \times 10^{-5}$.

V. DISCUSSION

The experimental evidence indicating that the distribution of muons from π-μ decay at rest in not isotropic[1-7] appears to be in direct conflict with other experiments[13-19] which are interpreted to indicate that the pion has zero spin. Some attempts[21-23] to reconcile these apparently conflicting experimental observations have been made by assuming a new particle with spin is causing the observed asymmetry effects. Simple models of this type have not been very successful, as discussed in Sec. I. In this paper we tried to develop a model of the pion which could satisfy both groups of experiments.[1-7, 13-19] In this model the pion is envisaged as a composite spin-1 particle which exists only in the $m_s = 0$ state. This model is successful in explaining the longitudinal polarization and small (or zero) magnetic moment observed in the π-μ decay asymmetric experiments. It can also explain the detailed-balance experiments,[13-15] the other magnetic moment experiment,[16] the observed ratio of the π-μ decay mode to the π-e decay mode,[17-18] and the polarization of the muon from π-decay[19]. The model is incomplete in that we have only considered a massless composite particle. However, even this crude model does have some definite experimental predictions which (if the theory is correct) should lead to conclusive proof that the pion has spin. Three such experiments are discussed in Sec. IV.

If these experiments show that the charged pion has spin, one will ask next if the neutral pion has spin. Evidence that the neutral pion has spin zero is based on a calculation[39-41] showing that a spin-1 particle cannot decay into two photons. This calculation is based in turn on the assumption that the photon is an exact boson and that two photons cannot exist in an antisymmetric state. We think that this assumption may be wrong since the only direct evidence of the statistics of the photon (blackbody-radiation experiments) can be satisfied if the photon is an approximate boson.[29]

The composite, massless, vector particle constructed in Appendix A does not obey exact Bose statistics. This feature will certainly carry over to a theory of the pion with mass. In fact all of the theories[34-37] in which the pion is envisioned as a composite particle formed of two or more fermions lead to non-Bose statistics.[44, 49, 50] The evidence of Bose statistics for the pion is even weaker than that of the photon. It was believed[51, 52] at one time that the absence of decay $K_2^0 \to \pi^+ + \pi^-$ was evidence for Bose statistics for the pion (Bose statistics require that the final state be symmetric under interchange of the two pions). Bose statistics plus *CP* invariance ruled out this decay mode. Since this decay mode has been observed[53] there is no evidence requiring exact Bose statistics for the pion. The non-Bose nature of these composite particles will only become apparent when there is an overlap of their constituent fermion wavefunctions. The study of decay-modes in which these composite particles must be in an antisymmetric state could test the statistics of the pion and photon. The recent observation[54] that $(\bar{p}p)$ annihilates into two pions could be evidence of non-Bose statistics of the pion if the $(\bar{p}p)$ system is in an *S* state.

It has been proposed that weak interactions are mediated by a vector meson.[28, 55, 56] If the pion is a vector particle, it would be a possible candidate for the intermediate vector meson since it fulfills many of the requirements, such as its decays ($\pi^\pm \to e^\pm + \nu_e$ and $\pi^\pm \to \mu^\pm + \nu_\mu$) and its appearance as a real particle in decays where this is energetically possible.

Finally, it may appear that there are regions in particle physics in which a pion with spin would imply disagreement with well established results, for example nuclear forces, current algebra + *PCAC* and others. The relevant point is that by the construction of the spin-1 pion in this paper, only the $m_s = 0$ component of the pion field exists and is therefore pertinent to these phenomena. Thus the spin manifests itself only in certain decays, as discussed in Section II. For example, the *PCAC* result relates the divergence $\partial_\mu A_\mu$ of the axial vector current to the asymptotic pion field φ, $\partial_\mu A_\mu \sim \varphi$. This relation would remain unmodified in the formulation of this paper, except that φ would have to be interpreted as the pion field whose particle interpretation leads to the longitudinally polarized particle.

