

SOME WARNINGS ABOUT CLASSICAL MAGNETIC POLES

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SUMMARY

The transformation properties of the electromagnetic fields - originated by the motion of a system composed of charges and poles - under strong time reversal are discussed. A modified version of the Cabibbo-Ferrari theory of charges and poles is presented. In this approach we use two independent electromagnetic fields and, therefore, the mixing gauge equation does not appear. It is concluded that there are physical reasons forbidding the identification, or even superposition, of the e.m. fields created by subluminal charges and poles. The purely formal analogies which gave origin to Parker's conjecture are discussed as well. A two-photon framework is necessary.

INTRODUCTION

Since the discovery in 1956 of P and C violations by the weak leptonic interactions⁽¹⁾ discrete symmetries have received considerable attention in elementary particle physics⁽²⁾, and some others possible violations have been reported: PC violations⁽³⁾, and T violation in the K_2^0 decays⁽⁴⁾ in the usual monopoles theories⁽⁵⁾. An excellent critical review on the methodology of the PCT theorem, in the framework of the Wightman formalism, was given by Santilli and Ktorides⁽⁶⁾. The implications of discrete space-time symmetries in classical electrodynamics have also been recently considered⁽⁷⁾.

On the other hand, in 1969 Parker⁽⁸⁾ conjectured, in the framework of his two dimensional extended relativity, the physical equivalence of a superluminal charge and a subluminal magnetic pole. Since then, this equivalence has been extensively used in tachyonic theories^(9,10). Parker's considerations on charges and poles were of classical nature and, in fact, a host of difficulties have been found⁽¹¹⁾ in the attempts to construct a tachyonic quantum field theory. Consequently, we shall limit our considerations to the framework of classical field theory and the standard theory of relativity.

We show that the behaviour of the electromagnetic fields created by charges and poles under strong time reversal oblige us to introduce two electromagnetic potentials. This is nothing more than the Cabibbo-Ferrari approach⁽¹²⁾. But we go a little further. In fact our conclusions indicate a non-equivalence between the e.m. fields created by charges and poles. This fact oblige us to eliminate the Cabibbo-Ferrari relation connecting "mixing gauges" and, therefore, a two photon theory of electromagnetism must be used if ever the monopoles are found.

Our reasoning is very simple and could be briefer. On the contrary, we have preferred to discuss some closely related topics which, like duality, will help the reader in a better understanding of the difference between the formal analogies and the physically based analogies that are used in the bibliography. We have acted so because it is our opinion that many points in relation with this theme, in particular Parker's conjecture, have not been clarified earlier because, on one hand, of the considerable confusion existing in the literature about the meaning of the discrete symmetries in a classical scheme and, on the other hand, because of the extended tendency of taking any formal analogy between formulae, as a physically significative one. Particularly, Pintacuda distinction between "strong" and "weak" transformations⁽¹³⁾ seems to us an important point, still waiting diffusion among the community of physicists.

The plan of this paper goes as follows: We briefly present in Part I Parker's conjecture. In Part II we discuss the duality transformations. In Part III we sketch the two potential approach of Cabibbo

and Ferrari. In Section IV we discuss the physical consequences of a strong transformation changing the sense of the velocities. In Part V we point out the difference between our approach and Cabibbo-Ferrari's. Parker's conjecture is dealt with and other consequences are discussed.

I. PARKER'S CONJECTURE

Parker⁽⁸⁾, in his extended two dimensional relativity was led to the transformation formulae,

$$\left. \begin{aligned} E_z - B_y &= \exp(\alpha) (E'_z - B'_y), & E_z + B_y &= -\exp(\alpha) (E'_z + B'_y) \\ E_y + B_z &= \exp(\alpha) (E'_y + B'_z), & E_y - B_z &= -\exp(\alpha) (E'_y - B'_z) \\ \exp(-\alpha) &= (v - c)^{1/2} (v + c)^{-1/2} \end{aligned} \right\} (1)$$

connecting the transversal e.m. fields measured in two inertial systems of relative speed v , $v > c$.

