revista mexicana de física 27 no. 4 (1981) 533-581

THE INTERACTING BOSON MODEL: ITS FORMULATION, APPLICATION, EXTENSION AND INTERPRETATION

Bruce R. Barrett*

Department of Physics and the Theoretical Physics Division,
University of Arizona, Tucson, Arizona 85721 †

and

Kernfysisch Versneller Instituut, Groningen, The Netherlands

ABSTRACT

The goal of this article is to review the present status of the Interacting Boson Model (IBM) for describing the collective properties of medium and heavy mass nuclei, with particular emphasis being given to the work on the IBM at the University of Arizona. First, a concise review of the basic phenomenological IBM, as developed by Arima and Iachello for only one kind of boson, is presented. Next, the extension of the IBM to both proton and neutron bosons is outlined. This latter model is known as the IBM-2. The application of the IBM-2 to the tungsten isotopes by the University of Arizona group is discussed, followed by their calculations for the mercury isotopes. In the case of the mercury isotopes an extended form of the IBM-2 is developed in order to treat the configuration mixing of two entirely different structures which occur in the same energy region. The relationship between the bosons and the underlying fermionic structure of the nucleus is discussed using the generalized seniority scheme of Talmi. Work by the Arizona group to calculate the phenomenological parameters of the IBM-2 using these generalized seniority ideas is described, along with their results,

* Work supported in part by the National Science Foundation (Grant No. PHY-7902654 A01) and by the Stichting voor Fundamental Onderzoek der Materie (FOM).

† Permanent Address.

which agree quite well with the empirical values. Efforts by the University of Arizona group to determine the influence of terms left out of the basic IBM, such as the g boson, using second-order perturbation theory are described. In conclusion, a discussion of the limitations as well as the usefulness of the IBM is given along with its exciting possibilities for the future of nuclear structure physics.

RESUMEN

El propósito del presente trabajo es el de hacer una revisión del estado actual del Modelo de Bosones en Interacción (MBI) utilizado hoy en día para describir las propiedades colectivas de núcleos pesados y semi-pesados. Se hace énfasis, en particular, al trabajo que sobre el MBI se viene realizando en la Universidad de Arizona. Primeramente se presenta una revisión concisa de las bases fenomenológicas del MBI con un solo tipo de bosones tal y como fueron desarrolladas por Arima y Iachello. Posteriormente se hace una presentación esquemática de la generalización del MBI que incluye tanto bosones de protón como de neutrón. Este último modelo se conoce como el MBI-2. Después se discuten las aplicaciones que el grupo de la Universidad de Arizona ha realizado a isótopos de tungsteno presentando en seguida los cálculos realizados para isótopos del mercurio. Para el caso de isótopos del mercurio se desarrolla una forma más general del MBI-2 que hace posible tratar la mezcla de configuraciones de dos estructuras completamente distintas que aparecen en la misma región de energías. Se discute también la relación entre los bosones y la estructura fermiónica nuclear subyacente utilizando el esquema de antigüedad (seniority) generalizado propuesto por Talmi. Se describe el trabajo realizado por el grupo de Arizona sobre el cálculo de los parámetros fenomenológicos del MBI-2 utilizando las ideas de antigüedad generalizada mencionadas anteriormente y se presentan los resultados obtenidos, los cuales concuerdan bastante bien con los valores empíricos. Se describen también los esfuerzos del grupo de la Universidad de Arizona para determinar, utilizando teoría de perturbaciones a segundo orden, la influencia de los términos no incluidos en el MBI básico, tales como el bosón g. Se discuten, en conclusión, tanto las limitaciones como la utilidad del MBI así como sus estimulantes posibilidades para el futuro de la física de la estructura nuclear.

I. INTRODUCTION

One of the most interesting features of medium and heavy mass nuclei is the presence of low-lying collective states. Much effort has been made in the previous thirty years to understand the nature of these collective properties in nuclei, mainly following the pioneering work of Rainwater[1] and Bohr and Mottelson[2] in terms of geometrical models. The problem with these geometrical models is that there is no well-defined procedure for making transitions between the different models, e.g., the transition from a spherical vibrator to a deformed

rotor. Attempts have also been made to describe these collective properties in terms of boson degrees of freedom[3], instead of fermion degrees of freedom. Unfortunately most of these boson methods involve infinite expansions, i.e.,boson operators of ever-increasing order[4].

Recently two boson methods have been developed which contain boson operators of finite order. One is the method of Janssen, Jolos and Dönau[5] which contains quadrupole operators that obey the commutation relations of a U(6) Lie algebra. Since their method utilizes only quadrupole operators, their expansion consists only of quadrupole (J=2) bosons. The second approach is that of Arima and Iachello[6-8], known as the Interacting Boson Model (IBM), which contains monopole (J=0) s bosons as well as quadrupole d bosons. The marked difference of the IBM from earlier boson models is that the total number of bosons is conserved, i.e., $n_s + n_d = N$ = constant. This is an extremely important feature, since it directly links the IBM to the number of valence fermions and thereby to the underlying single-particle or shell-model structure of the nucleus[9-11]. Consequently, the IBM provides us with a possible method of simultaneously interpreting nuclear collective properties in terms of a very simple model, which contains only a few parameters in a model space much smaller than the usual shell-model space, and understanding this model and its parameters in terms of the underlying fermionic structure.

The purpose of this article is to describe the Interacting Boson Model (IBM), its application to explain the collective properties of medium-to-heavy-mass nuclei and efforts to understand this model in terms of the nuclear shell model. Section II describes the IBM as originally developed by Arima and Iachello for even-even nuclei using only one kind of boson. The model was later expanded to treat both proton and neutron bosons by Otsuka et al.[9], as we also show in Section II. In Section III the IBM is applied to the isotopes of tungsten (W) and mercury (Hg) and the results are discussed in relationship to other models and to other nuclei investigated in this mass region using the IBM. The calculations for the Hg isotopes indicate an extension of the basic IBM to include configuration mixing. Section IV contains a description of how the bosons can be related to shell-model states. From

this connection we can understand the origin of the parameters in the IBM and how they should change as a function of the number of neutron and/or proton bosons. We can also investigate how the values of the model parameters are influenced by the restriction of the IBM to only s and d bosons by including the effects of a g boson, using ordinary perturbation theory. In Section V we discuss the limitations of the IBM, the problems which still face it, its extension to odd A nuclei, including the so-called supersymmetries, and its exciting possibilities for the future of nuclear structure theory.

II. THE INTERACTING BOSON MODEL

A. IBM-1: One kind of boson

The original Interacting Boson Model of Arima and Iachello[6-8] did not distinguish between proton and neutron bosons; we will refer to this model as the IBM-1. Like the shell model, the IBM-1 is a truncation scheme for restricting the nuclear wave function to a few important degrees of freedom. In the case of medium-to-heavy-mass nuclei, the low-lying collective properties cannot be described in terms of a few shell-model configurations but would require millions or billions of configurations. The concept of the IBM-1 is that the bosons represent collective configurations, which contain the important degrees of freedom of these low-lying properties.

As in the shell model, one starts with a number of valence particles outside closed major shells and assumes that the structure of the low-lying levels is dominated by excitations among these particles. Secondly, one assumes that the important particle configurations for the low-lying levels of even-even nuclei are those for identical particles paired together in states with total angular momentum J=0 and J=2. The final assumption is that these pairs can be treated as bosons. Hence, the number of bosons is equal to the number of pairs of particles outside the closed shell and is a strictly conserved quantity. Proton (neutron) bosons with angular momentum J=0 are denoted by $s_\pi(s_\nu)$, while those with angular momentum J=2 are denoted by $d_\pi(d_\nu)$. In order to take into account the particle-hole conjugation in the particle space, the number of proton, N_π, and neutron, N_ν, bosons is taken as the number of hole

pairs, if more than half of the shell is full. For example, $^{130}_{56}Ba_{74}$ has 6 protons outside the 50 closed shell or 3 proton-particle bosons and 8 neutron holes in the 82 closed shell or 4 neutron-hole bosons. In the IBM-1 we do not distinguish between proton and neutron bosons, so we will drop the subscripts π and ν in the rest of this subsection.

