

NON-TRIVIAL ZEROS OF THE RACAH ($6-j$) COEFFICIENT [†]

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(Recibido: mayo 18, 1974)

ABSTRACT: The non-trivial zeros of the ($6-j$) symbol are tabulated for arguments $\leq 37/2$. The significance of non-trivial zeros is discussed.

1. INTRODUCTION

The Racah coefficient, $W(abcd;ef)$, or as it is also called the ($6-j$) symbol, $\{a\ b\ e\} = (-1)^{a+b+c+d} W(abcd;ef)$, plays a fundamental role in the quantum theory of angular momentum; there is accordingly a detailed and extensive literature concerning applications as diverse as the experimental analysis of angular correlations in nuclear radiation, to unitary symmetry

[†] Supported in part by the National Science Foundation.

in particle physics. One aspect of this subject has, however, hardly been studied at all: this concerns the (non-trivial*) zeros of the Racah coefficient, their distribution and implications. As we shall illustrate below, one can expect such a study to be quite difficult; our aim in the present note is hence rather limited. We wish only to give the raw material for initiating such a study, a listing of the non-trivial zeros, greatly extending results hitherto available in the literature.

To motivate an interest in these non-trivial zeros we discuss in Section 2 the implications of two examples for which interpretations are known.

Section 3 contains the tabulation, an explanation of the notation employed in the tables, and a brief discussion of the method of calculation.

2. MOTIVATING EXAMPLES

An intuitive understanding of any quantum mechanical structure, such as the Racah coefficient, almost always involves examining the semi-classical limit, (terms up to order \hbar , but neglecting $(\hbar)^2$). This is the regime of large quantum numbers -that is, large angular momenta- and hence of asymptotic forms for the Racah function.^{1,2} Representing the Racah function by a tetrahedron, one asymptotic form results from taking one of the four vertices to approach infinity. Geometrically it is clear that we obtain a triangle oriented with respect to the direction along which the vertex went to infinity; in angular momentum terms the result is a $(3j-m)$ function (vector addition or Wigner coefficient) -a function defined on a triangle ("3j") and oriented in space ("m dependent"). A further limit, taking one vertex of the triangle to infinity, yields the Legendre (or more generally Jacobi) function- in angular momentum terms a vector oriented with respect to two directions, a $(lj-mm')$ symbol.

Alternately the two limits may be taken together, yielding for example Racah's asymptotic result:

$$[(2a+1)(2b+1)]^{1/2} W(abcd;ad) \sim (-1)^b P_b(\hat{a} \cdot \hat{d}). \quad (1)$$

* The Racah function involves six angular momenta, which may (in general) be represented as the six edges of a tetrahedron; each of the four faces involves a triangle of three angular momenta. A *trivial zero* of the Racah coefficient expresses the fact that the six arguments do *not* obey the geometrical constraints implied by the existence of a tetrahedron. Expressed differently, but equivalently, one or more of the four "triangle conditions" is violated.

Our discussion is, of course, quite heuristic, but it should suffice to indicate that the Racah coefficients are a generalization of Jacobi polynomials to arguments that are a form of discretized angle space.¹

Let us illustrate this view. Consider the Racah function:

$$W(a;cd;ad) = \left(\frac{(2a-2)!}{(2a+3)!} \frac{(2d-2)!}{(2d+3)!} \right)^{1/2} \times \\ \times 2 [3(x)(x+1) - 4a(a+1)d(d+1)] , \quad (2)$$

where:

$$x \equiv c(c+1) - a(a+1) - d(d+1) . \quad (3)$$

For large values of a, c, d (which form a triangle) we have the asymptotic relation:

$$(x/2ad) \sim [(c^2 - a^2 - d^2)/2ad] \equiv \cos \theta = \hat{a} \cdot \hat{d} . \quad (4)$$

One recognizes that the term in brackets, [...] in eq. (2) is just the Legendre polynomial $P_2(\cos \theta)$.

It is clear therefore that the Racah function is an oscillating function of its arguments and necessarily approximates all of the zeros of the Legendre (Jacobi) functions. Whether or not an exact zero is achieved (and how "often", i.e., the distribution) is basically a number-theoretic question, which to our knowledge has never been considered.*

a) First Example:

From the tables, one sees that the coefficient $W({3/2 \ 2 \ 3/2 \ 2; 3/2 \ 2})$ is (non-trivially) zero. This has the physical implication that *quadrupole radiation from an aligned state having $j=3/2$ to a ground state $j=3/2$ is isotropic*, despite the fact that triangle rules would permit a non-isotropic angular distribution. For this particular case the bracket [...] in eq. (3), above, is easily seen to be exactly zero.

One can give an "explanation" of this zero from the quasispin model. Consider two fermions in a $j=3/2$ shell. The interaction energy of a quadrupolar one-body potential in the two-particle state having $J=2$ then has the form:

* A similar number-theoretic question for the Wigner coefficients, for a physically motivated problem, was considered in ref. (3).

$$\begin{aligned} \langle E \rangle &= \langle (\frac{3}{2})^2 \nu=2 J=2 M | \sum_{i=1}^2 U_{\mu}^{k=2} | (\frac{3}{2})^2 \nu=2 J=2 M \rangle , \\ &\propto W(\frac{3}{2} 2 \frac{3}{2} 2; \frac{3}{2} 2) \end{aligned} \quad (5)$$

(Here the one-body potential is $\sum_{i=1}^2 U_{\mu}^{k=2}$, with $k=2$ denoting a quadrupole interaction; $\nu=2$ designates the state as having seniority 2.)

Using the concept of quasi-spin⁴, the same matrix element may be viewed in a totally different way: the even multipolar interactions transform as vectorial quasi-spin operators. For the particular case given in eq. (5) the initial (and final) state belongs to quasi-spin $S = (2j+1/4) - (\nu/2) = 0$, $S_z = (2j+1/4) - (n/2) = 0$. Thus the matrix element vanishes because the "triangle rule" is violated ($0+1 \rightarrow 0$). Note that it is the triangle rule for quasi-spin which is violated; for the Racah coefficient all the triangle rules are indeed satisfied so that the zero is non-trivial.

This explanation for the vanishing of $W(\frac{3}{2} 2 \frac{3}{2} 2; \frac{3}{2} 2)$ stems from de Shalit and Talmi,⁵ who based their result on explicit seniority calculations. The quasi-spin interpretation greatly simplifies the argument.

b) Second Example:

An example of quite a different sort -and considerably deeper- is provided by the zero $W(3535;33)$. This zero is related to the embedding in the rotation group R_7 (in Cartan's notation B_3) of the exceptional group G_2 .

This comes about in the following way. The generators of R_7 may be realized as the tensor operators $L=1, 3, 5$ acting in the seven dimensional space carried by $j=3$. Thus we take the generators of R_7 to be:

$$L=1: X_1^M | j=3, \mu \rangle = \sum_{\mu} C_{\mu M \mu'}^{313} | j=3, \mu' \rangle$$

or:

$$X_1^M \leftrightarrow C_{\mu M \mu'}^{313}, \quad (6)$$

$$L=3: X_3^M \leftrightarrow C_{\mu M \mu'}^{333}, \quad (7)$$

$$L=5: X_5^M \leftrightarrow C_{\mu M \mu'}^{353}, \quad (8)$$

This realization of the R_7 generators has the advantage that the commutation relations can be given explicitly in terms of the Racah coefficients:

$$[X_L^M, X_{L'}^{M'}] = \sum_M 2C_{MM'}^{LL'} [7(2L'+1)]^{1/2} W(3L, 3L'; 3L'') X_{L''}^{M''}. \quad (9)$$

where L'' is odd.