When $v = \infty$ is $\alpha = 0$, and therefore we have:

$$\left. \begin{aligned} E_z - B_y &= E'_z - B'_y, & E_z + B_y &= - (E'_z + B'_y) \\ E_y + B_z &= E'_y + B'_z, & E_y - B_z &= - (E'_y - B'_z) \end{aligned} \right\} (2)$$

If the primed indexes refer to the subluminal system, denoting the e.m. fields in this system by the symbol \downarrow , the solution of (2) can be written in the form:

$$\left. \begin{aligned} \uparrow E_y &= \downarrow B_z, & \uparrow E_z &= -\downarrow B_y \\ \uparrow B_y &= -\downarrow E_z, & \uparrow B_z &= \downarrow E_y \end{aligned} \right\} (3)$$

If, at the same time, we transform the space-time variables according with the formulae,

$$\left. \begin{aligned} x - ct &= -\exp(-\alpha) (x' - ct') \\ x + ct &= \exp(\alpha) (x' + ct') \end{aligned} \right\} , \quad (4)$$

Maxwell's equations remain invariant under the simultaneous substitution defined by (1) and (4).

Eq. (3) suggested to Parker the possibility "...that charged tachyons might have properties similar to those of magnetic monopoles".

Although Parker himself considered the connection between charged tachyons and poles as purely suggestive - the more so since it is well known that isotropy of space is not compatible with a real linear group of transformations—Parker's suggestions have been recently discussed in the framework of extended relativity. For example, it has been asserted that "only one electromagnetic charge is expected to exist, which behaves as electric when subluminal and as magnetic when superluminal"⁽⁹⁾. We are not interested, at least in this paper, in discussing the validity of the connection which identifies the duality bradyon-tachyon with the duality charge-pole⁽¹⁴⁾, because many critical remarks about this point and about the superlight transformations can be found elsewhere⁽¹⁰⁾. We consider as a much more reliable ground, in order to make our point about magnetic poles, the framework provided by the standard subluminal electrodynamics. We shall show that Parker's conjecture was guided by the formal analogy inherent in formulae (3), in which one apparently observes that a mere changing of referential system does transform a purely electric field ($\uparrow B_z = 0, \uparrow B_y = 0$) into a purely magnetic field ($\uparrow E_y = 0, \uparrow E_z = 0$). We shall see that there are intrinsic properties distinguishing the e.m. fields created by a charge from those originated by a pole. According to us this precious information distinguishing charges and poles cannot be lost by a mere changing of referential, no matter whether or not the new referential is a subluminal or a superluminal one.

II. CRITICAL REMARKS ON DUALITY TRANSFORMATIONS

It is known that in the presence of electric charges and poles Maxwell equations can be written in the form:

$$\left. \begin{aligned} \operatorname{div} \bar{\mathbf{E}} &= \rho_e , & \operatorname{div} \bar{\mathbf{B}} &= \rho_m \\ \operatorname{rot} \bar{\mathbf{B}} &= \dot{\bar{\mathbf{E}}} + \bar{\mathbf{j}}_e , & \operatorname{rot} \bar{\mathbf{E}} &= -\dot{\bar{\mathbf{B}}} - \bar{\mathbf{j}}_m \end{aligned} \right\} . \quad (5)$$

These equations show the formal symmetry,

$$\left. \begin{aligned} \bar{\mathbf{E}}' &= \bar{\mathbf{E}} \cos\theta + \bar{\mathbf{B}} \sin\theta , & \bar{\mathbf{B}}' &= -\bar{\mathbf{E}} \sin\theta + \bar{\mathbf{B}} \cos\theta \\ \rho_e' &= \rho_e \cos\theta + \rho_m \sin\theta , & \rho_m' &= -\rho_e \sin\theta + \rho_m \cos\theta \\ \bar{\mathbf{j}}_e' &= \bar{\mathbf{j}}_e \cos\theta + \bar{\mathbf{j}}_m \sin\theta , & \bar{\mathbf{j}}_m' &= -\bar{\mathbf{j}}_e \sin\theta + \bar{\mathbf{j}}_m \cos\theta \end{aligned} \right\} \quad (6)$$

that is usually referred to as "duality"⁽¹⁵⁾.

In particular, when $\theta = \pi/2$, Eqs. (6) gives us,

$$\bar{\mathbf{E}}' = \bar{\mathbf{B}} , \bar{\mathbf{B}}' = -\bar{\mathbf{E}} ; \rho_e' = \rho_m , \rho_m' = -\rho_e ; \bar{\mathbf{j}}_e' = \bar{\mathbf{j}}_m , \bar{\mathbf{j}}_m' = -\bar{\mathbf{j}}_e , \quad (7)$$

that is, an apparent interchanging of electric and magnetic entities.