In order to write down the appropriate operators for the bosons it is convenient to use the second quantized formalism, introducing the creation ($s^\dagger$, $d^\dagger_\mu$) and annihilation (s, d_μ) operators, for s and d bosons, respectively, where $\mu=0, \pm1, \pm2$. These operators satisfy the standard Bose commutation relations

$$
\begin{aligned}
&[s, s^\dagger] = 1, \quad [s, s] = 0, \quad [s^\dagger, s^\dagger] = 0 , \\
&[d_\mu, d^\dagger_\mu,] = \delta_{\mu\mu'}, \quad [d_\mu, d_\mu,] = 0, \quad [d^\dagger_\mu, d^\dagger_\mu,] = 0 , \\
&[s, d^\dagger_\mu] = 0, \quad [s^\dagger, d^\dagger_\mu] = 0, \quad [s, d_\mu] = 0 , \\
&[s^\dagger, d_\mu] = 0 .
\end{aligned}
\tag{1}
$$

The most general two-body Hamiltonian for a system of s and d bosons, which conserves the total number of bosons, can be written in terms of these operators in the form[6]

$$
\begin{aligned}
H = {} & \varepsilon_s (s^\dagger \cdot s) + \varepsilon_d (d^\dagger \cdot \tilde{d}) + \sum_{L=0,2,4} \frac{1}{2} (2L+1)^{1/2} C_L [[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}]^{(0)} \\
& + \frac{1}{\sqrt{2}} \tilde{V}_2 [[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times s]^{(2)} + [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} \\
& + \frac{1}{2} \tilde{V}_0 [[d^\dagger \times d^\dagger]^{(0)} \times [s \times s]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} \\
& + U_2 [[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times s]^{(2)}]^{(0)} + \frac{1}{2} U_0 [[s^\dagger \times s^\dagger]^{(0)} \times [s \times s]^{(0)}]^{(0)} ,
\end{aligned}
\tag{2}
$$

where $\tilde{d}_\mu = (-1)^{2+\mu} d_{-\mu} = (-1)^\mu d_{-\mu}$, which is a spherical tensor under rota-

tions, while $d^\dagger_\mu$ is already a spherical tensor. The [ ] denote the tensor product of two tensor operators, e.g., $[d^\dagger \times d^\dagger]^{(L)}_M = \sum_{\mu_1\mu_2} \langle 2\mu_1 2\mu_2 | LM \rangle d^\dagger_{\mu_1} d^\dagger_{\mu_2}$, while the ( ) denote the scalar product of two tensor operators, e.g., $(d^\dagger \cdot \tilde{d}) = (-1)^2 \sqrt{5}[d^\dagger \times d]^{(0)}_0 = \sqrt{5} \sum_{\mu_1\mu_2} \langle 2\mu_1 2\mu_2 | 00 \rangle d^\dagger_{\mu_1} \tilde{d}_{\mu_2} = \sum_{\mu_1} d^\dagger_{\mu_1} d_{\mu_1} =$ n_d = the number operator for d bosons. The above Hamiltonian contains nine parameters: the two single bosons energies ε_s and ε_d and the seven two-body terms $C_L(L=0,2,4)$, $\tilde{V}_L(L=0,2)$ and $U_L(L=0,2)$. Since the total number of bosons is conserved, $N = n_s + n_d$, the Hamiltonian can be rewritten so that the excitation energies are independent of n_s and depend upon only six parameters (see Refs. 12,13).

Another convenient form for writing the Hamiltonian for the excitation energies only is

$$H = \varepsilon' n_d + a_0(\underset{\sim}{P}^\dagger \cdot \underset{\sim}{P}) + a_1(\underset{\sim}{L} \cdot \underset{\sim}{L}) + a_2(\underset{\sim}{Q} \cdot \underset{\sim}{Q}) + a_3(\underset{\sim}{T}_3 \cdot \underset{\sim}{T}_3) + a_4(\underset{\sim}{T}_4 \cdot \underset{\sim}{T}_4), \quad (3)$$

where

$$\begin{aligned} n_d &= (d^\dagger \cdot \tilde{d}) \ , \\ \underset{\sim}{P} &= \frac{1}{2}(\tilde{d} \cdot \tilde{d}) - \frac{1}{2}(s \cdot s) \ , \\ \underset{\sim}{L} &= \sqrt{10}\,[d^\dagger \times \tilde{d}]^{(1)} \ , \\ \underset{\sim}{Q} &= [d^\dagger \times s + s^\dagger \times \tilde{d}]^{(2)} - \frac{\sqrt{7}}{2}[d^\dagger \times \tilde{d}]^{(2)} \ , \\ \underset{\sim}{T}_3 &= [d^\dagger \times \tilde{d}]^{(3)} \text{ and } \underset{\sim}{T}_4 = [d^\dagger \times \tilde{d}]^{(4)} \ . \end{aligned} \quad (4)$$

Note that this H contains only six independent parameters. This form of H is often convenient, since empirically only one or more terms in this H are required in order to describe reasonably well the low-lying excitation spectrum. The relationship between the parameters in Eqs. (2) and (3) is given in Ref. 12.

Another operator of interest is the one-body transition operator which has the second quantized form[12]

$$T^{(\ell)}_m = \alpha_2 \delta_{\ell 2} [d^\dagger \times s + s^\dagger \times \tilde{d}]^{(2)}_m + \beta_\ell [d^\dagger \times \tilde{d}]^{(\ell)}_m + \gamma_0 \delta_{\ell_0} \delta_{m_0} [s^\dagger \times s]^{(0)}_0 . \quad (5)$$

It should be noted that no multipole higher than four is possible if the operators $T^{(\ell)}$ are assumed to be at most one-body. Equation (5) yields transition operators for E0, M1, E2, M3 and E4 transitions, with appropriate values of the corresponding parameters. The B(Eℓ) and B(Mℓ) values are obtained in the usual way as

$$B(\ell; J_i \to J_f) = \frac{1}{2J_i + 1} |\langle J_f || T^{(\ell)} || J_i \rangle|^2 . \quad (6)$$

From the transition operators one can also calculate quadrupole moments, magnetic moments and isotope and isomer shifts. One can also construct two-particle transfer operators[6,13].

The Hamiltonian in the form of either Eq. (2) or (3) can be directly applied to describe the low-lying spectra of medium and heavy mass nuclei away from closed shells. Such studies have been carried out, for example, for the even samarium (Sm) isotopes[7]. It is important to emphasize that the Hamiltonians in Eqs. (2) and (3) are completely general and can be used along with the transition operators in a systematic manner to study the collective properties of any appropriate nucleus.

i) U(6) Symmetry and Dynamical Symmetries

There are a number of nuclei which exhibit special collective properties, which have already been explained and given specific names within the geometrical picture, such as vibrational properties and rotational properties. One of the very interesting features of the IBM-1 is that these geometrical cases follow directly as limiting cases of the IBM-1 which can be solved analytically. This is true since the second quantized operators of the IBM-1 (i.e., the $s^\dagger$, $d^\dagger_\mu$ and the s, $\tilde{d}_\mu$) can be combined pairwise (i.e., a creation operator with an annihilation operator) to form a set of 36 operators

$$G^{(k)}_m(\ell\ell') = [b^\dagger_\ell \times \tilde{b}_{\ell'}]^{(k)}_m , \text{ where } b^\dagger_{\ell\mu} \equiv (s^\dagger, d^\dagger_\mu) \quad (7)$$

$$\tilde{b}_{\ell\mu} \equiv (-1)^{\ell+\mu} b_{\ell,-\mu}$$

for $\ell, \ell' = 0, 2,$

which are closed under commutation, i.e.,

$$[G^k, G^{k'}] = \sum_{k''} C^{k''}_{kk'} G^{k''} , \tag{8}$$

where the $C^{k''}_{kk'}$ are numerical coefficients, and, consequently, form a Lie algebra[12,14]. One can verify that this is the Lie algebra of the group U(6) of unitary transformations in six dimensions. Since the Hamiltonian [Eq.(2)] is built out of these operators, it follows that it has the group structure of U(6). In other words, it can be regarded as a general rotation with constant norm ($n_s + n_d = N$ = constant) in the six dimensional space of the s and d bosons.

Generally it is possible to determine subgroups of some larger group, such as U(6), where a subgroup is defined by a subset of the generators of the larger group which also close under commutation. In the case of U(6) it has been found that three subgroup chains exist, when each chain is restricted to contain the angular momentum group, O(3)[14]. These three chains are

I. $U(6) \supset U(5) \supset O(5) \supset O(3) \supset O(2)$ (9a)

II. $U(6) \supset SU(3) \supset O(3) \supset O(2)$ (9b)

III. $U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2)$. (9c)

If the Hamiltonian H can be written in terms of only the Casimir operators of a <u>complete</u> chain of subgroups of U(6), then this H is <u>diagonal</u> in the representation of this subgroup chain and possesses what is known as a dynamical symmetry[12-14]. A dynamical symmetry is a systematic breaking of the symmetry of the larger group [e.g., U(6)] by the terms proportional to the Casimir operators of the subgroups. For example, in Chain I [Eq.(9a)], the U(6) symmetry is broken by the Casimir operators of U(5); in turn the U(5) symmetry is broken by the O(5) Casimir operator, and the O(5) symmetry is broken by the O(3) Casimir operator [see Eq. (10a)]. But the symmetries of each subgroup

are maintained in the sense that different representations of U(5) do not mix, etc. (see Fig.1). Since in the IBM-1 we have three subgroup chains, there are three possible dynamical symmetries. These three dynamical symmetries have been discussed extensively in previous papers (Refs. 6, 12-14), and the reader is referred to these articles for details of their construction and solution. Here we will only quote the results for the energy in each limit.

I. U(5) or vibrational limit

$$E([N], n_d, v, n_\Delta, L, M) = \varepsilon n_d + \alpha n_d(n_d+4) + 2\beta v(v+3) + 2\gamma L(L+1) \tag{10a}$$

II. SU(3) or rotational limit

$$E([N], (\lambda,\mu), K, L, M) = (\tfrac{3}{4}\kappa - \kappa')L(L+1) - \kappa[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)] \tag{10b}$$

III. O(6) or γ-unstable limit

$$E([N]\ \sigma, v, n_\Delta, L, M) = \tfrac{1}{4}A(N-\sigma)(N+\sigma+4) + Bv(v+3) + CL(L+1)\ . \tag{10c}$$

In the above energy formulas the symbols in parentheses on the left-hand side denote the quantum numbers which are needed to specify uniquely the states of each subgroup chain. For example, in the U(5) chain N is the total number of bosons, which is the U(6) quantum number; n_d is the number of d-bosons, which is the U(5) quantum number; v is the d-boson seniority, which is the O(5) quantum number; L is the angular momentum [O(3)] ; and M is the z-projection of L[O(2)]. The quantum number n_Δ is an extra quantum number required in order to fully descompose O(5) in going to O(3). Arima and Iachello chose this quantum number n_Δ to be the number of boson triplets coupled to zero angular momentum. The constants $\varepsilon,\alpha,\beta,\gamma$; κ,κ'; and A,B,C label boson energy parameters appropriate for each of the three limiting cases and represent particular linear combinations of the parameters in Eqs. (2) and (3). Figures 1-3 illustrate the energy spectra which correspond to the U(5), SU(3), and O(6) limiting cases, respectively.