Consider now the commutation relation for $[X_5, X_5]$. Because the Racah coefficient $W(3535; 33)$ is zero, we see from eq. (9) that the operator X_3 does not enter. Since $[X_1^M, X_L^{M'}] \subset X_L$, it follows that the subset $\{X_1, X_5\}$ closes under commutation; this is the well-known example (due to Racah and to Jahn) which elucidates the embedding $R_7 \supset G_2$.

It would be of considerable interest to discuss the remaining exceptional groups by similar explicit results. This problem seems to be surprisingly difficult, however.*

Our examples considered only two of the roughly 900 zeros given in the tables. Judd⁷, in his lectures on *Atomic Theory* at Canterbury University, has discussed two additional examples of non-trivial zeroes of the Racah coefficient, both vanishings being directly related to vanishings of fractional parentage coefficients in the atomic g shell. (No such systematic relationship exists in general, however, since Judd points out that a third fractional parentage coefficient vanishes in the same shell, but is not directly connected with any non-trivial (6-j) zero.)

It is hoped that these tables will stimulate further explanations and/or implications, which are all too few at the present.

3. EXPLANATION OF TABLES

The non-trivial zeros of the (6-j) symbol $\left\{ \begin{smallmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{smallmatrix} \right\}$ have been calculated for arguments $j_i, l_i \leq 18.5$, $i=1,2,3$. Using the symmetries of the (6-j) symbol we have ordered the arguments j_1, j_2, j_3, l_1, l_2 , and l_3 in a speedometric fashion with j_1 the slowest varying, and l_3 the most rapidly changing, variable. In this table we have $j_1 \geq j_2 \geq j_3$ and $j_1 \geq (l_1, l_2, l_3)$.

An entry in a given row, signifies that the (6-j) symbol $\left\{ \begin{smallmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{smallmatrix} \right\}$, where

* For F_4 there are some partial results (H. Freudenthal, private communication). Wadzinski (ref. 6) has discussed the embedding $U_{26} \supset F_4$.

J1	J2	J3	L1	L2	L3	J1	J2	J3	L1	L2	L3
12.5	11.5	10.0	8.5	4.5	9.0	13.5	13.0	6.5	6.0	7.5	8.0
12.5	11.5	10.0	8.5	6.5	7.0	13.5	13.0	7.5	6.0	11.5	13.0
12.5	11.5	10.0	10.0	11.0	2.5	13.5	13.0	7.5	7.0	8.5	7.0
12.5	12.0	1.5	9.5	4.5	11.5	13.5	13.0	10.5	10.5	5.0	10.5
12.5	12.0	1.5	7.0	7.5	12.0	13.5	13.0	11.5	6.0	7.5	13.0
12.5	12.0	7.5	3.5	5.0	11.5	13.5	13.0	11.5	7.0	10.5	7.0
12.5	12.0	7.5	3.5	9.5	11.5	13.5	13.0	7.0	5.5	2.5	13.0
12.5	12.0	7.5	7.0	1.5	12.0	13.5	13.5	11.0	9.0	3.0	12.5
12.5	12.0	8.5	2.5	9.0	11.5	14.0	8.0	8.0	3.5	7.5	7.5
12.5	12.0	9.5	8.5	3.0	10.5	14.0	8.5	8.5	4.5	8.0	7.0
12.5	12.5	5.0	5.5	4.5	9.0	14.0	9.0	8.0	4.5	7.5	7.5
12.5	12.5	8.0	7.5	6.5	9.0	14.0	10.0	5.0	3.5	6.5	9.5
12.5	12.5	8.0	8.5	3.5	11.0	14.0	10.0	5.0	6.0	10.0	7.0
12.5	12.5	8.0	8.5	7.5	6.0	14.0	10.0	7.0	6.0	10.0	5.0
12.5	12.5	9.0	5.0	5.0	11.5	14.0	10.0	10.0	7.0	9.0	6.0
12.5	12.5	9.0	8.5	1.5	12.0	14.0	10.5	4.5	4.5	8.0	8.0
13.0	8.0	6.0	3.5	7.5	7.5	14.0	10.5	4.5	6.5	9.0	10.0
13.0	8.5	6.5	3.5	7.0	7.0	14.0	10.5	5.5	3.5	6.0	9.0
13.0	9.0	5.0	5.5	8.5	8.5	14.0	10.5	8.5	2.0	9.5	10.5
13.0	9.0	8.0	2.5	8.5	7.5	14.0	10.5	9.5	2.0	8.5	10.5
13.0	9.0	9.0	6.0	8.0	6.0	14.0	10.5	9.5	7.5	10.0	5.0
13.0	9.5	8.5	3.5	9.0	7.0	14.0	11.0	4.0	8.0	11.0	8.0
13.0	10.0	8.0	1.0	8.0	10.0	14.0	11.0	5.0	4.5	7.5	7.5
13.0	10.0	8.0	3.5	8.5	7.5	14.0	11.0	6.0	2.5	7.5	10.5
13.0	10.0	8.0	6.0	7.0	7.0	14.0	11.0	6.0	4.5	9.5	10.5
13.0	10.5	3.5	3.0	4.5	10.5	14.0	11.0	6.0	6.0	9.0	10.0
13.0	10.5	3.5	4.0	6.5	8.5	14.0	11.0	6.0	6.0	9.0	10.0
13.0	10.5	3.5	5.0	6.5	10.5	14.0	11.0	8.0	6.0	5.0	10.0
13.0	10.5	3.5	6.5	9.0	8.0	14.0	11.0	8.0	6.0	6.0	9.0
13.0	10.5	3.5	7.5	10.0	8.0	14.0	11.0	8.0	8.0	11.0	4.0
13.0	10.5	4.5	3.0	3.5	10.5	14.0	11.0	9.0	2.0	9.0	10.0
13.0	10.5	6.5	5.0	3.5	10.5	14.0	11.5	6.5	5.0	4.5	10.5
13.0	10.5	6.5	5.5	7.0	7.0	14.0	11.5	8.5	7.0	7.5	7.5
13.0	10.5	9.5	7.5	8.0	6.0	14.0	11.5	11.5	2.5	11.0	10.0
13.0	10.5	9.5	7.5	10.0	6.0	14.0	11.5	11.5	9.5	11.0	4.0
13.0	10.5	10.5	8.0	7.5	6.5	14.0	12.0	3.0	7.5	9.5	9.5
13.0	10.5	10.5	8.5	10.0	4.0	14.0	12.0	5.0	9.0	11.0	12.0
13.0	11.0	3.0	9.0	11.0	9.0	14.0	12.0	6.0	3.0	8.0	12.0
13.0	11.0	4.0	3.0	4.0	10.0	14.0	12.0	6.0	5.0	5.0	10.0
13.0	11.0	4.0	4.0	6.0	8.0	14.0	12.0	6.0	5.0	9.0	10.0
13.0	11.0	5.0	5.0	9.0	11.0	14.0	12.0	7.0	6.0	6.0	9.0
13.0	11.0	9.0	5.0	5.0	11.0	14.0	12.0	7.0	6.0	8.0	9.0
13.0	11.0	9.0	7.5	3.5	10.5	14.0	12.0	8.0	3.0	8.0	12.0
13.0	11.0	9.0	7.5	7.5	6.5	14.0	12.0	11.0	2.5	10.5	10.5
13.0	11.0	9.0	8.0	9.0	5.0	14.0	12.0	11.0	4.5	10.5	8.5
13.0	11.0	9.0	9.0	9.0	3.0	14.0	12.0	11.0	6.0	8.0	9.0