The transformations formulae (6) are relevant in the theory of magnetic poles⁽⁹⁾ since Eq. (6) and (7) do show a formal symmetry in the roles played by electric and magnetic properties. A similar formal symmetry was reported here in relation with Eq. (3), equation which led Parker to the physical identification of the fields created by charges and poles. Before showing that these formal analogies are not sufficient for concluding the physical identification of the e.m. fields created by charges and poles, let us point out some remarks in relation with Eq. (6).

These transformations have been recently considered by Mignani et al⁽⁹⁾ in relation with the Hertz tensor form of Maxwell equations⁽¹⁶⁾,

$$\square H^{\mu\nu} = J^{\mu\nu} , \quad (8)$$

\underline{H} being the antisymmetric Hertz tensor and \underline{J} the antisymmetric source tensor. This formulation of Maxwell equations present some advantages for the introduction of the Cabibbo-Ferrary relation,

$$\partial^\nu A^\mu - \partial^\mu A^\nu + E^{\mu\nu\lambda\sigma} \partial_\lambda B_\sigma = 0 \quad , \quad (9)$$

where A^μ and B^μ are the two electromagnetic potentials, defined by,

$$A^\mu = \partial_\lambda H^{\lambda\mu} \quad , \quad B^\mu = \partial_\lambda \tilde{H}^{\lambda\mu} \quad (10)$$

and \tilde{H} being defined by,

$$\tilde{H} = 1/2 E^{\lambda\mu\nu\sigma} H_{\nu\sigma} \quad .$$

In this formulation, the equation,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + E^{\lambda\rho\sigma} \partial_\rho B_\sigma \quad , \quad (11)$$

provides the value of the e.m. field tensor F , which value is invariant under the gauge transformation,

$$H^{\mu\nu} \rightarrow H^{\mu\nu} + \chi^{\mu\nu} \quad , \quad \square \chi^{\mu\nu} = 0 \quad , \quad (12)$$

from which one easily gets the Cabibbo-Ferrari "null-field conditions", given by Eq. (9), restricting the values of the e.m. fields created by charges and poles.

According to us, the role played by duality in the formulation of Hertz's framework is very confusing for the following reasons:

II.1. The one-parameter group of transformations defined by Eq. (6) is not the more adequate group of symmetries of Eq. (5), because the two-parameter group defined by the equations,

$$\left. \begin{aligned} \bar{E}' &= a\bar{E} + b\bar{B} \quad , \quad \bar{B}' = -b\bar{E} + a\bar{B} \\ \rho_e' &= a\rho_e + b\rho_m \quad , \quad \rho_m' = -b\rho_e + a\rho_m \\ \bar{J}_e' &= a\bar{J}_e + b\bar{J}_m \quad , \quad \bar{J}_m' = -b\bar{J}_e + a\bar{J}_m \end{aligned} \right\} \quad , \quad (13)$$

$$a^2 + b^2 \neq 0$$

is a group of symmetries of Eq. (5), as well. This group contains the duality group of Eq. (6) when $a^2 + b^2 = 1$. As far as we know there are no physical arguments for giving preference to the standard Eq. (6) over the more general formulae (13).

11.2. Both Eqs. (6) and (13) possess a more formal than physical content, like Parker's suggestions in relation with Eq. (3). Indeed, since Eq. (5) are linear, it is obvious that to a certain linear superposition of the physical sources $\rho_e, \rho_m, \vec{j}_e, \vec{j}_m$, there corresponds another linear superposition of the fields which satisfy Eq. (5) as well. Nevertheless, neither Eqs. (6) nor (13) have a real physical meaning, since we cannot assume that the linear combinations of ρ_e and ρ_m appearing in Eqs. (6) and (13) do indicate the possibility of substituting the sources of \vec{E} (ρ_e) by the sources of \vec{B} (ρ_m). The physical reasons, showing clearly why the formal analogies suggested when looking at Eqs. (6) and (13) cannot be given full physical meaning, shall become clear in Section IV.

III. THE TWO POTENTIAL APPROACH TO MAXWELL EQUATIONS

The covariant form of the Maxwell-Lorentz equations for a system composed of charges and poles is,

$$\left. \begin{aligned} \partial_\nu F^{\mu\nu} &= j_e^\mu \\ \partial_\nu \tilde{F}^{\mu\nu} &= j_m^\mu \\ m_0 \cdot \frac{dv^\mu}{d\tau} &= e F^{\mu\nu} \cdot v_\nu + g \tilde{F}^{\mu\nu} \cdot v_\nu \end{aligned} \right\} \quad (14)$$

j_e and j_m being the electric and magnetic sources of the fields and (e, g) the charges of the test particle.