As mentioned earlier, nuclei with vibrational U(5)-like proper-

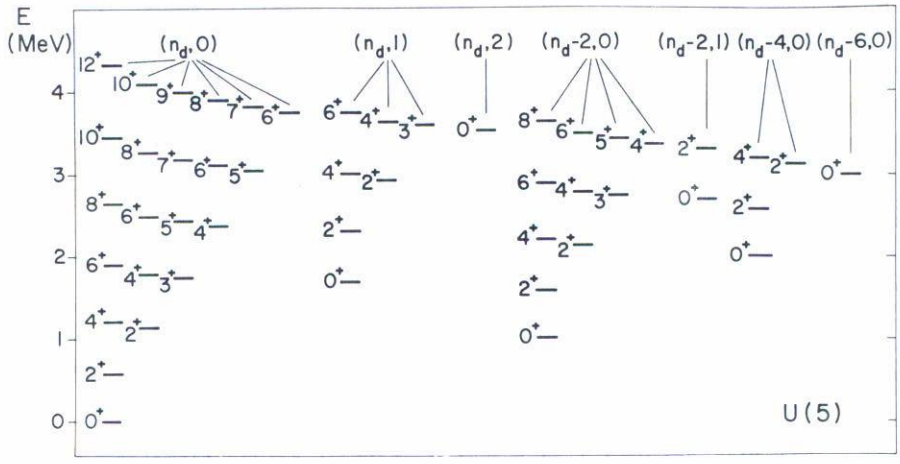

Fig.1 A typical spectrum with U(5) symmetry and N = 6. In parentheses are the values of v and n_Δ.

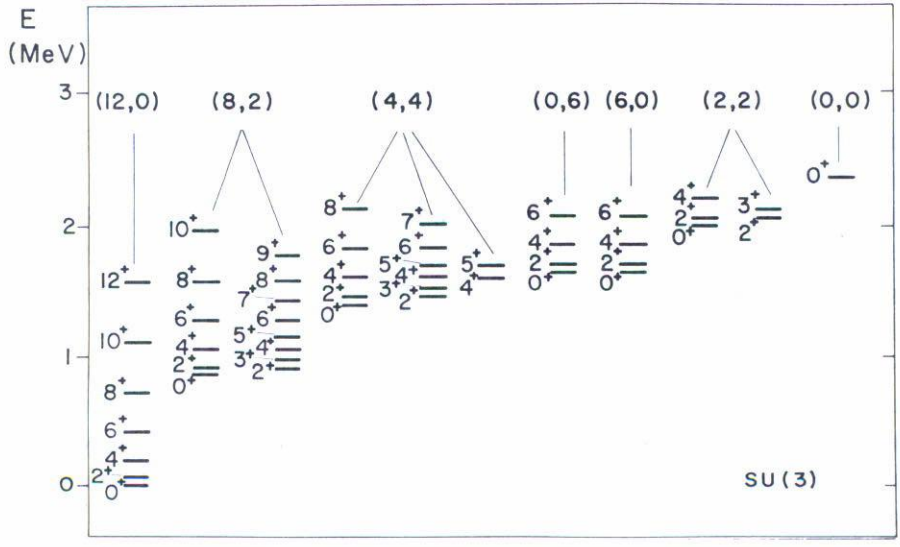

Fig.2 A typical spectrum with SU(3) symmetry and N = 6. In parentheses are the values of λ and μ.

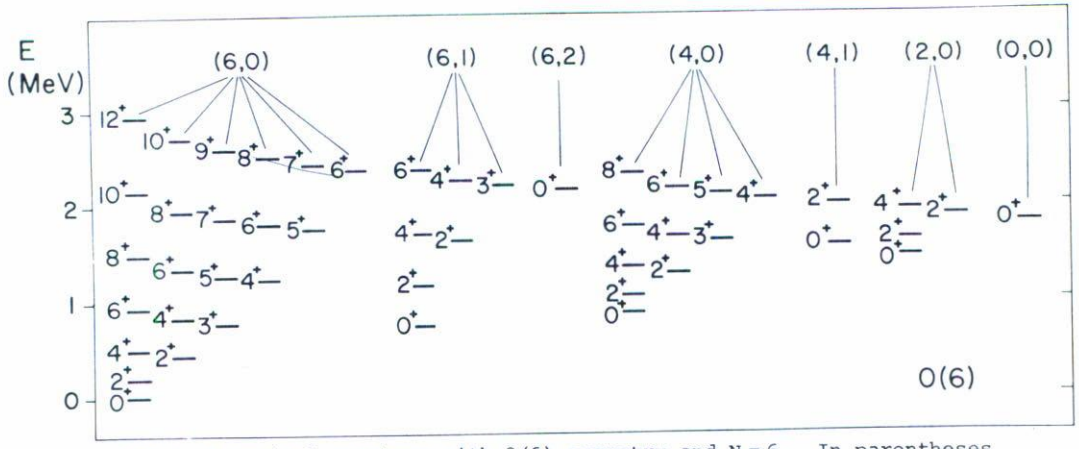

Fig.3 A typical spectrum with O(6) symmetry and N = 6. In parentheses are the values of σ and n_Δ.

ties and rotational SU(3)-like properties have already been known for some time and investigated using geometrical models. One of the interesting predictions of the IBM-1 is that a third limiting case exists, namely the O(6) limit, which appears to resemble the γ-unstable model of Wilets and Jean[15] and has been observed experimentally among some of the Pt isotopes[16]. Figures 4-6 show examples of experimental spectra which exhibit the limiting cases U(5), SU(3) and O(6), respectively. In general, examples of U(5) - like nuclei are found at the beginning and the very end of shells, examples of SU(3) - like nuclei are found in the middle of shells, and examples of O(6) - like nuclei past the middle of shells.

As stated previously, most nuclei do not belong to any of these limiting cases but are somewhere in between two of them. For example, the Sm isotopes start out vibrational-like [U(5)] at the beginning of the 82 neutron shell and become rotational-like [SU(3)] as the neutron number increases towards the middle of the shell, as seen in Fig.7(Exp.). Using the general IBM-1 Hamiltonian, Eq. (3), Scholten et al.[7] have been able to reproduce this transition from U(5) to SU(3) symmetry in the Sm isotopes [Fig. 7(Th.)]. They are also able to reproduce the correct trends with respect to N_ν in the B(E2) transition rates, two-neutron separation energies, quadrupole moments and isotope and isomer shifts[7,13].

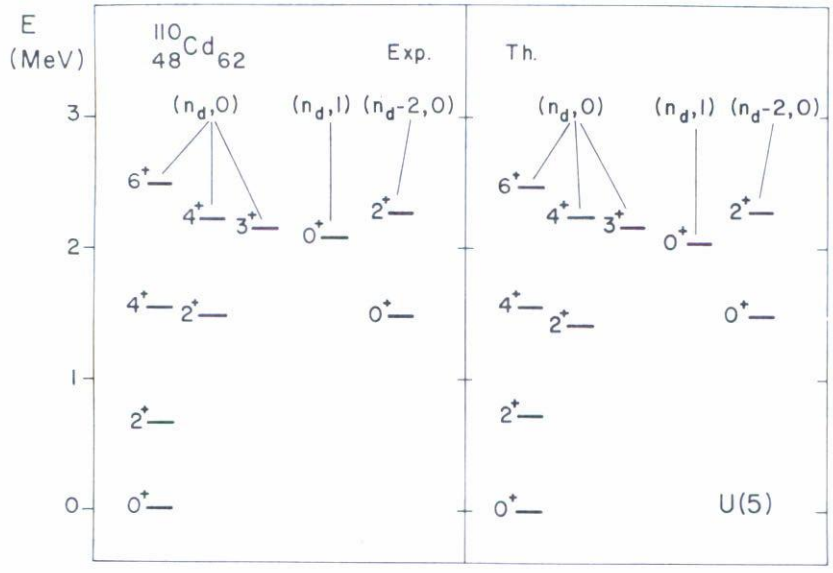

Fig.4 An example of a spectrum with U(5) symmetry: $^{110}_{48}Cd_{62}$, $N_\pi = 1$, $N_\nu = 6$, $N = 7$.

