EQUALITY BETWEEN NEUTRINO-NUCLEON AND ANTINEUTRINO-NUCLEON CROSS SECTIONS AT HIGH ENERGIES FOR FIXED FINAL HADRONIC MASS

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(Recibido: septiembre 2, 1974)

ABSTRACT: Under a weak hypothesis we show that, for fixed final hadronic mass with strangeness $S = 0$, in the high energy limit, the total cross sections for ν-N and $\overline{\nu}$-N are equal unless the $\Delta S = 0$ (charged) weak current violates the usual charge-symmetry requirement.

In this note we prove the following theorems: when the (anti) neutrino energy E *tends to infinity*,

$$d\sigma(\nu_l + p \to l^- + X)/dW = d\sigma(\overline{\nu}_l + n \to l^+ + X)/dW \ , \qquad (1)$$

* Work supported in part by CONACYT (México)

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$$d\sigma(\nu_l + n \to l^- + X)/dW = d\sigma(\overline{\nu}_l + p \to l^+ + X)/dW \quad , \qquad (2)$$

$$d\sigma(\nu_l + \begin{bmatrix} p \\ n \end{bmatrix} \to \nu_l + X)/dW = d\sigma(\overline{\nu}_l + \begin{bmatrix} p \\ n \end{bmatrix} \to \overline{\nu}_l + X)/dW \quad , \qquad (3)$$

$-X$ is the sum of all final hadronic states with mass W and strangeness $S = 0-$ if

(i) these cross sections are calculated at the lowest order of weak interactions (Fermi coupling or intermediate weak boson theories),

(ii) $W_3(W, q^2) < O(1/q^2)$ at fixed W when $-q^2 \to \infty$ (Ref. 1) and

(iii) the $\Delta S = 0$ (charged) weak current satisfies charge symmetry.

This last assumption is necessary to prove eqs. (1) and (2). Our conventions are taken from ref. (2). q^2 is the (momentum transfer)^2.

Before going to the details of the demonstration, let us see first its experimental implications. Choose $E = 1.7$ GeV such that the maximum mass of the produced hadrons is 2 GeV. We now use the experimental result[3].

$$\sigma(\nu_\mu + N \to \mu^- + X) = (0.74 \pm 0.02) . E . 10^{-38} \text{cm}^2/\text{nucleon}, \qquad (4)$$

$$\sigma(\overline{\nu}_\mu + N \to \mu^+ + X)/\sigma(\nu_\mu + N \to \mu^- + X) = 0.37 \pm 0.02 \;, \qquad (5)$$

where N is an isoscalar target consisting of $\frac{1}{2}(n+p)$ and $10 \text{ GeV} \geqslant E \geqslant 1 \text{ GeV}$. From eq. (4) we have in particular that, at $E = 1.7$ GeV,

$$\sigma(\nu_\mu + N \to \mu^- + X) \simeq 1.25 \times 10^{-38} \text{cm}^2/\text{nucleon} \; . \qquad (6)$$

If we were to assume that, when E has any value higher than 1.7 GeV,

$$\int_0^{2\,\text{GeV}} [d\sigma(\nu_\mu + N \to \mu^- + X)/dW]\, dW \equiv \sigma(\nu_\mu \to \mu^- ;\; W < 2 \text{ GeV}) \qquad (7)$$

remains constant[4], then we would have to conclude that, in contradistinction, $\sigma(\overline{\nu}_\mu \to \mu^+ ;\; W < 2 \text{ GeV})$ is far from its asymptotic value, at $E \simeq 2$ GeV, because eqs. (1) and (2) imply

$$\sigma(\overline{\nu}_\mu \to \mu^+ ;\ W < 2\ \text{GeV})/\sigma(\nu_\mu \to \mu^- ;\ W < 2\ \text{GeV}) \underset{E\to\infty}{=} 1\ , \qquad (8)$$

to be compared with eq. (5). Therefore the observed ratio, eq. (5) at $E = 1.7$ GeV, would be, in part, a consequence of the fact that $\sigma(\nu_\mu \to \mu^- ;\ W < 2\ \text{GeV})$ gets its asymptotic value at lower E than $\sigma(\overline{\nu}_\mu \to \mu^+ ;\ W < 2\ \text{GeV})$. If, on the contrary, $\sigma(\nu_\mu \to \mu^- ;\ W < 2\ \text{GeV})$ does not get its asymptotic value for $E \sim 2$ GeV, then the flux factor would have to be modified in ref. (4) as well as the evaluation of $\sigma(\nu_\mu + N \to \mu^- + X)$ when $E \gtrsim 20$ GeV.

It has also been realized[5] that most of the models for the weak production of the nucleon resonances predict eqs. (1) and (2), with W the resonance mass. Here we show that conditions (i)-(iii) define the class of such models.