It is known⁽¹⁷⁾ that neither in the case of a puntiform magnetic source nor in the case of a macroscopic bundle of them is it possible to have,

$$\vec{B} = \text{rot } \vec{A} \quad ,$$

at least if \vec{A} is assumed to be a well behaved vector field. Indeed, if \vec{A} is regular and single valued, Stokes's theorem gives:

$$\int \text{rot } \vec{A} \cdot d^2\vec{s} = \int \text{div}(\text{rot} \cdot \vec{A}) d^3v = 0 \quad . \quad (15)$$

On the other hand, Maxwell's equations and Stokes' theorem imply,

$$\int \vec{B} \cdot d^2\vec{s} = \int \text{div } \vec{B} \cdot d^3v = \int \rho_m \cdot d^3v \neq 0 \quad ,$$

that is, a different result to Eq. (15) is obtained. If, for instance, our system contains only a magnetic pole, the a-priori imposition of a Coulombian behaviour of \vec{B} and, at the same time, $\text{div } \vec{B} = 0$ conducts us to a possible solution of the form,

$$\vec{A} \sim \cotg\theta/r \cdot \vec{u}_p \quad ,$$

having a singularity for $\theta = 0$ and $\theta = \pi$. In this way the spherical symmetry of the source is destroyed⁽¹⁷⁾. The singularities of \vec{A} are nothing more than Dirac-Schwinger strings⁽¹⁸⁾.

We know, today, that the Dirac approach to Eq. (14) is not the unique one. In fact, both the quantization of the electrical charge and the need of not introducing additional degrees of freedom for the electromagnetic fields are conditions satisfied in the Cabibbo-Ferrari theory⁽⁷⁾. The two potential approach of these authors is not only free of the unphysical strings, but it seems to be the obvious mathematical context for treating the linear equations (14). These equations being linear, the solution of them can be obviously expressed as a linear combination of the solution of the equations,

$$\partial_\nu F^{\mu\nu} = j_e^\mu \quad , \quad \partial_\nu \tilde{F}^{\mu\nu} = 0 \quad , \quad (16)$$

and

$$\partial_\nu F^{\mu\nu} = 0 \quad , \quad \partial_\nu \tilde{F}^{\mu\nu} = j_m^\mu \quad . \quad (17)$$

Both of the equations (16) and (17) permit the introduction of a regular

and well behaved potential, which we denote by A_e and A_m .

To avoid the use of two independent potentials, Cabibbo-Ferrari were led to introduce not only the ordinary gauge transformations,

$$A_e^\mu \rightarrow A_e^\mu + \partial^\mu \Lambda \quad ; \quad A_m^\mu \rightarrow A_m^\mu + \partial^\mu \Lambda' \quad ,$$

They were obliged, as well, to assume that A_e and A_m were physically identical and, therefore, that the e.m. fields obtained from A_e and A_m were entities with identical physical properties. This being so, they were able to introduce additional gauge transformations, mixing A_e and A_m ,

$$A_e \rightarrow A_e + A'_e \quad ; \quad A_m \rightarrow A_m + A'_m \quad . \quad (18)$$

Since the unique e.m. field tensor F introduced by them ($A_e \leftrightarrow A_m \leftrightarrow B$) is given by Eq. (11), its value will not be changed if the "mixing gauges" are connected by the "null-field condition",

$$\partial_\mu A'_{ev} - \partial_\nu A'_{e\mu} + E_{\mu\nu\rho\sigma} \partial^\rho A_m'^\sigma = 0 \quad . \quad (19)$$

This Eq. is interpreted as a constrain which limits the additional degrees of freedom associated with the use of two e.m. potentials.

It will be shown, in Section V, that there are physical reasons preventing the mixing of the e.m. fields appearing in Eq. (9) and (19) and that, therefore, both conditions must disappear. In this way A_e and A_m become independent and two electromagnetic field tensors F_e and F_m have to be used (two-photon theory).

IV. THE TRANSFORMATION OF THE ELECTROMAGNETIC FIELDS UNDER STRONG TIME REVERSAL. WHEN $V/C \ll 1$

Let us suppose that we have a system A composed of charges and poles moving in a certain way in relation to a certain referential. Consider the system B obtained from the system A by changing exclusively the sense of the velocities of all the sources (charges and poles)

contained in \underline{A} (classical time reversal). This kind of transformation, in which ρ_e and ρ_m remain unchanged and both \vec{j}_e and \vec{j}_m change in sign, is a "strong transformation" in the terminology introduced by Pinta-cuda⁽¹³⁾.