13.0	11.0	10.0	6.0	10.0	6.0	14.0	12.0	11.0	9.0	5.0	10.0
13.0	11.0	10.0	8.0	7.0	7.0	14.0	12.0	11.0	9.0	9.0	12.0
13.0	11.5	4.5	4.0	7.5	11.5	14.0	12.0	12.0	9.5	7.5	7.5
13.0	11.5	7.5	4.0	4.5	11.5	14.0	12.5	2.5	3.5	5.0	11.0
13.0	11.5	9.5	3.5	7.0	11.0	14.0	12.5	2.5	6.0	7.5	10.5
13.0	12.0	2.0	3.0	4.0	11.0	14.0	12.5	2.5	8.0	9.5	10.5
13.0	12.0	2.0	8.0	9.0	11.0	14.0	12.5	2.5	10.5	12.0	11.0
13.0	12.0	2.0	11.0	12.0	12.0	14.0	12.5	2.5	12.0	12.0	11.0
13.0	12.0	5.0	4.5	3.5	10.5	14.0	12.5	5.5	2.0	5.5	12.5
13.0	12.0	5.0	5.0	5.0	9.0	14.0	12.5	6.5	5.0	8.5	9.5
13.0	12.0	5.0	6.5	7.5	6.5	14.0	12.5	6.5	6.5	7.0	8.0
13.0	12.0	8.0	7.0	4.0	10.0	14.0	12.5	7.5	4.5	8.0	9.0
13.0	12.0	8.0	7.5	8.5	7.5	14.0	12.5	7.5	6.5	3.0	12.0
13.0	12.0	8.0	8.0	8.0	6.0	14.0	12.5	7.5	6.5	5.0	10.0
13.0	12.0	9.0	2.5	7.5	11.5	14.0	12.5	7.5	7.5	8.0	7.0
13.0	12.0	9.0	5.0	5.0	12.0	14.0	12.5	7.5	9.0	10.5	4.5
13.0	12.0	12.0	3.0	11.0	10.0	14.0	12.5	7.5	9.0	10.5	9.0
13.0	12.0	12.0	5.0	5.0	12.0	14.0	12.5	10.5	4.5	10.0	9.0
13.0	12.0	12.0	8.0	8.0	11.0	14.0	12.5	11.5	7.5	5.0	12.0
13.0	12.0	12.0	11.0	12.0	2.0	14.0	12.5	11.5	9.5	6.0	9.0
13.0	12.5	1.5	4.5	5.0	12.0	14.0	13.0	2.0	12.0	13.0	13.0
13.0	12.5	2.5	3.0	3.5	10.5	14.0	12.5	7.5	6.5	5.0	10.0
13.0	12.5	5.5	7.5	8.0	6.0	14.0	12.5	7.5	7.5	8.0	7.0
13.0	12.5	6.5	6.0	3.5	10.5	14.0	12.5	7.5	7.5	8.0	10.0
13.0	12.5	8.5	2.5	3.5	10.5	14.0	12.5	7.5	7.5	8.0	12.5
13.0	12.5	8.5	6.5	8.0	11.0	14.0	12.5	7.5	9.0	10.5	4.5
13.0	12.5	8.5	8.5	3.5	10.0	14.0	13.0	6.0	3.0	13.0	13.0
13.0	12.5	8.5	10.0	6.0	6.0	14.0	13.0	6.0	8.0	9.0	6.0
13.0	12.5	11.5	3.0	4.0	10.5	14.0	13.0	2.0	3.5	4.5	10.5
13.0	12.5	12.5	10.5	8.0	3.5	14.0	13.0	11.0	3.0	6.0	13.0
13.0	12.5	12.5	6.0	4.0	10.0	14.0	13.0	13.0	11.0	4.5	10.5
13.0	13.0	7.0	7.0	5.0	9.0	14.0	13.0	13.0	12.0	13.0	2.0
13.0	13.0	12.0	8.5	4.5	11.5	14.0	13.5	1.5	5.5	6.0	13.0
13.5	10.5	5.0	3.0	7.0	9.5	14.0	13.5	1.5	5.5	6.0	13.0
13.5	11.0	4.5	6.0	9.5	10.0	14.0	13.5	1.5	8.0	8.5	13.5
13.5	11.0	5.5	3.0	6.5	9.0	14.0	13.5	1.5	3.5	3.0	12.0
13.5	11.0	7.5	1.0	7.5	11.0	14.0	13.5	1.5	6.0	7.5	12.5
13.5	11.0	9.5	4.0	4.5	8.0	14.0	13.5	1.5	2.5	7.0	13.0
13.5	11.0	9.5	7.0	10.5	5.0	14.0	13.5	1.5	5.5	4.0	13.0
13.5	12.0	11.5	9.0	11.5	4.0	14.0	13.5	1.5	8.0	4.5	13.5
13.5	12.5	3.0	6.0	10.0	12.5	14.0	13.5	1.5	9.5	3.0	12.0
13.5	12.5	3.0	11.0	12.5	2.5	14.0	13.5	1.5	9.5	7.0	8.0
13.5	12.5	5.0	4.0	12.5	9.5	14.0	13.5	1.5	10.5	10.0	5.0
13.5	12.5	8.0	6.0	12.5	9.0	14.0	13.5	1.5	12.5	10.0	9.0
13.5	12.5	8.0	8.0	12.5	9.0	14.0	14.0	2.0	2.5	2.5	12.5
13.5	12.5	8.0	9.5	12.5	9.0	14.0	14.0	2.0	6.0	8.0	9.0
13.5	12.5	8.0	11.5	12.5	9.0	14.0	14.0	2.0	6.0	8.0	9.0
13.5	12.5	8.0	12.5	12.5	9.0	14.0	14.0	2.0	6.0	8.0	9.0