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APPENDIX A

Construction of $m_s = 0$ Field from Neutrino Field

We will use the representation with

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} , \quad \boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}$$

$$\boldsymbol{\gamma} = i \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix} , \ \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \ \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{A.1}$$

The Dirac equation for $m = 0$ is

$$\gamma_\mu p_\mu \psi = 0 . \tag{A.2}$$

Letting

$$\psi = U(\mathbf{n}) \exp\big(i(\mathbf{p} \cdot \mathbf{x} - pt)\big) , \tag{A.3}$$

we obtain four normalized solutions for U for positive $\mathbf{n}(= \mathbf{p}/p)$:

$$U_{+1}^{+1}(\mathbf{n}) = [\tfrac{1}{2}(1+n_3)]^{\frac{1}{2}} \begin{pmatrix} 1 \\ (n_1 + in_2)/(1+n_3) \\ 0 \\ 0 \end{pmatrix} , \tag{A.4}$$

$$U_{-1}^{-1}(n) = \left[\tfrac{1}{2}(1+n_3)\right]^{\frac{1}{2}} \begin{pmatrix} (-n_1+in_2)/(1+n_3) \\ 1 \\ 0 \\ 0 \end{pmatrix} , \qquad \text{(A.5)}$$

$$U_{-1}^{+1}(n) = \left[\tfrac{1}{2}(1+n_3)\right]^{\frac{1}{2}} \begin{pmatrix} 0 \\ 0 \\ (-n_1+in_2)/(1+n_3) \\ 1 \end{pmatrix} , \qquad \text{(A.6)}$$

$$U_{+1}^{-1}(n) = \left[\tfrac{1}{2}(1+n_3)\right]^{\frac{1}{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ (n_1+in_2)/(1+n_3) \end{pmatrix} . \qquad \text{(A.7)}$$

The subscript on U refers to spin state while the superscript refers to energy state. Thus, the helicity operator

$$S = \sigma \cdot n = (1/p_4)(\gamma_2\gamma_3 p_1 + \gamma_3\gamma_1 p_2 + \gamma_1\gamma_2 p_3) \qquad \text{(A.8)}$$

has eigenvalues of $+1$ for the spinors of (A.4) and (A.7) and -1 for the spinors of (A.5) and (A.6). Similarly the energy operator ($W_{op} = \alpha \cdot n$) has eigenvalues of $+1$ for (A.4) and (A.6) and -1 for (A.5) and (A.7).

For negative momentum $(-n)$ we obtain the relations

$$U_{+1}^{+1}(-n) = U_{-1}^{-1}(n) ,$$

$$U_{-1}^{-1}(-n) = U_{+1}^{+1}(n) ,$$

$$U_{-1}^{+1}(-n) = U_{+1}^{-1}(n) ,$$

$$U_{+1}^{-1}(-n) = U_{-1}^{+1}(n) . \qquad \text{(A.9)}$$

As mentioned in Sec. III we will use the notation that ν_1 is the neutrino (positive energy state) with spin *parallel* to its momentum and ν_2 is the neutrino (positive energy state) with spin *antiparallel* to its momentum. Let $a_1(\boldsymbol{k},\boldsymbol{n})$, $c_1(\boldsymbol{k},\boldsymbol{n})$, $a_2(\boldsymbol{k},\boldsymbol{n})$, and $c_2(\boldsymbol{k},\boldsymbol{n})$ be the annihilation operators with momentum $\boldsymbol{k}\boldsymbol{n}$ for ν_1, $\overline{\nu}_1$, ν_2, and $\overline{\nu}_2$ respectively. Then, the general neutrino field in terms of particles and antiparticles (not holes) is

$$\psi(\mathbf{x},t) = \int d\boldsymbol{k}\ [\, a_1(\boldsymbol{k},\boldsymbol{n})\, U_{+1}^{+1}(\boldsymbol{n}) \exp\big((i(\boldsymbol{k}\cdot\mathbf{x} - \boldsymbol{k}t)\big) +$$

$$+\, c_1^\dagger(\boldsymbol{k},\boldsymbol{n})\, U_{-1}^{-1}(-\boldsymbol{n}) \exp\big(-i(\boldsymbol{k}\cdot\mathbf{x} - \boldsymbol{k}t)\big) +$$

$$+\, a_2(\boldsymbol{k},\boldsymbol{n})\, U_{-1}^{+1}(\boldsymbol{n}) \exp\big(i(\boldsymbol{k}\cdot\mathbf{x} - \boldsymbol{k}t)\big) +$$

$$+\, c_2^\dagger(\boldsymbol{k},\boldsymbol{n})\, U_{+1}^{-1}(-\boldsymbol{n}) \exp\big(-i(\boldsymbol{k}\cdot\mathbf{x} - \boldsymbol{k}t)\big)] \quad , \qquad \text{(A.10)}$$

where † is used to designate Hermitian conjugate.