Let us prove the theorems - eqs. (1), (2) and (3). The double differential cross section for $\nu(\overline{\nu})$ scattering on unpolarized target can be written[2] as:

$$d^2\sigma^{\nu,\overline{\nu}}/dQ^2 dW^2 = (G^2/4\pi M^2 E^2)\{2(m^2+Q^2)\, W_1^{\nu,\overline{\nu}} + (4EE' - Q^2 - m^2)\, W_2^{\nu,\overline{\nu}} +$$

$$+ (m^2/M^2)(Q^2+m^2)\, W_4^{\nu,\overline{\nu}} - (2Em^2/M)\, W_5^{\nu,\overline{\nu}} \pm (W_3^{\nu,\overline{\nu}}/M)\,[E(m^2-Q^2) - E'(m^2+Q^2)]\}\ , \qquad (9)$$

where the W_i's are the structure functions defined in ref. (2), $W_i \equiv W_i(W, q^2)$, $E(E')$ is the initial (final) lepton energy, $m(M)$ is the lepton (nucleon) mass, $Q^2 \equiv -q^2 > 0$, $(\nu/M) = (p \cdot q/M)$ is the energy transfer in the laboratory frame and $G = 1.026\, M_p^{-2}$, the Fermi coupling constant. Integrating eq. (9) with respect to Q^2, we obtain:

$$d\sigma^{\nu,\overline{\nu}}/dW^2 = (G^2/2\pi M E^2)\{2\int_0^{\xi_m} (m^2 + 2M\xi)\, W_1^{\nu,\overline{\nu}}\, d\xi +$$

$$+ \int_0^{\xi_m} [4E^2 - (2E(W^2-M^2)/M) - m^2 - 2M\xi - 4E\xi]\, W_2^{\nu,\overline{\nu}}\, d\xi + \qquad (10)$$

$$+ (m^2/M^2)\int_0^{\xi_m} (m^2 + 2M\xi)\, W_4^{\nu,\overline{\nu}}\, d\xi - (2Em^2/M)\int_0^{\xi_m} W_5^{\nu,\overline{\nu}}\, d\xi \pm$$

$$\pm \int_0^{\xi_m} [(m^2/2M^2)(W^2 - m^2) + ((m^2 + W^2 - M^2 - 4ME)/M)\,\xi + 2\xi^2]\, W_3^{\nu,\overline{\nu}}\, d\xi\}\ ,$$

where $\xi = Q^2/2M$ and $\xi_m = [1/1+(M/2E)]\,[E - ((W^2 - M^2)/2M^2)]$.

In the limit $E \to \infty$, with W fixed, $\xi_m = E$ and we must be careful in evaluating the integrals in eq. (10). This is so because the integrals can diverge if the structure functions W_i do not vanish sufficiently rapidly as $Q^2 \to \infty$.

To study the asymptotic behavior of the W_i's, we use the following inequalities derived from positivity requirements[2]

$$0 \leqslant |W_3| \sqrt{\nu^2 + M^2 Q^2}/2M^2 \leqslant W_1 \leqslant W_2 \,[1 + (\nu^2/M^2 Q^2)] .$$

In the limit $Q^2 \to \infty$, W fixed, these inequalities become:

$$|W_3| \leqslant (4M^2/Q^2)\, W_1 \leqslant W_2 \ . \tag{11}$$

With these inequalities in mind we can take the limit of eq. (10) when $E \to \infty$, W fixed. If $W_3(W, q^2) < O(1/q^2)$ for a given W, (assumption (ii)), we find the integrals $\int_0^E \xi^2 W_3\, d\xi$, $\int_0^E \xi W_3 d\xi$ and $\int_0^E W_3 d\xi$ are less divergent than $O(E^2)$, $O(E)$ and $O(E^0)$ respectively such that the corresponding contributions to eq. (10) are *smaller than* $O(E^0)$ and $O(E^{-2})$ respectively. Therefore, using eqs. (11), we conclude that *eq. (10) will be independent of* W_3 *in the limit* $E \to \infty$, W *fixed*

Due to the trivial equality,

$$W_i(\overline{\nu}_l + \binom{p}{n} \to \overline{\nu}_l + X_0) = W_i(\nu_l + \binom{p}{n} \to \nu_l + X_0)$$

the proof of eq. (3) follows immediately. Assuming that the $\Delta S = 0$ (charged) weak current satisfies charge symmetry, i.e. $W_i^{\nu\,(p,n)} = W_i^{\overline{\nu}\,(n,p)}$ for $\Delta S = 0$ transitions[6], the proof of eqs. (1) and (2) is immediate.

Up to here we have assumed Fermi coupling. Our result is independent of assumption (ii) if intermediate weak bosons of mass M_W exist. In this case, the structure functions must be replaced by:

$$W_i(W, q^2) \to W_i(W, q^2)/[1 + (Q^2/M_W^2)]^2$$

giving a better convergence of the integrals in eq. (10).

Finally it is interesting to note that our result is independent of scaling[7]. Furthermore, any possible experimental deviation from eqs. (1), (2) and (3), provided assumption (ii)[1] is satisfied, would mean that the strangeness-conserving weak hadron current is not charge symmetric[6].

ACKNOWLEDGMENT

It is a pleasure to thank L. Heiko for several helpful comments.

REFERENCES

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3. T. Heichten *et al.*, Phys. Letters 46B (1973) 274.
4. A similar assumption is made in A. Benvenuti *et al.*, Phys. Rev. Letters 32 (1974) 125.
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6. Possible violations of the charge symmetry in ν and $\overline{\nu}$ scattering (with $S = 0$ hadronic final states) are considered by A. de Rújula and S.L. Glashow (Phys. Rev. D9 (1974) 180), but for rather large $W^2 (> M_c^2$, with $M_c \simeq 2-4$ GeV).
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RESUMEN

Bajo una hipótesis débil, mostramos que para una masa hadrónica final fija con extrañeza $S = 0$, en el límite de altas energías, las secciones totales para $\nu - N$ y $\bar{\nu} - N$ son iguales, a menos que la corriente débil (cargada) $\Delta S = 0$ viole el requisito común de simetría de carga.