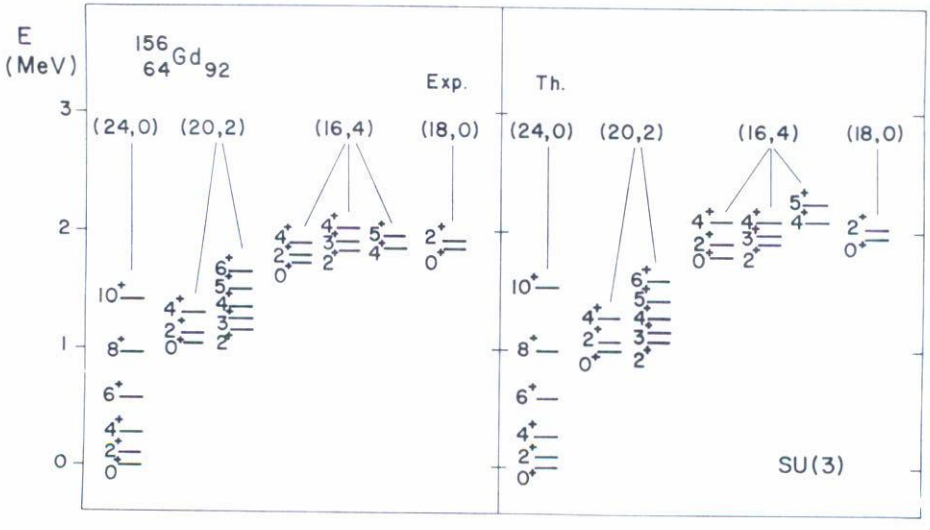

Fig.5 An example of a spectrum with SU(3) symmetry: $^{156}_{64}Gd_{92}$, $N_\pi = 7$, $N_\nu = 5$, $N = 12$.

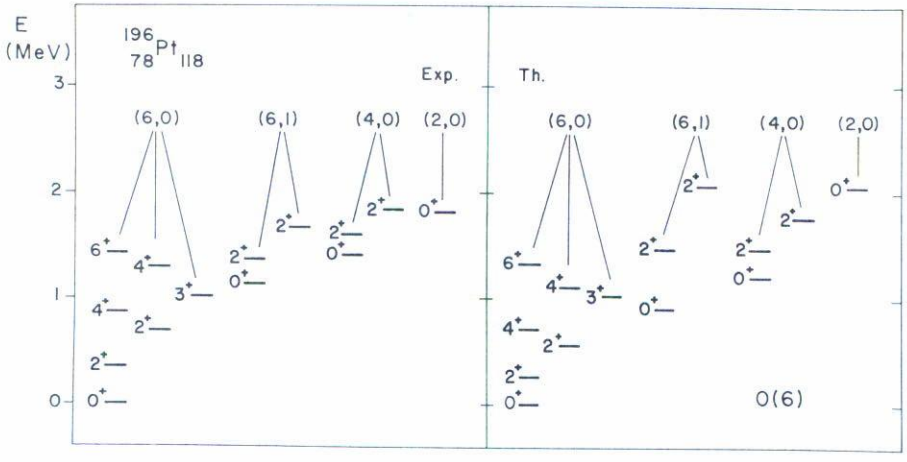

Fig.6 An example of a spectrum with O(6) symmetry: $^{196}_{78}Pt_{118}$, $N_\pi = 2$, $N_\nu = 4$, $N = 6$.

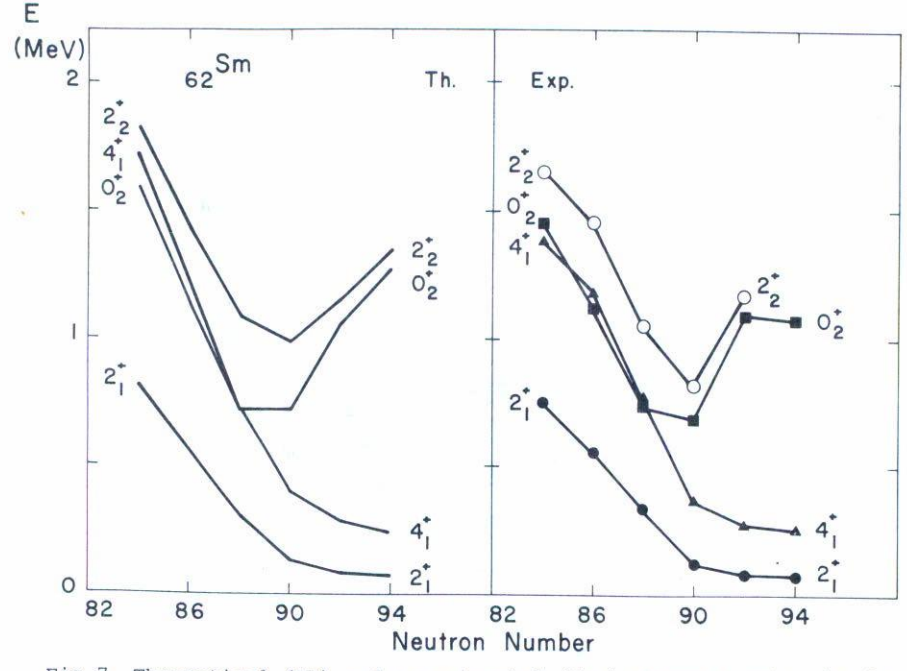

Fig.7. Theoretical (Th) and experimental (Exp) energy spectra in the samarium isotopes.

A computer program, called PHINT, has been written by Scholten[17] and is available on request. The program PHINT solves the full IBM-1 Hamiltonian [Eq.(3)].

B. IBM-2: Proton and Neutron Bosons

As Talmi has pointed out for many years[11], it is the proton-neutron interaction which is mainly responsible for causing nuclei to deform. As experimental evidence for this fact, he observes that the energy splitting between the 0^+ ground state and the first 2^+ excited state in the even tin (Sn) isotopes is more or less constant throughout the entire 50 to 82 neutron shell (see Fig.8). For the even isotones with $N_\nu = 82$ one finds a similar result. Hence, the interaction among only valence neutrons or only valence protons in semi-magic nuclei (only one closed shell) is not sufficient to cause nuclei to deform. On the other hand, nuclei such as barium (Ba) and xenon (Xe), which

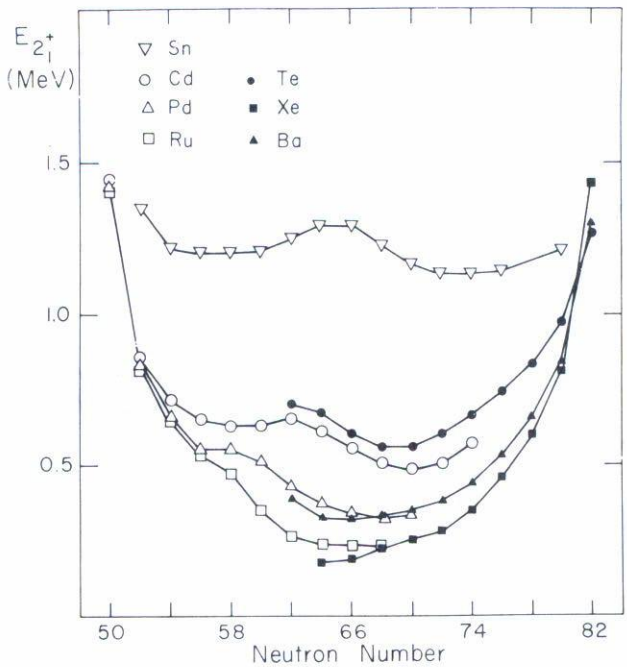

Fig.8 The energy of the 2_1^+ state of the Sn, Cd, Pd, Ru, Te, Xe and Ba isotopes in the 50-82 neutron shell.

have both active valence protons and valence neutrons, show a strong decrease in the $2^+ - 0^+$ energy splitting as the neutron hole number increases from the closed shell value of 82, as seen in Fig.8 for the Ba isotopes. As the number of neutron holes increases, the proton-neutron interaction increases in strength, causing the Ba and Xe isotopes near the middle of the 50 to 82 neutron shell to become strongly deformed, as seen from their rotational-like spectra in this mass region.

So one wants to distinguish between neutron and proton bosons and wants to introduce a term into the basic Hamiltonian involving an interaction between the neutron and the proton bosons. A completely general form for such a Hamiltonian is[8,9,13]

$$H = H_{\pi\pi} + H_{\nu\nu} + V_{\pi\nu} \quad , \tag{11}$$

where $H_{\rho\rho}$ ($\rho = \pi, \nu$) is the Hamiltonian for identical bosons and is of the form of Eq.(2), while $V_{\pi\nu}$ represents the interaction between the proton and neutron bosons.

In the IBM-2 the eigenstates are direct products of proton-boson states and neutron-boson states. In terms of group theory they represent the direct product of $U(6) \times U(6)$, so that the eigenstates are labeled by $[N_\pi] \times [N_\nu]$, where $[N]$ denotes the completely symmetric representation of U(6) containing N bosons.

The problem now is which, of all possible proton-neutron states, are the most important ones. Based on previous calculations using the IBM-1[8,13], we believe that the most significant IBM-2 proton-neutron states are those which have the same structure as the equivalent IBM-1 states, namely the totally symmetric states, labeled by $[N_\pi + N_\nu]$. Another way of saying this is that as much as possible we would like the IBM-2 Hamiltonian to be "invariant under rotations" in the proton-neutron space. Since this is a two-dimensional space, it transforms according to SU(2) and can be described using a formalism known as F-spin[13,18], which is similar to but not identical to isospin, where $F_z = \frac{N_\pi - N_\nu}{2}$ and $F = \frac{N}{2}, \frac{N-1}{2}, \ldots, F_z$. In terms of F-spin, the IBM-1 states correspond to the IBM-2 states of maximum F-spin, i.e., $F = \frac{N}{2} = \frac{N_\pi + N_\nu}{2}$.