J1	J2	J3	L1	L2	L3	J1	J2	J3	L1	L2	L3
14.0	14.0	5.0	5.0	3.0	12.0	15.0	12.5	10.5	8.5	5.0	11.0
14.0	14.0	5.0	7.5	7.5	7.5	15.0	12.5	10.5	8.5	4.0	8.0
14.0	14.0	8.0	7.5	2.5	12.5	15.0	12.5	12.5	10.5	12.0	4.0
14.0	14.0	9.0	9.0	7.0	8.0	15.0	13.0	3.0	11.5	11.0	9.0
14.0	14.0	10.0	7.0	4.0	13.0	15.0	13.0	10.0	4.0	7.0	12.0
14.0	14.0	10.0	9.5	1.5	13.5	15.0	13.0	5.0	3.0	6.0	11.0
14.0	14.0	12.0	7.5	5.5	12.5	15.0	13.0	5.0	4.0	4.0	12.0
14.0	14.0	14.0	12.0	9.0	6.0	15.0	13.0	5.0	6.0	8.0	8.0
14.0	14.0	6.0	4.5	9.5	9.0	15.0	13.0	8.0	2.0	8.0	12.0
14.5	11.0	5.5	5.5	10.0	9.5	15.0	13.0	8.0	8.0	8.0	8.0
14.5	11.5	5.0	10.0	11.0	11.5	15.0	13.0	12.0	6.5	6.5	12.5
14.5	11.5	7.0	4.5	8.5	8.0	15.0	13.0	12.0	8.0	12.0	6.0
14.5	11.5	11.0	4.5	8.5	10.0	15.0	13.5	2.5	10.0	11.5	11.5
14.5	11.5	11.0	10.0	5.0	11.5	15.0	13.5	5.5	5.5	6.0	10.0
14.5	12.0	5.5	3.5	8.0	11.5	15.0	13.5	5.5	4.5	11.0	6.0
14.5	12.0	5.5	7.0	11.5	12.0	15.0	13.5	7.5	6.5	4.0	12.0
14.5	12.0	7.5	3.5	9.0	10.5	15.0	13.5	9.5	9.5	6.0	11.0
14.5	12.0	7.5	6.5	12.0	10.5	15.0	13.5	9.5	9.5	10.0	6.0
14.5	12.0	10.5	3.5	9.0	10.5	15.0	13.5	11.5	11.5	10.0	10.0
14.5	12.0	10.5	4.5	9.0	9.5	15.0	13.5	13.5	11.0	8.5	7.5
14.5	12.0	10.5	5.0	10.5	8.0	15.0	13.5	13.5	13.5	10.0	8.0
14.5	12.0	10.5	6.5	12.0	7.5	15.0	13.5	2.0	5.5	6.5	12.5
14.5	12.0	11.5	3.5	9.0	11.5	15.0	14.0	2.0	7.5	8.5	12.5
14.5	12.0	11.5	7.0	5.5	12.0	15.0	14.0	2.0	13.0	14.0	14.0
14.5	12.5	4.0	1.5	4.5	12.0	15.0	14.0	3.0	7.0	9.0	14.0
14.5	12.5	4.0	5.0	8.0	11.5	15.0	14.0	3.0	8.0	8.0	13.0
14.5	12.5	4.0	4.5	9.0	9.5	15.0	14.0	5.0	1.0	5.0	14.0
14.5	12.5	6.0	8.0	12.0	10.5	15.0	14.0	5.0	4.0	6.0	11.0
14.5	12.5	10.5	5.0	10.5	8.0	15.0	14.0	5.0	7.0	8.0	8.0
14.5	12.5	7.0	1.0	7.0	12.5	15.0	14.0	5.0	7.5	9.5	12.5
14.5	12.5	7.0	5.5	8.5	8.0	15.0	14.0	8.0	7.5	9.5	7.5
14.5	12.5	10.0	5.0	10.0	8.5	15.0	14.0	8.0	8.0	9.0	9.0
14.5	12.5	11.0	4.5	7.5	12.0	15.0	14.0	9.0	5.0	9.0	10.0
14.5	13.0	4.5	5.0	8.5	13.0	15.0	14.0	9.0	6.5	9.5	8.5
14.5	13.0	6.5	3.5	7.0	10.5	15.0	14.0	9.0	7.0	3.0	14.0
14.5	13.0	8.5	5.0	4.5	13.0	15.0	14.0	9.0	8.0	4.0	12.0
14.5	13.0	10.5	3.5	10.0	10.5	15.0	14.0	9.0	9.5	6.5	10.5
14.5	13.0	12.5	1.5	12.0	12.5	15.0	14.0	9.0	11.0	12.0	4.0
14.5	13.0	12.5	5.0	11.5	9.0	15.0	14.0	9.0	9.5	6.5	10.5
14.5	13.0	12.5	6.0	7.5	12.0	15.0	14.0	13.0	10.5	3.5	13.5
14.5	13.0	12.5	8.5	12.0	5.5	15.0	14.0	13.0	11.0	8.0	8.0
14.5	13.0	12.5	10.0	12.5	4.0	15.0	14.0	13.0	12.5	9.5	9.5
14.5	13.5	9.0	6.5	10.5	9.0	15.0	14.0	13.0	12.5	7.5	7.5
14.5	14.0	5.5	5.0	6.5	10.0	15.0	14.0	14.0	7.5	12.5	7.5
14.5	14.0	9.5	4.5	9.0	10.5	15.0	14.0	14.0	13.0	14.0	2.0
14.5	14.0	9.5	9.0	10.5	6.0	15.0	14.0	14.0	6.5	5.0	11.0

14.5	14.0	13.5	9.0	6.5	11.0	15.0	14.5	8.5	8.5	7.0	9.0
14.5	14.5	3.0	8.5	7.5	13.0	15.0	14.5	10.5	9.5	6.0	10.0
14.5	14.5	8.0	10.0	8.0	8.5	15.0	14.5	12.5	11.5	10.0	6.0
14.5	14.5	13.0	9.5	4.5	13.0	15.0	14.5	12.5	12.5	9.0	8.0
14.5	14.5	14.0	8.0	7.0	12.5	15.0	14.5	12.5	13.5	9.0	9.0
15.0	9.5	8.5	1.5	9.0	9.0	15.0	15.0	10.0	6.0	5.0	14.0
15.0	9.5	9.5	5.5	8.0	8.0	15.0	15.0	12.0	9.5	4.5	12.5
15.0	10.0	6.0	3.0	8.0	8.0	15.5	9.5	8.0	2.0	9.0	8.5
15.0	10.0	6.0	5.0	9.0	9.0	15.5	10.5	7.0	3.5	9.5	9.0
15.0	10.5	5.5	6.0	9.5	9.5	15.5	11.0	5.5	3.0	7.5	9.0
15.0	10.5	6.5	3.5	6.0	10.0	15.5	11.0	5.5	5.0	8.5	10.0
15.0	10.5	8.5	5.5	6.0	10.0	15.5	11.0	5.5	5.0	9.5	8.0
15.0	11.0	6.0	6.0	8.0	10.0	15.5	11.0	5.5	6.5	11.0	7.5
15.0	11.0	7.0	5.0	8.0	8.0	15.5	11.0	7.5	1.5	8.0	10.5
15.0	11.0	9.0	3.0	11.0	11.0	15.5	11.0	7.5	3.5	9.0	8.5
15.0	11.0	11.0	5.0	10.0	8.0	15.5	11.0	7.5	6.5	11.0	5.5
15.0	11.5	11.0	8.0	10.0	6.0	15.5	11.0	10.5	5.5	8.0	9.5
15.0	11.5	5.5	4.5	9.0	10.0	15.5	11.5	6.0	5.0	9.0	7.5
15.0	11.5	6.5	6.0	7.5	9.5	15.5	11.5	6.0	5.5	8.5	9.0
15.0	11.5	7.5	5.5	7.0	9.0	15.5	11.5	6.0	9.0	11.0	10.5
15.0	11.5	7.5	7.5	11.0	5.0	15.5	11.5	7.0	4.5	5.5	11.0
15.0	11.5	11.5	8.5	9.0	7.0	15.5	11.5	10.0	5.5	8.5	9.0
15.0	12.0	4.0	2.0	4.0	12.0	15.5	12.0	4.5	10.0	3.0	14.0
15.0	12.0	4.0	4.5	7.5	9.5	15.5	9.5	8.0	2.0	9.0	8.5
15.0	12.0	4.0	5.5	7.5	11.5	15.5	11.5	6.0	3.5	9.5	9.5
15.0	12.0	4.0	6.0	9.0	9.0	15.5	11.5	6.0	5.0	9.0	7.5
15.0	12.0	4.0	9.0	11.0	12.0	15.5	11.5	6.0	6.5	10.0	6.5
15.0	12.0	6.0	5.0	9.0	9.0	15.5	12.0	9.5	1.0	9.5	12.0
15.0	12.0	6.0	8.0	11.0	11.0	15.5	12.0	10.5	8.0	5.5	11.0
15.0	12.0	7.0	5.0	11.0	12.0	15.5	12.5	4.0	4.5	6.5	12.0
15.0	12.0	7.0	6.0	8.0	8.0	15.5	12.5	5.0	3.5	5.5	11.0
15.0	12.0	10.0	3.0	10.0	10.0	15.5	12.5	5.0	9.5	5.5	11.0
15.0	12.0	10.0	3.0	11.0	11.0	15.5	12.5	12.0	9.5	5.5	11.0
15.0	12.0	10.0	5.0	9.0	9.0	15.5	13.0	4.5	9.0	12.5	12.0
15.0	12.0	10.0	7.5	11.5	5.5	15.5	13.0	5.5	5.5	10.0	12.5
15.0	12.0	11.0	5.0	7.0	12.0	15.5	13.0	7.5	6.0	4.5	12.0
15.0	12.0	11.0	6.5	11.5	6.5	15.5	13.0	11.5	6.0	11.5	8.0
15.0	12.2	11.0	9.0	4.0	12.0	15.5	13.5	3.0	4.0	6.0	11.5
15.0	12.5	3.5	7.0	9.5	9.5	15.5	13.5	5.0	4.5	5.5	11.0
15.0	12.5	4.5	3.0	6.5	11.5	15.5	13.5	5.0	7.0	11.0	13.5
15.0	12.5	4.5	4.5	7.0	9.0	15.5	13.5	6.0	4.0	9.0	13.5
15.0	12.5	4.5	9.5	12.0	7.0	15.5	13.5	8.0	8.0	9.0	7.5
15.0	12.5	4.5	4.5	8.0	9.0	15.5	13.5	8.0	9.0	9.0	8.5
15.0	12.5	6.5	7.5	10.0	6.0	15.5	13.5	9.0	4.0	6.0	13.5
15.0	12.5	7.5	2.0	8.5	12.5	15.5	13.5	11.0	7.0	5.0	13.5
15.0	12.5	8.5	2.0	7.5	12.5	15.5	13.5	13.0	8.0	11.0	7.5