We define annihilation operators for the composite $m_s = 0$ particle in terms of neutrino operators. The two different operators are analogous to the right ($m_s = +1$) and left ($m_s = -1$) circularly-polarized-photon operators,[29] but here they both have $m_s = 0$,

$$\lambda(p,\boldsymbol{n}) = \int_{p/2}^{\infty} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, c_2(|\tfrac{1}{2}p - \boldsymbol{k}|, -\boldsymbol{n})\, a_1(|\tfrac{1}{2}p + \boldsymbol{k}|, \boldsymbol{n}) +$$

$$+ \int_{-p/2}^{p/2} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, c_1(|\tfrac{1}{2}p + \boldsymbol{k}|, \boldsymbol{n})\, a_1(|\tfrac{1}{2}p - \boldsymbol{k}|, \boldsymbol{n}) +$$

$$+ \int_{p/2}^{\infty} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, c_1(|\tfrac{1}{2}p + \boldsymbol{k}|, \boldsymbol{n})\, a_2(|\tfrac{1}{2}p - \boldsymbol{k}|, -\boldsymbol{n}) \qquad \text{(A.11)}$$

and

$$\zeta(p,n) = \int_{p/2}^{\infty} dk\,\phi^\dagger(k)\,c_1(|\tfrac{1}{2}p - k|, -n)\,a_2(|\tfrac{1}{2}p + k|, n) +$$

$$+ \int_{-p/2}^{p/2} dk\,\phi^\dagger(k)\,c_2(|\tfrac{1}{2}p + k|, n)\,a_2(|\tfrac{1}{2}p - k|, n) +$$

$$+ \int_{p/2}^{\infty} dk\,\phi^\dagger(k)\,c_2(|\tfrac{1}{2}p + k|, n)\,a_1(|\tfrac{1}{2}p - k|, -n)\ , \qquad (A.12)$$

where $\phi(k)$ is as yet an unspecified function of k.

Processes such as emission and absorption of this composite particle would be represented by the interaction Hamiltonian

$$H_{\text{int}} = \text{const.}\ (\psi_n^\dagger O_{\text{int}} \psi_n)(\psi_{\underline{\nu}}^\dagger O_{\text{int}} \psi_\nu) + \text{Hermitian conjugate}, \qquad (A.13)$$

or we can introduce V_{int} by

$$H_{\text{int}} = \text{const.}\ (\psi_n^\dagger O_{\text{int}} \psi_n)\ V_{\text{int}} \qquad (A.14)$$

Comparing Eqs. (A.9), (A.10), and (A.11), one notes that the spinor combinations that go along with $\lambda(p,n)$ of Eq. (A.11) are of the form

$$[U_{+1}^{-1}(n)]^\dagger\, O_{\text{int}}\, U_{+1}^{+1}(n)\ ,$$

$$[U_{+1}^{+1}(n)]^\dagger\, O_{\text{int}}\, U_{+1}^{+1}(n)\ , \qquad (A.15)$$

$$[U_{+1}^{+1}(n)]^\dagger\, O_{\text{int}}\, U_{+1}^{-1}(n)\ .$$

The possible choices for O_{int} are

$$O_S = \gamma_4 \ ,$$

$$O_V = \gamma_4 \gamma_\mu \ ,$$

$$O_T = i\gamma_4(\gamma_\lambda \gamma_\mu - \gamma_\mu \gamma_\lambda) \ ,$$

$$O_A = i\gamma_4 \gamma_\mu \gamma_5 \ ,$$

$$O_P = \gamma_4 \gamma_5 \ . \tag{A.16}$$

The only non-vanishing terms resulting from substituting Eq. (A.16) into Eq. (A.15) can be put in the form