Consequently, the only difference between the IBM-1 and the IBM-2 is that in the IBM-2 one also has states of lower (or mixed) U(6) symmetry, such as $[N_\pi + N_\nu - 1,1]$, $[N_\pi + N_\nu - 2,2]$, etc. A sufficient condition for the IBM-2 Hamiltonian to be F-spin invariant is for $V_{\pi\pi} = V_{\nu\nu} = V_{\pi\nu}$. Since this is in general not true, it will be necessary to separate the totally symmetric states from the states of mixed U(6) symmetry, as will be described below.

The general approach[8,9,11] is to include two terms in $V_{\pi\nu}$. The first is a quadrupole-quadrupole interaction between the proton and neutron bosons of the form $\kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)}$, where κ is the interaction strength parameter and

$$Q_\rho^{(2)} = (s_\rho^\dagger \times \tilde{d}_\rho + d_\rho^\dagger \times s_\rho)^{(2)} + \chi_\rho (d_\rho^\dagger \times \tilde{d}_\rho)^{(2)} \qquad (\rho=\pi,\nu) \qquad (12)$$

is the boson quadrupole operator. The parameter χ_ρ is the ratio of the seniority conserving part of Q_ρ to the seniority non-conserving part. The second term in $V_{\pi\nu}$ is a Majorana force of the form

$$M_{\pi\nu} = \xi_2 (s_\nu^\dagger \times d_\pi^\dagger - d_\nu^\dagger \times s_\pi^\dagger)^{(2)} \cdot (s_\nu \times \tilde{d}_\pi - \tilde{d}_\nu \times s_\pi)^{(2)} + \sum_{k=1,3} \xi_k (d_\nu^\dagger \times d_\pi^\dagger)^{(k)} \cdot (\tilde{d}_\nu \times \tilde{d}_\pi)^{(k)} \quad . \qquad (13)$$

The purpose of the Majorana force is to remove states with mixed U(6) symmetry, as discussed above, by shifting them up in energy. A large Majorana force guarantees that the low-lying states in the IBM-2 are nearly totally symmetric in the proton and neutron variables.

A quadrupole-quadrupole form is taken for the basic proton-boson-neutron-boson interaction for two reasons. First, it is a manifestation of the strong quadrupole-quadrupole interaction between protons and neutrons in the fermion space. Second, it is the lowest-order multipole-multipole force which mixes seniority[11].

Thus, the general form taken for the IBM-2 Hamiltonian[8,9,13] is

$$H = E_0 + \varepsilon(n_{d_\pi} + n_{d_\nu}) + \kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} + M_{\pi\nu} + V_{\nu\nu} + V_{\pi\pi} \quad , \qquad (14)$$

where $\varepsilon_\nu = \varepsilon_\pi = \varepsilon$ for simplicity and E_0 is a constant for a given nucleus, depending at most quadratically on N_ν and N_π and contributing only to

the binding energy. From the earlier discussion (e.g., the Sn isotopes) we expect the dominant term in $V_{\rho\rho}$ to be the d-boson conserving term

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho} \frac{1}{2} \sqrt{2L+1} \left[(d_\rho^\dagger \times d_\rho^\dagger)^{(L)} (\tilde{d}_\rho \times \tilde{d}_\rho)^{(L)} \right]^{(0)} , \quad \rho=\nu,\pi. \qquad (15)$$

Since the Hamiltonian (14) does not contain completely general forms for the interactions among the bosons, both for identical and non-identical bosons, it does not possess the appealing symmetry properties and subgroup chains, such as those found earlier for the IBM-1 Hamiltonian (2). However, there exist limits in which numerical solutions similar to the previous three IBM-1 cases, i.e., U(5), SU(3) and O(6), are obtained[8,13]. For example, when the number of proton and neutron bosons is small and near the beginning or the very end of a shell, so that $\varepsilon \langle n_d \rangle$ is large compared with $\kappa \langle Q_\pi \cdot Q_\nu \rangle$, U(5) or vibrational-like results are realized in the IBM-2. Here we have used $\langle\ \rangle$ to denote the expectation value of an operator. When the number of proton and neutron bosons is large (near the middle of a shell) and $\chi_\pi \simeq \chi_\nu \simeq -\sqrt{7}/2$, so that $\kappa \langle Q_\pi \cdot Q_\nu \rangle$ is large compared with $\varepsilon \langle n_d \rangle$, the IBM-2 results are SU(3) or rotational-like in structure. Finally, when the number of proton and neutron bosons is between the middle of the shell and the end of the shell and $\chi_\pi \simeq -\chi_\nu \simeq 0$, the IBM-2 Hamiltonian yields an O(6)-like spectrum. A direct relationship between the IBM-1 and IBM-2 parameters can be derived using the F-spin formalism[13].

A computer program called NPBOS, written by Otsuka and Scholten[19], determines the eigenenergies and eigenstates of the IBM-2 Hamiltonian (14) for different choices of the parameters. In general, there could be ten variable parameters to be determined for each nucleus studied. This number is however usually reduced to six by assuming that only $V_{\nu\nu}(V_{\pi\pi})$ contributes to relative splittings in isotopes (isotones) and that the contribution of C_{4_ρ} is negligible. The remaining six parameters are ε, κ, χ_ν, χ_π, C_{0_ρ} and C_{2_ρ} ($\rho = \pi$ or ν). After the first isotope (isotone) is described, $\chi_\pi(\chi_\nu)$ is determined and is then assumed to be the same for all the remaining isotopes (isotones), leaving only five parameters per nucleus.

The goal is to use the program NPBOS to determine empirically

the values of these six parameters which yield the best description of the low-lying spectra of medium- to heavy-mass nuclei and which at the same time vary smoothly with changes in the neutron- and proton-boson number. It is important, for example, that the set of parameters for one set of isotopes (constant Z) be quite similar to the set of parameters for the neighboring series of isotopes (i.e., nuclei with $Z \pm 2$). The values of these parameters should not vary in a random manner, if the IBM-2 is a truly meaningful description of the properties of nuclei in this mass region, since according to the IBM-2 these parameters are related to the underlying fermionic structure[8-11,13,20].

Therefore, it is definitely of interest to determine empirically these six parameters as well as possible in the mass regions $50 \leq N$ or $Z \leq 82$, $82 \leq N \leq 126$, $Z > 82$ and $N > 126$ and then to compare their values with results obtained from calculations based on microscopic theories involving the fermionic degrees of freedom. Work along this line has been carried out and is continuing and will be discussed in the next two sections.

III. APPLICATIONS OF THE IBM-2

Numerous applications of the IBM-2 to different nuclei have already been carried out, and the reader is referred to References 8, 13 and 21 for the details regarding some of these calculations. Here we present only the IBM-2 calculations performed at the University of Arizona for the tungsten[22] and mercury[23,24] isotopes.

A. The tungsten (W) isotopes:

The proton number for W is $Z = 74$, so it has four proton-boson holes, i.e., $N_\pi = 4$. The neutrons are in the 82-126 shell, so that $0 < N_\nu \leq 11$, where particle bosons are counted from the beginning to the middle of the shell and hole bosons from the middle to the end of the shell.

As discussed in Section II-B, it is assumed that $\varepsilon_\pi = \varepsilon_\nu = \varepsilon$ for simplicity. This might seem an oversimplification, since the proton bosons and neutron bosons are in different shells. However, calculations using this assumption have led to reasonable results, not only for W, but also for other nuclides[8,13,21].

The C_{0_ν} and C_{2_ν} terms are included in the $V_{\nu\nu}$ interaction but not at all in the $V_{\pi\pi}$ interaction, since for most of the region fitted, $N_\nu > N_\pi$, so that the $V_{\pi\pi}$ term is not excepted to be very important.

In the Majorana term $\xi_2 = 0.04$ MeV and $\xi_1 = \xi_3 = -0.02$ MeV for the entire isotopic chain. The Majorana term is used primarily to push up in energy those states with large non-symmetric parts. Since the low-lying collective states are largely symmetric, the influence of the Majorana term on these states is expected to be minimal[8,13].

With these simplifications, the Hamiltonian used in the fit to the W isotopes becomes

$$H = \varepsilon(n_{d_\pi} + n_{d_\nu}) + \kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} + M_{\pi\nu} + V_{\nu\nu} \quad , \tag{16}$$

so there are only six free parameters: ε, κ, χ_π, χ_ν, C_{0_ν} and C_{2_ν}. After one isotope is fitted, χ_π is established and is kept constant for the remaining isotope fits, so there are only five free parameters thereafter.

The experimentally determined energy levels for the even-even W isotopes span the range in neutron number from N = 96 to 114, but predictions can be made beyond this region by a smooth extrapolation of the above parameters.

Figure 9 shows the results of the Duval-Barrett[22] calculations of the energy levels for the isotopic chain $^{168}_{74}W_{94}$ to $^{192}_{74}W_{118}$. Figure 10 gives a detailed comparison with the experimental data[25-27] according to the quasi-ground state rotational band and the quasi-γ and β vibrational bands. Figure 11 contains graphs of the parameters used.