J1	J2	J3	L1	L2	L3	J1	J2	J3	L1	L2	L3
15.5	13.5	13.0	10.5	7.5	9.0	16.0	14.0	6.0	4.5	7.5	10.5
15.5	14.0	2.5	5.0	6.5	12.0	16.0	14.0	13.0	9.0	13.0	6.0
15.5	14.0	2.5	12.5	14.0	12.5	16.0	14.5	3.5	10.0	12.5	14.5
15.5	14.0	3.5	4.0	5.5	11.0	16.0	14.5	3.5	10.5	12.0	9.0
15.5	14.0	6.5	6.0	5.5	11.0	16.0	14.5	4.5	6.0	8.5	11.5
15.5	14.0	8.5	7.5	3.0	13.5	16.0	14.5	4.5	6.0	9.5	14.5
15.5	14.0	8.5	8.0	7.5	9.0	16.0	14.5	7.5	8.0	8.5	8.5
15.5	14.0	8.5	10.5	12.0	4.5	16.0	14.5	9.5	6.0	4.5	14.5
15.5	14.0	10.5	6.0	10.5	9.0	16.0	14.5	9.5	9.0	8.5	8.5
15.5	14.0	10.5	7.5	11.0	7.5	16.0	14.5	11.5	11.5	9.0	9.0
15.5	14.0	10.5	9.0	5.5	11.0	16.0	14.5	11.5	12.5	4.0	10.0
15.5	14.0	12.5	7.5	12.0	7.5	16.0	14.5	11.5	13.0	13.5	4.5
15.5	14.0	12.5	10.5	6.0	10.5	16.0	14.5	12.5	10.0	3.5	14.5
15.5	14.0	12.5	10.5	8.0	8.5	16.0	15.0	2.0	14.0	15.0	15.0
15.5	14.0	12.5	11.0	10.5	6.0	16.0	15.0	4.0	4.0	7.0	15.0
15.5	14.0	12.5	12.5	14.0	2.5	16.0	15.0	5.0	6.0	8.0	11.0
15.5	14.0	13.5	11.0	13.5	4.0	16.0	15.0	7.0	4.0	4.0	15.0
15.5	14.5	10.0	5.5	5.5	14.0	16.0	15.0	12.0	12.5	10.5	7.5
15.5	14.5	12.0	10.5	4.5	12.0	16.0	15.0	15.0	11.5	12.5	5.5
15.5	15.0	1.5	9.0	9.5	15.0	16.0	15.0	15.0	11.5	14.5	7.5
15.5	15.0	3.5	1.0	3.5	15.0	16.0	15.0	15.0	12.5	9.5	7.5
15.5	15.0	3.5	5.0	5.5	11.0	16.0	15.0	15.0	14.0	15.0	2.0
15.5	15.0	7.5	4.0	7.5	12.0	16.0	15.5	4.0	4.0	5.5	15.5
15.5	15.0	7.5	5.5	8.0	10.5	16.0	15.5	2.5	8.0	8.5	11.5
15.5	15.0	7.5	7.5	9.0	8.5	16.0	15.5	5.5	4.0	2.5	15.5
15.5	15.0	7.5	10.5	11.0	5.5	16.0	15.5	8.5	9.0	3.5	14.5
15.5	15.0	9.5	9.0	1.5	15.0	16.0	15.5	11.5	5.0	7.5	14.5
15.5	15.0	9.5	9.0	6.5	10.0	16.0	15.5	11.5	11.0	12.5	5.5
15.5	15.0	9.5	9.5	8.0	8.5	16.0	15.5	14.5	11.0	13.5	5.5
15.5	15.0	11.5	12.0	12.5	4.0	16.0	15.5	14.5	12.5	8.0	13.0
15.5	15.0	11.5	9.0	9.5	9.0	16.0	15.5	15.5	11.5	10.0	8.0
15.5	15.0	11.5	9.0	11.5	5.0	16.0	15.5	15.5	13.5	12.0	5.0
15.5	15.0	11.5	10.5	3.0	13.5	16.0	16.0	5.0	15.0	13.5	4.5
15.5	15.0	11.5	12.5	14.0	4.5	16.0	16.0	8.0	6.5	2.5	15.5
15.5	15.0	11.5	13.0	13.5	3.0	16.0	16.0	8.0	9.5	8.5	8.5
15.5	15.0	12.5	9.0	4.5	14.0	16.0	16.0	11.0	8.0	4.0	15.0
15.5	15.0	7.0	11.0	5.5	11.0	16.0	16.0	14.0	10.5	4.5	14.5
15.5	15.5	8.0	9.5	3.5	15.0	16.0	16.0	16.0	12.5	4.5	8.5
15.5	15.5	11.0	10.5	1.5	15.0	16.0	16.0	7.0	2.5	7.5	10.0
15.5	15.5	11.0	10.5	10.5	7.5	16.0	11.0	7.5	3.0	8.5	11.0
15.5	15.5	11.0	11.0	11.0	9.0	16.0	11.0	8.5	3.0	7.5	11.0
15.5	15.5	15.0	10.0	9.0	7.5	16.0	11.0	9.5	2.5	11.0	10.5
15.5	15.5	15.0	10.0	6.0	13.5	16.0	11.0	10.5	2.5	11.0	9.5