We stress that this way of considering T-reversal is not the usual one of Quantum Field Theory, in which theory a pseudoscalar character is ascribed to ρ_m ⁽¹⁹⁾. The difference is that our classical T-reversal is a strong transformation, since only transformations of geometrical objects in Minkovsky space is permitted: in our case the transformation is $\vec{v} \rightarrow -\vec{v}$. On the other hand, T-reversal, as usually employed in the CPMT theorem of Quantum Field Theory⁽⁶⁾ is a weak transformation, in which the signs of the charges and poles are involved as well⁽¹⁹⁾. At a classical level - as strongly remarked by Aharoni and others⁽²⁰⁾ - a changing of the sign of ρ_e or ρ_m under strong transformations has no sense at all. In fact, it is up to us to choose a transformation under which only the sense of the velocities of the sources is reversed.

Although the transformation $\vec{v} \rightarrow -\vec{v}$, considered by us, is a global transformation and, therefore, its implications have an approximate value, we are going to see that its consequences are far reaching.

Indeed, being clear the meaning attached to our time inversion operation, suppose that we call (\vec{E}_e, \vec{B}_e) and (\vec{E}_m, \vec{B}_m) the electromagnetic fields created, respectively, by the electric and magnetic sources of system \underline{A} . Then, the fields created in the system \underline{B} are,

$$(\vec{E}_e, -\vec{B}_e) \quad , \quad (-\vec{E}_m, \vec{B}_m) .$$

This situation shows that, had we written $(\vec{E}_e + \vec{E}_m, \vec{B}_e + \vec{B}_m)$ for the total electromagnetic field created by the system \underline{A} , then this total field would not have a defined behaviour under the T inversion operation on the sources. The only way of ascertain the behaviour of the total field (\vec{E}_t, \vec{B}_t) under our transformation would be the resolution of it in terms of its magnetic and electric parts (\vec{E}_e, \vec{B}_e) , (\vec{E}_m, \vec{B}_m) .

From all this it follows that the electromagnetic field created by a system in which only magnetic source poles are present can be

physically distinguished from the electromagnetic field created by another system in which only electric charges are present. Indeed, if under the transformation $\vec{v} \rightarrow -\vec{v}$ the electric field is found to change in sign, we can be sure that the sources are of magnetic nature. If, on the contrary, it is the magnetic field which is observed to change in sign, the sources are purely electrical. Therefore, by mixing \vec{E}_e , \vec{B}_e with \vec{E}_m , \vec{B}_m physical information is lost: the information which tell us what kind of source -electric or magnetic- was responsible for the creation of a particular (\vec{E}, \vec{B}) . This is precious information, which in no way can be permitted to be lost by permitting the linear combinations $\vec{E}_e + \vec{E}_m$ and $\vec{B}_e + \vec{B}_m$.

Since, according to us, (\vec{E}_e, \vec{B}_e) and (\vec{E}_m, \vec{B}_m) are couples of physical magnitudes intrinsically different -in fact our strong v-reversal can distinguish between them, as explained above - the use of a single electromagnetic field tensor $F^{\mu\nu}$ is no longer permitted. Therefore, Eq. (9) cannot be safely written, since the mixings $\vec{E}_e + \vec{E}_m$ and $\vec{B}_e + \vec{B}_m$ are implicitly contained in it. For identical physical reasons the null-field condition (19) cannot be maintained.

The strong v-reversal approach to Eq. (5) has obliged us, therefore, to assume a Cabibbo-Ferrari two-potential description. These two potentials have to be independent and must be kept unmixed. To each of them is associated a different kind of photon in an eventual quantification of our classical theory. The duplicity of the neutrino fields comes immediately to mind. The above arguments can be equally applied to any physical system A in which two kinds of sources s_1 and s_2 are present, sources which possess the following characteristics: When s_1 is at rest, only the field E_1 is observed; when s_1 is in motion we observe E_1 and the relativistic effect, proportional to the velocity of the source, $E'_1 \cdot E_1$, E'_1 acts on a test particle of type s_1 via a direct coupling and a coupling with the velocity of the test particle. On the other hand, when the source s_2 is at rest only the field E_2 is observed; when s_2 is in motion we observe a field E'_2 , as well, proportional to the velocity of the source s_2 . E_2, E'_2 acts on a test particle of type s_1 via a coupling with the velocity of s_1 and a direct coupling respec-

tively. It is for this physical reason that we can say that (E_1, E_2) and (E_z, E_1') are couples of similar fields.