Perhaps the most striking feature of the energy spectra is the sharp rise in the γ and β bands at neutron number N = 108, which may be due to a subshell closure in the $i_{13/2}$ Nilsson level and/or a reversal in the deformation. This is supported by such effects as (1) a maximum in the quadrupole momentum of the 2_1^+ state for N = 108, (2) a minimum in the $2_1^+ - 0_1^+$ splitting for N = 108, and (3) a large change in the two-neutron separation energy after N = 108. Fitting this sharp rise has led to a dip in the value of χ_ν at N = 108.

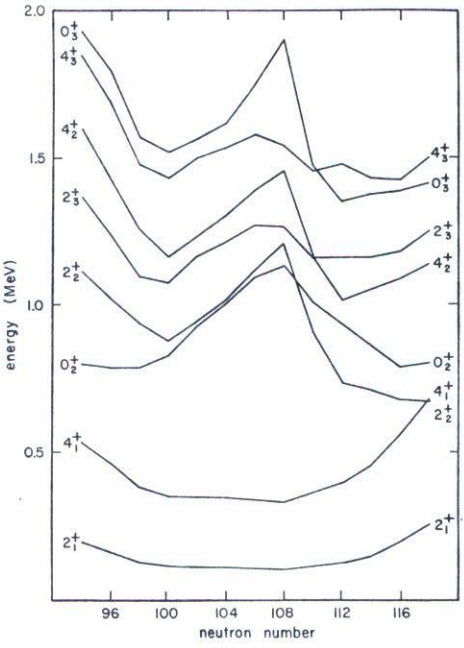

Fig.9 Calculated energy spectra of the tungsten isotopes showing the low-lying 0^+, 2^+ and 4^+ states, with respect to the 0^+ ground state.

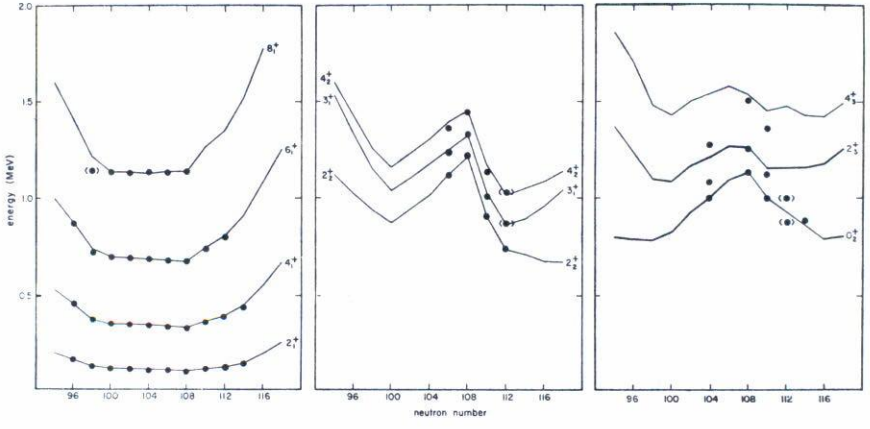

Fig.10 Comparison between the calculated (solid lines) and experimental (black dots) energy levels of the tungsten isotopes in (1) the ground state band, (2) the quasi-γ band, and (3) the quasi-β band, respectively.The experimental points are from Refs. 25-27.

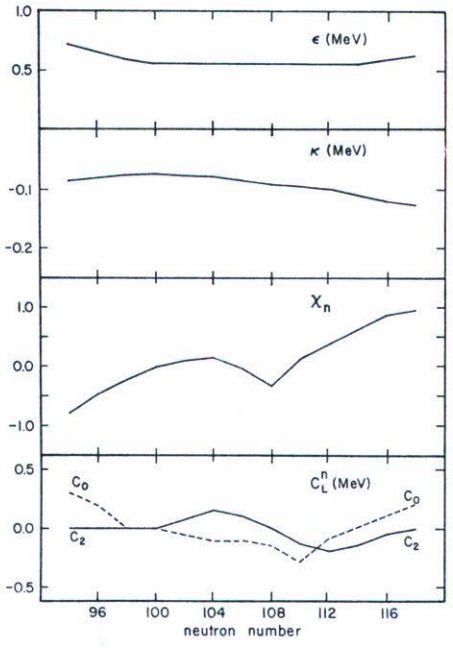

Fig.11. The IBM-2 parameter set used for the tungsten isotopes. The value used for χ_π was -1.6.

In general, the agreement with experiment for the ground-state-band and γ-band energy levels is quite good. The agreement with the β-band energies is not so successful, notably in $^{186}_{74}W_{112}$ and $^{178}_{74}W_{104}$. The IBM-2 predicts the β-band energy levels for these two isotopes to be much farther apart than experiment seems to indicate. In the case of $^{186}_{74}W_{112}$ one should note that the experimental β-band energies are listed as questionable.

Duval and Barrett[22] also determined the electromagnetic transition rates between the different energy states, using the wave functions obtained from the previous calculation and the computer program NPBEM[19]. The most general single-boson transition operator of angular momentum ℓ can be written as

$$T^{(\ell)} = T_\pi^{(\ell)} + T_\nu^{(\ell)} \quad , \tag{17}$$

where $T_\rho^{(\ell)}$ is of the form of Eq. (5). One can use the transition operator (17) to calculate all moments from $\ell = 0$ to $\ell = 4$ but results only for $\ell = 2$ will be given here. The results of other moments, i.e., $\rho(E0)$ and isotope and isomer shifts, are given in Ref. 22.

For $\ell = 2$ the E2 transition operator can be written in the form

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \quad , \tag{18}$$

where Q_ρ is given by Eq. (12). In principle, the parameters χ_π and χ_ν in the quadrupole operators Q may be different from those used in the quadrupole operators in the Hamiltonian Eq. (16) ; however, Duval and Barrett have taken them to be the same in their calculations, so as to reduce the number of free parameters. This also seems a natural choice. The parameters e_π and e_ν have units of eb and indicate the proton-boson and neutron-boson effective charges, respectively. As a further simplification, they use a constant $e_\pi = e_\nu$ for all nuclei. The value of the constant is determined by fitting one of the experimentally known transition rates. Using $e_\pi = e_\nu = 0.126$ eb (determined by fitting the $2_1^+ \rightarrow 0_1^+$ transition in $^{182}_{74}W_{108}$), they obtain the results shown in the following figures and in Ref. (22). Results are presented using the more conventional reduced transition rates, i.e., the B(E2) values given by Eq. (6). Figures 12 and 13 show the absolute B(E2) values for the $2_1^+ \rightarrow 0_1^+$, $4_2^+ \rightarrow 0_1^+$, and $2_3^+ \rightarrow 0_1^+$ transitions. These are typical examples of the results obtained. Many more such B(E2) values and also branching ratios are reported in Ref. (22). It should be noted that the theory reproduces all of the trends of the known data[28-30], including the fact that the $2_3^+ \rightarrow 0_1^+$ transition is 10^{-2} times weaker than the $2_1^+ \rightarrow 0_1^+$ transition. It should also be remembered than no attempt was made to fit any of the B(E2) values while determining the parameters in the Hamiltonian.

Since the E2 transition operator is, in fact, a quadrupole operator, it is also possible to calculate the quadrupole moment for a nucleus in a state of angular momentum J, using

$$Q_J = \left(\frac{16\pi}{5}\right)^{1/2} \begin{pmatrix} J2J \\ -J0J \end{pmatrix} \langle J \,||\, T^{(E2)} \,||\, J \rangle \quad . \tag{19}$$

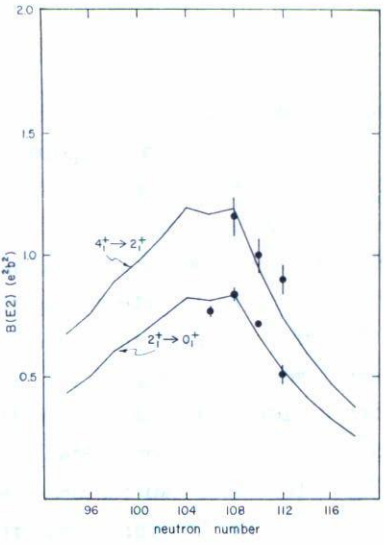

Fig. 12 Comparison between calculated and experimental B(E2) values for the $2_1^+ \rightarrow 0_1^+$ and $4_1^+ \rightarrow 2_1^+$ transitions. The experimental points are from Ref. 28 .