$$[U_{+1}^{+1}(n)]^\dagger \gamma_4 \gamma_\mu U_{+1}^{+1}(n) \ . \tag{A.17}$$

Similarly, for $\zeta(p,n)$ of Eq. (11) we have only terms of the form

$$[U_{-1}^{+1}(n)]^\dagger \gamma_4 \gamma_\mu U_{-1}^{+1}(n) \ . \tag{A.18}$$

For convenience let

$$u(n) = U_{+1}^{+1}(n)$$

and

$$v(n) = U_{-1}^{+1}(n) \ . \tag{A.19}$$

It should be noted that $u(n)$ and $v(n)$ refer to positive energy states with spin parallel and antiparallel to the direction of propagation respectively as

$$Su(n) = u(n) \ , \quad Sv(n) = -v(n) \ . \tag{A.20}$$

Combining Eqs. (A.11), (A.12), (A.17), and (A.18), we obtain for V_{int} the four-vector

$$V_\mu = \text{const.}\int_0^\infty dp\, p^{\frac{1}{2}} \{[\lambda(p,n)\, u^\dagger(n)\, \gamma_4\gamma_\mu u(n) +$$

$$+ \zeta(p,n) v^\dagger(n)\, \gamma_4\gamma_\mu v(n)] \exp\left(i(\mathbf{p}\cdot x - pt)\right) -$$

$$- [\lambda^\dagger(p,n)\, u^\dagger(n)\, \gamma_4\gamma_\mu u(n) + \zeta^\dagger(p,n)\, v^\dagger(n)\, \gamma_4\gamma_\mu v(n)] \exp\left(-i(\mathbf{p}\cdot \mathbf{x} - pt)\right)\} \ . \tag{A.21}$$

We see that $\mathbf{V}$ and $\phi_V = -iV_4$ are real as $\mathbf{V}^\dagger = V$, and $\phi_V^\dagger = \phi_V$. From Eqs. (A.1), (A.4), (A.6), and (A.19), we obtain

$$iu^\dagger(n)\, \gamma_4\gamma_\mu u(n) = iv^\dagger(n)\, \gamma_4\gamma_\mu v(n) = n$$

and

$$u^\dagger(n)\, \gamma_4\gamma_4 u(n) = v^\dagger(n)\, \gamma_4\gamma_4 v(n) = 1 \ . \tag{A.22}$$

From Eq. (A.22) we thus see that the particle described by these fields is longitudinally polarized, whereas the photon is transversely polarized.[57]

The field equations for this composite particle are (see Appendix A of ref. 30 for the method of derivation):

$$\nabla \cdot \mathbf{V} + (\partial\phi_V/\partial t) = 0 \tag{A.23}$$

$$\nabla \times \mathbf{V} = 0 \ , \tag{A.24}$$

$$(\partial\mathbf{V}/\partial t) + \nabla\phi_V = 0 \ . \tag{A.25}$$

Combining Eqs. (A.23)–(A.25) results in

$$\Box\, \mathbf{V} = 0 \ , \tag{A.26}$$

showing that this particle is massless.

We now inquire about the transformation of V_μ under parity P, charge conjugation C, and rotations about $\boldsymbol{n}$ operations.

The parity operator was defined[30] such that

$$P a_1(\boldsymbol{k}, n) P^{-1} = \epsilon_p a_2(\boldsymbol{k}, -n) \ , \tag{A.27}$$

$$P a_2(\boldsymbol{k}, n) P^{-1} = \epsilon_p a_1(\boldsymbol{k}, -n) \ , \tag{A.28}$$

$$P c_1(\boldsymbol{k}, n) P^{-1} = \epsilon_p^* c_2(\boldsymbol{k}, -n) \ , \tag{A.29}$$

$$P c_2(\boldsymbol{k}, n) P^{-1} = \epsilon_p^* c_1(\boldsymbol{k}, -n) \ , \tag{A.30}$$

and the charge conjugation operator such that

$$C a_1(\boldsymbol{k}, n) C^{-1} = \epsilon_c c_2(\boldsymbol{k}, n) \ , \tag{A.31}$$

$$C a_2(\boldsymbol{k}, n) C^{-1} = \epsilon_c c_1(\boldsymbol{k}, n) \ , \tag{A.32}$$

$$C c_1(\boldsymbol{k}, n) C^{-1} = \epsilon_c^* a_2(\boldsymbol{k}, n) \ , \tag{A.33}$$

$$C c_2(\boldsymbol{k}, n) C^{-1} = \epsilon_c^* a_1(\boldsymbol{k}, n) \ . \tag{A.34}$$