Our contention is that the fields appearing in a same couple cannot be mixed together, since they transform in a different way under the transformations $\bar{v} \rightarrow -\bar{v}$. Therefore, even if they are similar fields, because of the similar way of being coupled to the test particles of type s_1 , the transformation $\bar{v} \rightarrow -\bar{v}$ is able of distinguishing between them and, accordingly, the fields appearing in the same couple must be treated as different physical entities. This is not strange since, for example, E_1 is originated by a physical source at rest and E_2' by the motion of a physical source s_2 that, in principle, has no physical relation at all with s_1 . The intrinsic difference between the sources (and the fact that E_2' can be eliminated in a certain referential, while E_1 cannot be eliminated in this way) is, in fact, reflected by the physical distinction that one has to keep between E_1 and E_2' on one hand, and between E_z and E_1' on the other hand. In the particular case of the e.m. fields created by charges and poles, the above couples are (\bar{E}_e, \bar{E}_m) and (\bar{B}_e, \bar{B}_m) and the fact that \bar{E}_e and \bar{E}_m are different physical entities forces us to the introduction of two electromagnetic tensors F_e and F_m . As we have explained above, F_e and F_m cannot be superposed. On the other hand, the considerations of this section concerning the sources s_1 and s_2 have little relation with Maxwell's equations, since we have introduced E_1' and E_2' as relativistic effects originated by the motion of an observer, which observed in motion the sources s_1 and s_2 .

V. CONCLUSIONS

Whether or not magnetic poles exist in Nature is a question that only experiments can answer⁽²¹⁾. This work does not imply any restriction on the future possibility of finding them, although the absence of crossing symmetry and non-analyticity of the S-matrix have been reported⁽²²⁾ as consequences following the existence of monopoles in the framework of quantum field theory.

Our considerations have been classical, since the operational ground of strong transformations is of classical nature. On the other hand, the study of magnetic poles at a classical level is by no means an academic issue, this matter being a subject of recent research⁽²³⁾; in particular the unified field theories of Boal-Moffat⁽²⁴⁾ seem to forbid the existence of magnetic poles.

Our approach is similar to that of Cabibbo-Ferrari and others⁽⁷⁾, since two electromagnetic potentials have been introduced. Nevertheless our considerations have to differ from theirs because, as we have explained, the transformation $\bar{v} \rightarrow -\bar{v}$ is able to distinguish between the e.m. field created by charges and poles. Therefore we cannot consider as equivalent two physical entities, like F_e and F_m , having different ways of transformation under $\bar{v} \rightarrow -\bar{v}$. The null-field condition has been eliminated, since it connects, linearly, fields like \bar{E}_e and \bar{E}_m having different physical properties. Since we consider the two electromagnetic potentials A_e and A_m , what we propose here is a two photon theory.

In our theory, the motion of a dyon (e,g) under prescribed external fields F_e and F_m would be given by the completely symmetric and covariant equation,

$$m_0 \frac{dv^\mu}{d\tau} = e F_e^{\mu\nu} \cdot V_\nu + e \tilde{F}_m^{\mu\nu} \cdot V_\nu + g F_e^{\mu\nu} \cdot V_\nu + g F_m^{\mu\nu} \cdot V_\nu \quad , \quad (20)$$

which corrects Eq. (14), in which only one kind of e.m. field F_e was used: the e.m. field created exclusively by electric sources. As can be seen from Eq. (20), the use of two independent e.m. fields does really complicate the study of the system composed by charges and poles, as noticed in 1966 by Carithers et al.⁽⁴⁾. But the arguments given above lead, inevitably, to this duplicity of e.m. fields if a monopole is ever found.

Our approach can shed some light on Parker's conjecture. The formal identification suggested by Parker between F_e and F_m (as explained in Section I and II) is not possible, simply because F_e and F_m are different physical objects. Acting otherwise would be similar to

identify, in strong interactions theory, the polar and axial parts of a physical current. Parker's conjecture was enunciated in 1969. According to us, the suggestions contained in the scientific literature prior to 1969 -particularly those by Zocher-Torok⁽⁴⁾ and Carithers et al⁽⁴⁾ - were sufficient for solving the conjecture in a way different to that proposed by Parker: only the considerable confusion about the meaning of strong and weak symmetries has prevented an earlier clarification on these matters. For this reason, the clarifying ideas of Pintacuda⁽¹³⁾, Aharoni and Luders⁽²⁰⁾ about strong and weak symmetries merit much more diffusion than they actually seem to have.

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