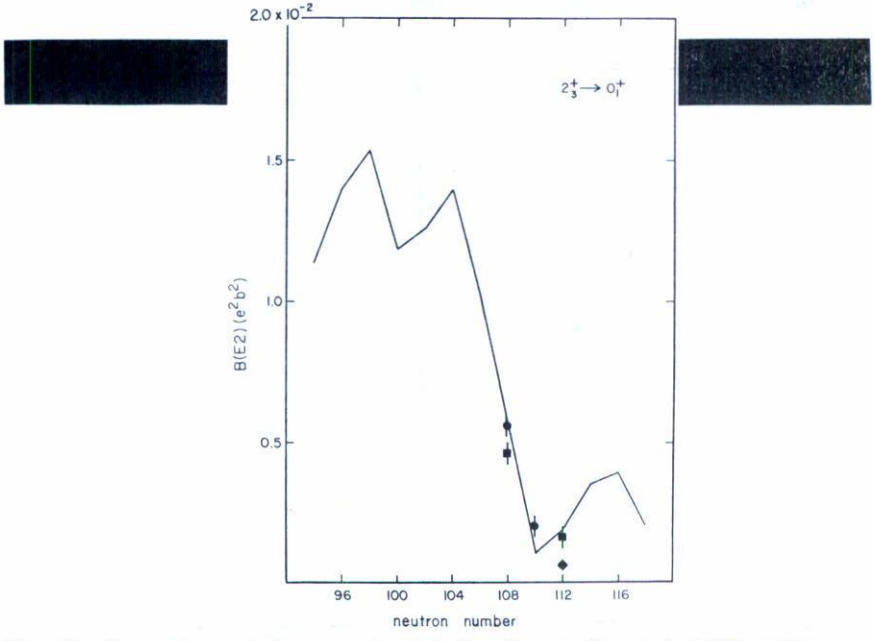

Fig. 13 Comparison between calculated and experimental B(E2) values for the $2_3^+ \rightarrow 0_1^+$ transition. The circles are from Ref. 28 , the squares from Ref. 29 and the diamond from Ref. 30 .

Using the IBM-2 wavefunctions and the E2 transition operator given by Eq.(18), Duval and Barrett obtain the quadrupole moments shown in Fig.14 for $J = 2_1^+$ and $J = 2_2^+$. Note that the parameters e_π and e_ν in the $T^{(E2)}$ operator have already been determined from fitting the B(E2) data and as before, χ_ν and χ_π are the same numbers used in the Hamiltonian (16), so there are no new free parameters in determining the quadrupole moments. The IBM-2 predicts the correct sign in both of the above cases, and the agreement with experiment is very good for $Q_{2_1^+}$. But, in the case of $Q_{2_2^+}$, the IBM-2 values differ dramatically from the recently determined experimental numbers[31] for $^{182}_{74}W_{108}$ and $^{184}_{74}W_{110}$. Experiment indicates a sharp decrease in $Q_{2_2^+}$ for these two isotopes, which is not predicted by the IBM-2. This decrease is, however, predicted by the pairing-plus-quadrupole model of Kumar and Baranger[32], but for other properties associated with the 2_2^+ state (i.e., energy and E2 transitions), the IBM-2 agrees much better with experiment[22].

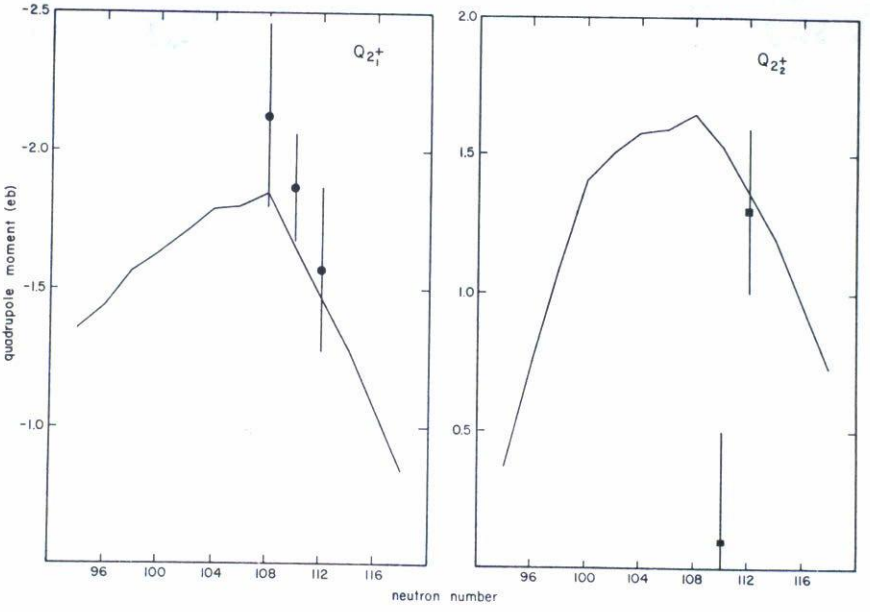

Fig.14 Comparison between calculated and experimental quadrupole moments for the 2_1^+ and 2_2^+ states. The experimental points (the circles and squares) are from Ref.31

The program NPBOS also gives the ground-state energies of the 0_1^+ states, from which one can determine the two-neutron separation energies[22]. Figure 15 shows a comparison of the theoretical results with experiment. The agreement with experiment is quite good.

To summarize this subsection, we have seen that the IBM-2 yields results for the tungsten isotopes which are in generally quite good agreement with experiment, particularly for the energy levels and E2 transitions. The IBM-2 also allows us to make a large number of theoretical predictions where data are not now available. It would be worthwhile for experimentalists to perform further investigations of the tungsten isotopes in order to compare them with the theoretical predictions.

As was stated in Section II, the goal of the empirical IBM-2 is to determine smoothly varying values of the parameters ε, κ, χ_π and χ_ν which reproduce the experimental data for all even-even nuclei in a given mass region. In Figure 16 we show the values of these parameters for the neighboring osmium and platinum isotopes[33] and see that they are, for the most part, consistent with the parameters for the tungsten isotopes, illustrated in Fig.11. Also the isotopes of xenon, barium and cerium, whose valence bosons occupy the 50-82 shell, have been recently fitted[34] with a consistent set of parameters. These parameter values plus those for gadolinium, samarium, neodymium and thorium[35] are also shown in Fig.16. So the goal of obtaining self-consistent sets of the IBM-2 parameters is borne out by the current research, but further investigations of the isotopes of other nuclides need to be carried out in the future.

B. The mercury (Hg) isotopes:

The Hg isotopes are particularly fascinating since their experimental excitation spectra indicate the coexistence of two different structures in the same energy region[36]. Generally there is configuration mixing between these structures. Recently Duval and Barrett[23] have developed an extension of the IBM-2, which allows them to describe in a completely general and quantitative manner the mixing of two different configurations of any arbitrary structure.

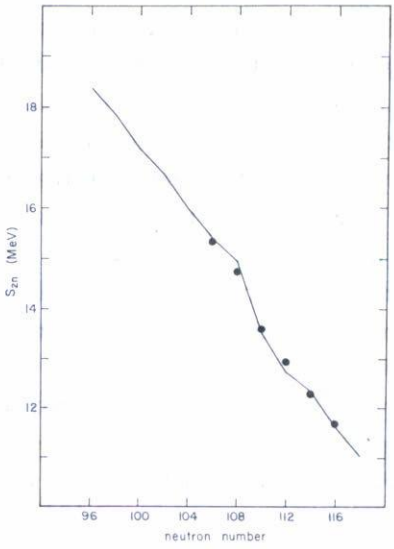

Fig.15 Comparison between calculated and experimental two-neutron separation energies, S_{2n}. The experimental points (solid circles) are from Ref.27.

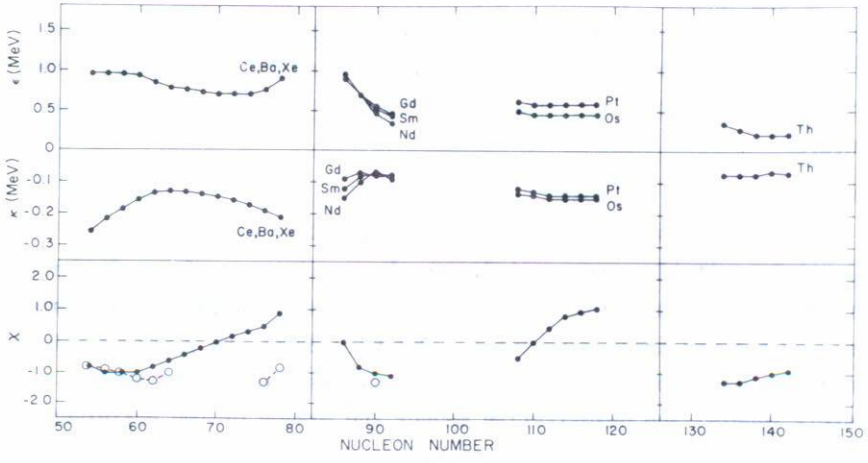

Fig.16 Summary of the IBM-2 parameters ε, κ, χ_υ and χ_π appearing in Eqs.(12) and (14), as determined from phenomenological calculations as given in Refs. 33-35. The open circles denote values for χ_π.

The Duval-Barrett method[23] consists of first describing the general features of the two different configurations in terms of two different IBM-2 calculations and then mixing the results of these two calculations using an appropriate IBM-2 mixing Hamiltonian. Their technique is most easily understood in terms of a specific application, which they chose to do for the mercury isotopes in the region $^{184}_{80}Hg_{104}$ to $^{188}_{80}Hg_{108}$. These are particularly interesting isotopes to study since they simultaneously show one set of states which are distinctly rotational in character and a second set which are vibrational in structure. Such simultaneous rotational and vibrational structures are a common feature of nuclei which have either their proton number or their neutron number near a closed shell value, another example being the tin isotopes[37]. Although previous descriptions of this phenomenon in the Hg isotopes have been given[36], in the Duval-Barrett technique the rotational and vibrational characters of the different configurations arise quite naturally through the IBM-2 formalism.

This effect is illustrated in Fig. 17 which shows the energy spectra of the Hg isotopes[27]. One clearly sees the two different configurations, namely the vibrational states, which change very little from isotope to isotope, and the rotational levels which come very low in energy at $^{184}_{80}Hg_{104}$.