The transformation of $\lambda(p, n)$ of Eq. (A.11) is

$$P\lambda(p,n) P^{-1} = \int_{p/2}^{\infty} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, P c_2(|\tfrac{1}{2}p - \boldsymbol{k}|, -n) P^{-1} P a_1(|\tfrac{1}{2}p + \boldsymbol{k}|, n) P^{-1} +$$

$$+ \int_{-p/2}^{p/2} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, P c_1(|\tfrac{1}{2}p + \boldsymbol{k}|, n) P^{-1} P a_1(|\tfrac{1}{2}p - \boldsymbol{k}|, n) P^{-1} +$$

$$+ \int_{p/2}^{\infty} d\boldsymbol{k}\, \phi^\dagger(\boldsymbol{k})\, P c_1(|\tfrac{1}{2}p + \boldsymbol{k}|, n) P^{-1} P a_2(|\tfrac{1}{2}p - \boldsymbol{k}|, -n) P^{-1} \ . \tag{A.35}$$

By use of Eqs. (A.27)-(A.30) we obtain

$$P\lambda(\boldsymbol{p},\boldsymbol{n})P^{-1}=\zeta(\boldsymbol{p},-\boldsymbol{n}) \;. \qquad (A.36)$$

In a similar manner, the transformation equations for the other operators are obtained:

$$P\,\zeta(\boldsymbol{p},\boldsymbol{n})P^{-1}=\lambda(\boldsymbol{p},-\boldsymbol{n}) \;, \qquad (A.37)$$

$$C\lambda(\boldsymbol{p},\boldsymbol{n})C^{-1}=-\,\zeta(\boldsymbol{p},\boldsymbol{n}) \;, \qquad (A.38)$$

$$C\,\zeta(\boldsymbol{p},\boldsymbol{n})C^{-1}=-\,\lambda(\boldsymbol{p},\boldsymbol{n}) \;. \qquad (A.39)$$

Operating on Eq. (A.21) then results in

$$P\,V_{\mu}(\boldsymbol{x},t)P^{-1}=-\,V_{\mu}(-\,\boldsymbol{x},t) \qquad (A.40)$$

and

$$C\,V_{\mu}(\boldsymbol{x},t)C^{-1}=-\,V_{\mu}(\boldsymbol{x},t) \;. \qquad (A.41)$$

Under a rotation of the coordinate system through an angle θ about $\boldsymbol{n}$, the composite particle operators transform so that

$$R_{\theta}\lambda(\boldsymbol{p},\boldsymbol{n})R_{\theta}^{-1}=\lambda(\boldsymbol{p},\boldsymbol{n}) \;, \qquad (A.42)$$

$$R_{\theta}\,\zeta(\boldsymbol{p},\boldsymbol{n})R_{\theta}^{-1}=\zeta(\boldsymbol{p},\boldsymbol{n}) \;, \qquad (A.43)$$

as expected for a particle with the $m_s = 0$ along $\boldsymbol{n}$.

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57. Compare Eq. (A.22) with Eq. (A.6) of Ref. 30.

RESUMEN

Aunque generalmente se cree que el pion es una partícula con espín cero y por lo tanto esféricamente simétrica, hay un grupo de experimentos que indican fuertemente que la distribución de muones del decaimiento π-μ en reposo, no es isotrópico. La asignación de espín cero al pion cargado se basa en la interpretación de otro grupo de experimentos. En este artículo hemos tratado de resolver esta paradoja formando un modelo del pion que puede satisfacer ambos grupos de experimentos. El modelo consiste de un pion compuesto, formado de dos partículas sin masa con espín $\frac{1}{2}$. Este pion compuesto es una partícula vectorial y, como el fotón, no existe en los tres estados m_s; existe sólo en el estado $m_s = 0$. Este crudo modelo satisface los resultados de ambos grupos de experimentos, pero es deficiente en cuanto a que sólo un pion sin masa se ha desarrollado hasta ahora. Sin embargo, el modelo predice resultados experimentales que podrían probar conclusivamente que el pion tiene espín. Se reexamina el argumento que muestra que el pion neutral tiene espín cero, y se muestra que una suposición diferente respecto a las estadísticas del fotón podrían permitir que el π^0 sea una partícula vectorial que decae en dos fotones.