15.5	15.5	15.0	12.0	6.0	13.5	16.5	11.5	8.0	3.0	8.0	10.5
16.0	9.0	8.0	2.5	8.5	8.5	16.5	11.5	9.0	5.0	11.0	7.5
16.0	10.0	7.0	4.0	9.0	9.0	16.5	11.5	9.0	6.5	10.5	7.0
16.0	10.5	7.5	4.0	8.5	8.5	16.5	11.5	10.0	2.5	10.5	10.0
16.0	10.5	7.5	5.0	7.5	10.5	16.5	12.0	8.5	4.5	11.0	9.5
16.0	10.5	7.5	5.0	10.5	9.5	16.5	12.0	8.5	5.0	10.5	8.0
16.0	10.5	7.5	8.0	10.5	10.5	16.5	12.0	8.5	6.5	10.0	7.5
16.0	10.5	10.5	6.0	10.5	7.5	16.5	12.0	9.5	6.5	10.5	10.5
16.0	10.5	10.5	6.5	9.0	8.0	16.5	12.5	5.0	6.5	7.0	10.5
16.0	10.5	10.5	8.0	10.5	7.5	16.5	12.5	5.0	3.0	7.0	10.5
16.0	11.0	7.0	2.0	8.0	10.0	16.5	12.5	5.0	5.0	8.0	11.5
16.0	11.0	8.0	4.0	11.0	10.0	16.5	12.5	5.0	8.0	12.0	8.5
16.0	11.0	10.0	4.0	11.0	9.0	16.5	12.5	6.0	9.0	12.0	11.5
16.0	11.0	10.0	6.5	8.5	8.5	16.5	12.5	6.0	3.5	8.5	11.0
16.0	11.0	11.0	7.5	10.5	6.5	16.5	12.5	9.0	6.5	5.5	12.0
16.0	11.5	8.5	2.5	8.0	11.0	16.5	12.5	9.0	6.5	7.5	10.0
16.0	11.5	10.5	5.0	11.5	7.5	16.5	12.5	10.0	2.0	11.0	12.5
16.0	12.0	5.0	5.0	9.0	9.0	16.5	12.5	11.0	2.0	10.0	12.5
16.0	12.0	5.0	8.0	11.0	11.0	16.5	12.5	11.0	4.5	11.5	9.0
16.0	12.0	9.0	4.0	10.0	9.0	16.5	13.0	4.5	9.0	12.5	9.0
16.0	12.0	9.0	5.0	9.0	9.0	16.5	13.0	6.0	3.5	8.0	10.5
16.0	12.0	9.0	6.0	11.0	7.0	16.5	13.0	6.0	6.0	11.0	10.5
16.0	12.0	9.0	7.5	10.5	6.5	16.5	13.0	6.5	3.5	8.0	10.5
16.0	12.0	9.0	8.0	9.0	9.0	16.5	13.0	10.5	4.5	11.0	9.5
16.0	12.0	12.0	9.0	11.0	6.0	16.5	13.0	10.5	4.5	11.0	9.5
16.0	12.5	4.5	9.5	12.0	12.0	16.5	13.0	10.5	8.0	6.5	11.0
16.0	12.5	5.5	5.0	8.5	8.5	16.5	13.0	10.5	8.0	7.5	10.0
16.0	12.5	8.5	6.0	10.5	7.5	16.5	13.0	10.5	8.0	12.5	6.0
16.0	12.5	9.5	8.5	7.0	11.0	16.5	13.0	10.5	8.5	10.0	7.5
16.0	12.5	11.5	4.0	11.5	9.5	16.5	13.5	5.0	8.5	10.5	13.0
16.0	13.0	5.0	4.5	8.5	11.5	16.5	13.5	5.0	11.0	13.0	10.5
16.0	13.0	5.0	8.5	12.5	11.5	16.5	13.5	6.0	5.0	7.0	10.5
16.0	13.0	9.0	1.0	9.0	13.0	16.5	13.5	10.0	8.0	10.5	10.5
16.0	13.0	9.0	5.0	10.0	9.0	16.5	14.0	3.5	2.5	5.0	12.5
16.0	13.0	9.0	7.0	11.0	7.0	16.5	14.0	3.5	6.0	8.0	11.0
16.0	13.0	9.0	7.5	8.5	8.5	16.5	14.0	3.5	8.0	11.0	10.5
16.0	13.0	9.0	8.5	7.5	10.5	16.5	14.0	3.5	10.5	13.0	10.5
16.0	13.0	11.0	4.0	11.0	10.0	16.5	14.0	4.5	8.5	9.0	12.5
16.0	13.0	13.0	8.5	10.5	7.5	16.5	14.0	4.5	7.5	11.0	11.5
16.0	13.0	13.0	10.0	9.0	8.0	16.5	14.0	6.5	8.0	10.5	7.0
16.0	13.0	13.0	11.5	10.5	7.5	16.5	14.0	6.5	7.5	11.0	12.5
16.0	13.5	12.5	10.0	8.5	8.5	16.5	14.0	7.5	8.0	10.5	7.0
16.0	13.5	12.5	12.5	8.0	11.0	16.5	14.0	7.5	3.0	9.5	14.0
16.0	13.5	12.5	11.5	13.0	4.0	16.5	14.0	7.5	6.0	9.5	9.0
16.0	14.0	3.0	9.0	11.0	11.0	16.5	14.0	7.5	6.5	7.0	10.5
16.0	14.0	4.0	9.0	11.0	13.0	16.5	14.0	9.5	3.0	7.0	14.0
16.0	14.0	5.0	10.5	13.5	10.5	16.5	14.0	9.5	8.0	7.5	10.0
16.0	14.0	13.0	11.5	13.0	12.0	16.5	14.0	11.5	10.0	11.0	7.0

J1	J2	J3	L1	L2	L3	J1	J2	J3	L1	L2	L3
17.5	14.0	6.5	1.5	7.0	13.5	18.0	15.5	8.5	7.5	11.0	9.0
17.5	14.0	6.5	4.0	9.5	15.0	18.0	15.5	8.5	8.0	10.5	10.5
17.5	14.0	6.5	7.0	12.5	13.0	18.0	15.5	8.5	9.0	12.5	7.5
17.5	14.0	7.5	4.5	11.0	13.5	18.0	15.5	10.5	4.0	7.5	15.5
17.5	14.0	8.5	2.5	9.0	12.5	18.0	15.5	12.5	6.0	12.5	10.5
17.5	14.0	11.5	3.0	11.5	12.0	18.0	15.5	12.5	9.0	13.5	7.5
17.5	14.5	9.0	2.0	10.0	14.5	18.0	15.5	15.5	11.0	12.5	7.5
17.5	14.5	10.0	2.0	9.0	14.5	18.0	15.5	15.5	11.5	14.0	6.0
17.5	14.5	14.0	2.5	12.5	14.0	18.0	15.5	15.5	13.5	15.0	4.0
17.5	15.0	7.5	4.0	8.5	12.0	18.0	16.0	3.0	10.5	12.5	12.5
17.5	15.0	9.5	2.0	9.5	14.0	18.0	16.0	5.0	2.5	6.5	15.5
17.5	15.0	9.5	7.0	9.5	10.0	18.0	16.0	5.0	6.5	10.5	15.5
17.5	15.0	9.5	10.5	14.0	6.5	18.0	16.0	5.0	9.0	13.0	16.0
17.5	15.0	13.5	7.5	13.0	13.5	18.0	16.0	7.0	3.0	9.0	16.0
17.5	15.0	13.5	5.5	13.0	10.5	18.0	16.0	9.0	3.0	7.0	16.0
17.5	15.0	13.5	8.0	13.5	8.0	18.0	16.0	9.0	8.5	11.5	8.5
17.5	15.0	13.5	9.5	14.0	6.5	18.0	16.0	13.0	2.5	11.5	15.5
17.5	15.0	13.5	10.0	14.5	9.0	18.0	16.0	13.0	6.5	7.5	15.5
17.5	15.0	14.5	10.0	10.5	9.0	18.0	16.0	13.0	9.0	5.0	16.0
17.5	15.5	6.0	5.5	4.5	13.0	18.0	16.0	13.0	10.0	15.0	8.0
17.5	15.5	9.0	4.5	9.5	12.0	18.0	16.0	15.0	5.0	14.0	12.0
17.5	15.5	10.0	5.5	10.5	11.0	18.0	16.0	15.0	6.0	14.0	11.0
17.5	15.5	13.0	5.5	12.5	11.0	18.0	16.0	15.0	7.5	12.5	10.5
17.5	15.5	14.0	10.0	10.0	9.5	18.0	16.0	15.0	7.5	15.5	10.5
17.5	15.5	15.0	4.0	12.0	14.5	18.0	16.0	15.0	10.0	13.0	8.0
17.5	15.5	15.0	11.5	14.5	7.0	18.0	16.0	15.0	11.0	15.0	6.0
17.5	15.5	15.0	14.5	14.5	5.0	18.0	16.0	16.0	13.0	10.0	9.0
17.5	16.0	4.5	7.0	10.5	16.0	18.0	16.5	2.5	12.5	14.0	14.0
17.5	16.0	6.5	5.5	9.0	12.5	18.0	16.5	3.5	7.0	9.5	15.5
17.5	16.0	8.5	10.5	13.0	7.5	18.0	16.5	3.5	15.0	15.5	15.5
17.5	16.0	10.5	7.0	4.5	16.0	18.0	16.5	5.5	2.5	6.0	15.0
17.5	16.0	11.5	6.0	15.5	16.0	18.0	16.5	5.5	4.5	7.0	13.0
17.5	16.0	12.5	7.5	6.0	15.5	18.0	16.5	7.5	5.0	8.5	12.5
17.5	16.0	15.5	6.0	11.5	16.0	18.0	16.5	12.5	2.5	12.0	15.0
17.5	16.0	15.5	7.0	10.5	16.0	18.0	16.5	13.5	10.5	14.0	7.0
17.5	16.0	15.5	7.5	12.0	15.5	18.0	16.5	14.5	5.0	13.5	12.5
17.5	16.0	15.5	10.5	13.0	7.5	18.0	16.5	14.5	7.5	12.0	11.0
17.5	16.0	15.5	13.0	15.5	4.0	18.0	16.5	15.5	13.0	9.5	9.5
17.5	16.5	5.0	2.0	6.0	16.5	18.0	17.0	2.0	16.0	17.0	17.0
17.5	16.5	6.0	2.0	5.0	16.5	18.0	17.0	3.0	9.0	11.0	17.0
17.5	16.5	9.0	7.0	10.0	10.5	18.0	17.0	4.0	5.0	8.0	17.0
17.5	16.5	13.0	6.0	8.0	15.5	18.0	17.0	8.0	3.0	8.0	15.0
17.5	16.5	14.0	1.5	13.5	16.0	18.0	17.0	8.0	5.0	4.0	17.0
17.5	16.5	14.0	4.0	13.0	13.5	18.0	17.0	8.0	6.0	9.0	12.0
17.5	16.5	14.0	7.0	13.0	10.5	18.0	17.0	9.0	4.0	9.0	14.0

17.5	16.5	14.0	9.0	13.5	8.0	18.0	17.0	11.0	9.0	12.0	9.0
17.5	16.5	14.0	11.0	14.0	6.5	18.0	17.0	13.0	14.0	7.0	14.0
17.5	16.5	16.0	15.0	8.0	11.5	18.0	17.0	14.0	6.0	13.0	12.0
17.5	17.0	3.5	14.5	16.0	15.5	18.0	17.0	15.0	6.5	11.5	15.5
17.5	17.0	7.0	2.0	5.5	16.0	18.0	17.0	15.0	9.0	14.0	9.0
17.5	17.0	11.5	8.0	11.5	10.0	18.0	17.0	15.0	13.0	15.5	4.5
17.5	17.0	11.5	10.0	12.5	11.0	18.0	17.0	17.0	14.5	12.5	6.5
17.5	17.5	11.0	7.0	5.0	16.5	18.0	17.0	17.0	16.0	17.0	2.0
17.5	17.5	14.0	13.5	7.5	14.0	18.0	17.5	10.5	11.0	9.5	9.5
17.5	17.5	15.0	11.5	4.5	16.0	18.0	17.5	12.5	12.0	9.5	9.5
18.0	10.0	9.0	4.0	10.0	10.0	18.0	17.5	13.5	12.0	9.5	9.5
18.0	10.0	10.0	4.0	10.0	9.0	18.0	17.5	14.5	13.5	6.0	14.0
18.0	10.5	9.5	4.0	9.5	9.5	18.0	17.5	17.5	9.5	15.0	10.0
18.0	11.0	11.0	6.0	10.0	9.0	18.0	18.0	2.0	13.5	15.0	4.5
18.0	11.5	10.5	6.0	9.5	9.5	18.0	18.0	11.0	6.0	6.0	17.0
18.0	12.0	7.0	3.5	9.5	9.5	18.0	18.0	11.0	13.0	10.0	10.0
18.0	12.0	7.0	8.0	12.0	12.0	18.0	18.0	12.0	9.0	4.0	17.0
18.0	12.0	12.0	8.0	12.0	7.0	18.0	18.0	13.0	11.0	10.0	10.0
18.0	12.5	12.5	8.5	11.0	8.0	18.0	18.0	13.0	13.5	7.0	12.5
18.0	13.5	5.5	5.5	10.0	10.0	18.0	18.0	14.0	12.0	3.0	17.0
18.0	13.5	6.5	7.5	13.0	11.0	18.0	18.0	14.0	12.5	6.5	14.5
18.0	13.5	7.5	3.0	9.5	12.5	18.0	18.0	16.0	15.5	15.5	16.0
18.0	14.0	6.0	2.0	7.0	13.0	18.0	18.0	18.0	15.5	11.5	7.5
18.0	14.0	6.0	5.5	9.5	9.5	18.0	18.0	18.0	16.0	14.0	5.0
18.0	14.0	6.0	7.0	8.0	14.0	18.5	12.5	8.0	4.0	11.0	10.5
18.0	14.0	7.0	8.0	12.0	12.0	18.5	13.0	6.5	3.5	9.0	10.5
18.0	14.0	8.0	3.0	9.0	12.0	18.5	13.0	6.5	5.0	8.5	13.0
18.0	14.0	8.0	7.0	6.0	14.0	18.5	13.0	6.5	7.5	13.0	8.5
18.0	14.0	11.0	1.0	11.0	14.0	18.5	13.0	6.5	8.0	11.5	13.0
18.0	14.0	11.0	8.5	13.5	6.5	18.5	13.0	6.5	8.0	12.5	11.0
18.0	14.0	14.0	10.5	8.5	8.5	18.5	13.0	6.5	4.0	10.5	10.0
18.0	14.0	14.0	11.0	13.0	6.0	18.5	13.0	6.5	5.0	6.5	13.0
18.0	14.5	9.5	8.0	9.5	9.5	18.5	13.0	11.5	7.5	13.0	6.5
18.0	14.5	9.5	11.0	13.5	6.5	18.5	13.5	6.0	2.5	6.5	13.0
18.0	14.5	14.5	13.0	13.5	6.5	18.5	13.5	11.0	9.0	13.0	6.5
18.0	15.0	4.0	8.0	11.0	11.0	18.5	14.0	6.5	6.5	12.0	11.5
18.0	15.0	6.0	5.0	10.0	14.0	18.5	14.0	7.5	5.0	7.5	12.0
18.0	15.0	6.0	6.0	11.0	14.0	18.5	14.0	7.5	7.5	12.0	7.5
18.0	15.0	7.0	7.0	7.0	13.0	18.5	14.0	13.5	4.0	13.5	11.0
18.0	15.0	13.0	6.0	13.0	10.0	18.5	14.0	13.5	10.5	13.0	6.5
18.0	15.0	13.0	10.5	9.5	9.5	18.5	14.0	13.5	11.0	13.5	9.0
18.0	15.0	15.0	12.0	12.0	7.0	18.5	14.0	6.0	6.0	10.0	14.5
18.0	15.0	15.0	12.0	14.0	7.0	18.5	14.5	6.0	6.5	9.5	13.0
18.0	15.5	4.5	4.5	8.0	14.0	18.5	14.5	6.0	7.5	12.5	12.0
18.0	15.5	7.5	4.0	10.5	15.5	18.5	14.5	10.0	6.0	9.0	14.5

J1	J2	J3	L1	L2	L3	J1	J2	J3	L1	L2	L3
18.5	14.5	10.0	7.5	6.5	13.0	18.5	14.0	2.5	5.0	6.5	14.0
18.5	15.0	4.5	3.0	6.5	13.0	18.5	14.0	6.5	5.0	7.5	14.0
18.5	15.0	4.5	5.0	7.5	14.0	18.5	14.0	7.5	4.0	6.5	13.0
18.5	15.0	6.5	6.5	8.0	12.5	18.5	14.0	9.5	3.5	7.0	17.5
18.5	15.0	7.5	2.5	9.0	14.5	18.5	14.0	10.5	10.0	6.5	13.0
18.5	15.0	8.5	6.5	6.0	13.5	18.5	14.0	11.5	8.5	14.0	12.5
18.5	15.0	8.5	8.0	10.5	9.0	18.5	14.0	11.5	10.5	12.0	10.5
18.5	15.0	8.5	10.0	13.5	6.0	18.5	14.0	11.5	11.0	12.5	18.0
18.5	15.0	10.5	1.0	10.5	15.0	18.5	14.0	11.5	12.0	10.5	9.0
18.5	15.0	12.5	4.0	12.5	12.0	18.5	14.0	13.5	12.5	3.0	16.5
18.5	15.0	12.5	9.0	12.5	8.0	18.5	14.0	13.5	12.5	9.0	10.5
18.5	15.0	12.5	9.0	13.5	9.0	18.5	14.0	13.5	13.5	12.0	7.5
18.5	15.5	5.0	6.0	10.0	13.5	18.5	14.0	13.5	14.0	9.5	11.0
18.5	15.5	5.0	10.0	14.0	13.5	18.5	14.0	15.5	6.0	11.5	15.0
18.5	15.5	7.0	3.5	9.5	15.0	18.5	14.0	15.5	8.0	11.5	15.0
18.5	15.5	7.0	4.5	10.5	15.0						
18.5	15.5	8.0	8.0	14.0	15.5						
18.5	15.5	8.0	2.5	8.5	14.0						
18.5	15.5	8.0	6.5	6.5	13.0						
18.5	15.5	8.0	6.5	10.5	10.0						
18.5	15.5	8.0	10.0	13.0	6.5						
18.5	15.5	9.0	7.5	7.5	12.0						
18.5	15.5	9.0	8.0	9.0	10.5						
18.5	15.5	9.0	9.0	11.0	8.5						
18.5	15.5	9.0	11.0	14.0	5.5						
18.5	15.5	13.0	7.5	13.5	9.0						
18.5	15.5	14.0	3.5	11.5	15.0						
18.5	15.5	14.0	4.5	10.5	15.0						
18.5	15.5	14.0	6.0	15.0	11.5						
18.5	15.5	14.0	8.0	7.0	15.5						
18.5	15.5	14.0	10.5	14.5	8.0						
18.5	15.5	14.0	12.0	13.0	6.5						
18.5	15.5	15.0	10.5	12.5	8.0						
18.5	16.0	3.5	7.5	9.0	15.5						
18.5	16.0	5.5	5.0	6.5	13.0						
18.5	16.0	9.5	8.0	4.5	15.0						
18.5	16.0	9.5	8.0	15.5	15.0						
18.5	16.0	9.5	8.0	15.5	15.0						
18.5	16.0	11.5	9.0	11.5	9.0						
18.5	16.0	11.5	9.0	12.5	10.0						
18.5	16.0	11.5	11.0	11.5	11.0						
18.5	16.0	14.5	9.0	14.5	8.0						
18.5	16.0	14.5	12.0	6.5	13.0						
18.5	16.0	15.5	12.5	9.0	10.5						
18.5	16.5	4.0	3.0	6.0	15.5						
18.5	16.5	4.0	11.0	14.0	15.5						
18.5	16.5	6.0	6.0	11.0	16.5						
18.5	16.5	8.0	1.0	8.0	16.5						
18.5	16.5	8.0	3.5	8.5	14.0						
18.5	16.5	8.0	6.0	8.0	12.5						
18.5	16.5	8.0	7.5	10.5	10.0						
18.5	16.5	8.0	11.0	13.0	6.5						
18.5	16.5	10.0	8.5	15.5	14.0						
18.5	16.5	11.0	6.0	6.0	16.5						
18.5	16.5	11.0	9.5	6.5	13.0						
18.5	16.5	11.0	10.5	13.5	7.0						
18.5	16.5	12.0	7.5	12.5	10.0						
18.5	16.5	13.0	3.5	12.5	14.0						
18.5	16.5	13.0	6.0	14.0	12.5						
18.5	16.5	15.0	6.0	10.0	15.5						
18.5	16.5	15.0	12.5	7.5	12.0						
18.5	16.5	15.0	12.5	9.5	10.0						
18.5	16.5	15.0	14.0	8.0	12.5						
18.5	17.0	2.5	15.0	16.5	15.0						
18.5	17.0	3.5	10.5	13.0	16.5						
18.5	17.0	4.5	3.0	5.5	15.0						
18.5	17.0	6.5	6.0	8.5	12.0						
18.5	17.0	8.5	4.5	9.0	13.5						
18.5	17.0	10.5	9.5	3.0	16.5						
18.5	17.0	10.5	9.5	7.0	12.5						
18.5	17.0	10.5	10.5	10.5	9.5						
18.5	17.0	10.5	10.5	13.0	7.5						
18.5	17.0	10.5	13.5	15.5	4.5						
18.5	17.0	12.5	4.5	12.0	13.5						
18.5	17.0	12.5	12.0	14.5	6.0						
18.5	17.0	14.5	12.5	6.0	13.5						
18.5	17.0	16.5	3.0	15.5	15.0						
18.5	17.0	16.5	6.5	15.0	11.5						
18.5	17.0	16.5	7.5	15.0	10.5						
18.5	17.0	16.5	11.0	15.5	7.0						
18.5	17.0	16.5	14.0	7.0	13.5						
18.5	17.0	16.5	14.0	13.0	13.5						
18.5	17.0	17.0	13.0	7.0	13.5						
18.5	18.0	1.5	11.0	11.5	18.0						

arguments are the entries in the table, for example, is the non-trivial zero
 $\left\{ \begin{smallmatrix} 2 & 2 & 2 \\ 3/2 & 3/2 & 3/2 \end{smallmatrix} \right\}$ discussed as our first example.

The vanishing values of the $(6-j)$ symbols were calculated by a computer program which did its calculations with an arithmetic working only with powers of primes; that is, each number was decomposed into products of prime factors before being manipulated by the computer.

The program will be submitted to *Computer Physics Communications* (North Holland) so that extension of the tables may be made as required.

ACKNOWLEDGMENT

We wish to thank Dr. Vinaya Joshi for helpful discussions on this subject.

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RESUMEN

Se tabulan los ceros no triviales del símbolo $(6-j)$ para argumentos $\leq 37/2$. Se discute el significado de los ceros no triviales.