Duval and Barrett consider the simultaneous vibrational and rotational structures in the Hg isotopes to arise from two different boson configurations. The vibrational spectrum is assumed to come from the standard IBM-2 picture of the interaction of the one proton-hole boson ($N_\pi = 1$) with the valence neutron bosons (N_ν), as shown in Fig. 18(a). In this case, the effect of the $\kappa\, Q_\pi \cdot Q_\nu$ interaction is small, since $N_\pi = 1$, and the IBM-2 Hamiltonian (14) possesses more or less a U(5) or vibrational symmetry[8,13]. The rotational spectrum is assumed to arise from the excitation of a proton boson (i.e., a proton pair) above the 82 shell gap, so as to produce a configuration of two proton-hole bosons and one proton-particle boson ($N_\pi = 3$), as shown in Fig. 18(b). In this case there are three active proton bosons to interact with the active neutron bosons through the $\kappa\, Q_\pi \cdot Q_\nu$ term. This interaction strongly mixes states of different numbers of s and d bosons and has been shown to lead to a

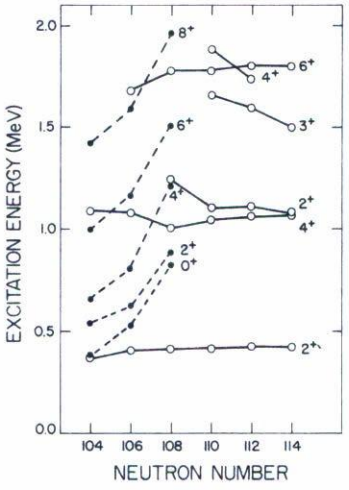

Fig.17 Experimental energy levels for $^{182}_{80}Hg_{102}$ to $^{192}_{80}Hg_{112}$. The solid lines connect states of vibrational character. The dashed lines connect states of rotational character. The data are from Ref.27.

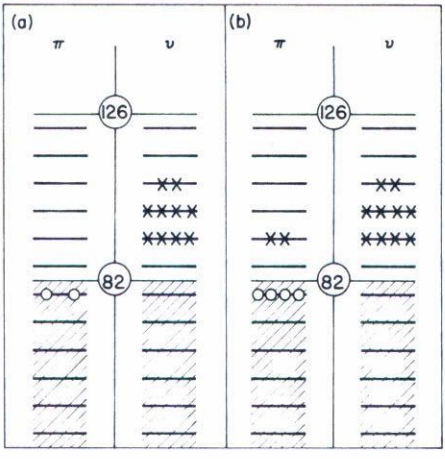

Fig.18 (a) The single-particle proton and neutron configurations for the even-even Hg isotopes.
(b) The single-particle proton and neutron configurations for the even-even Hg isotopes when a proton pair has been excited across the 82 shell gap.

rotational spectrum when several active proton and several active neutron bosons are present. So for $N_\pi = 3$ the IBM-2 Hamiltonian (14) possesses more or less a SU(3) or rotational symmetry[8,13].

One would also expect there to be some mixing between these different configurations. There is evidence for such mixing in the experimental data[38]. It can also be seen in Fig.17, e.g., the 4^+ states for N = 106 and 108. To account for this mixing, Duval and Barrett introduce a boson mixing Hamiltonian between the $N_\pi = 1$ configuration and the $N_\pi = 3$ configuration. In the specific case considered here, the most general mixing Hamiltonian that is two body in the fermion space and connects the two configurations is:

$$H_{mix} = \alpha(s_\pi^\dagger \times s_\pi^\dagger + s_\pi \times s_\pi)^{(0)} + \beta(d_\pi^\dagger \times d_\pi^\dagger + \tilde{d}_\pi \times \tilde{d}_\pi)^{(0)} \quad . \qquad (20)$$

In the case of the mixing of configurations in other nuclei, H_{mix} may have a different form.

The complete IBM-2 calculation of the Hg isotopes now involves two separate calculations (one for $N_\pi = 1$, another for $N_\pi = 3$) plus a mixing calculation. In the former two independent calculations, the IBM-2 Hamiltonian (14) is diagonalized in the appropriate space. Each of these calculations would appear to involve several different parameters. The number of parameters is reduced by insisting that the neutron number dependent parameters be the same for each isotope in <u>both</u> calculations. The $V_{\pi\pi}$ term is not used since N_π is small, i.e., one or three. Also, the value for χ_π in each calculation is kept fixed for <u>all</u> isotopes. The parameters α and β are kept <u>constant</u> for <u>all</u> isotopes. Another parameter, which gives the amount of energy needed to excite a proton boson into the next major shell, is also needed. This parameter, called Δ, is also kept fixed at a value of 4 MeV for all isotopes.

The results of the Duval-Barrett calculations are given in Figs.19(a) and (b). Figure 19(a) shows an unmixed calculation, i.e., $\alpha = \beta = 0$. In this case two separate calculations were done and an amount Δ was added to the $N_\pi = 3$ eigenvalues. Figure 19(b) shows the result of including mixing among the states in Fig.19(a). The mixing is seen to cause the states to "move" in exactly the manner described by the experimental states.

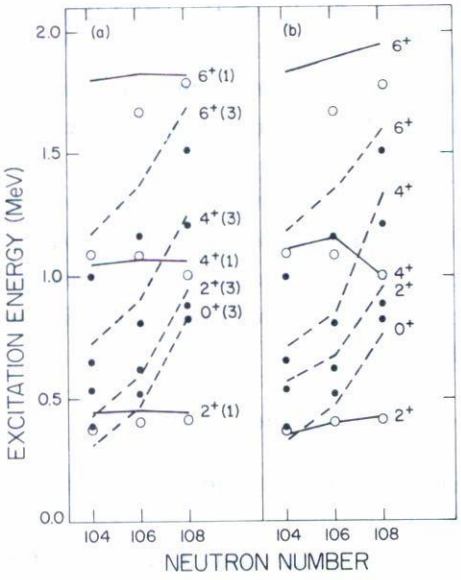

Fig. 19. Comparison of theoretical versus experimental energy levels (a) for no configuration mixing, and (b) for configuration mixing, with parameters α=0.1 MeV and β=0.2 MeV [See Eq. (20)]. Solid lines are the N_π=1 configurations, labeled by (1); dashed lines are the N_π=3 configurations, labeled by (3). Only the first two 0^+ bands are shown. Experimental states are ordered in increasing angular momentum. Open circles are vibrational levels. Solid circles are rotational levels.

Using the wave functions from the previous calculations, Duval and Barrett also determined the B(E2) values by considering an E2 transition operator of the form

$$T^{(E2)} = e_1(Q_{1_\pi} + Q_{1_\nu}) + e_3(Q_{3_\pi} + Q_{3_\nu}) \quad , \tag{21}$$

where e_1 has units of e-barns and gives the effective charge of the proton and neutron bosons in the $N_\pi = 1$ configuration. The proton and neutron bosons are treated as having the same effective charge for simplicity. Similarly, e_3 is the effective boson charge in the $N_\pi = 3$ configuration. The Q operators are the same as those given in Eq. (12). The subscripts 1 and 3 refer to the proton-boson space in which the operator acts.

TABLE I.

COMPARISON OF EXPERIMENTAL AND THEORETICAL B(E2) VALUES

	B(E2; $J_i \to J_f$)						B(E2; $J_i \to J_f$)/B(E2; $J_i' \to J_f'$)							
	$2_1^+ \to 0_1^+$		$4_1^+ \to 2_1^+$		$6_1^+ \to 4_1^+$		$\frac{2_2^+ \to 2_1^+}{2_2^+ \to 0_1^+}$		$\frac{4_1^+ \to 2_2^+}{4_1^+ \to 2_1^+}$		$\frac{4_2^+ \to 2_1^+}{4_2^+ \to 2_1^+}$		$\frac{6_2^+ \to 4_2^+}{6_2^+ \to 4_1^+}$	
	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.
$^{188}_{80}Hg_{108}$	-	.41	-	.62	-	.70	15[c]	11.1	-	.16	81[d]	19.5	1.35[d]	4.8
$^{186}_{80}Hg_{106}$	.28[a]	.39	.6[a]	.63	1.9[a]	1.06	>1200[c]	6.8	<.9[c]	.54	1.17[c]	4.55	30[c]	28.8
$^{184}_{80}Hg_{104}$	.39[b]	.39	1.13[b]	.88	2.1[b]	1.19	-	1.48	-	.28	-	6.27	-	84.2

[a] Ref. 39

[b] Ref. 40

[c] Ref. 41

[d] Ref. 42

Table I compares the results of their theoretical calculations for B(E2) transition rates and branching ratios with experiment. For absolute B(E2) values they had only to determine the values of e_1 and e_3. For the sake of simplicity they assumed that the ratio e_1/e_3 is given by κ_1/κ_3, where κ_1 and κ_3 are the strengths of the quadrupole interaction in the $N_\pi = 1$ and $N_\pi = 3$ configurations, respectively. This is a reasonable assumption, since both the effective charge and the quadrupole interaction strength are proportional to the mean square proton radius. Thus by fitting to only one of the experimental B(E2) values, they determined both e_1 and e_3 . For the branching ratios, however, only the ratio e_1/e_3 is important, so there are no new free parameters in this case.

For the most part, the theoretical calculations of energy levels and B(E2) values compare very well with experiment, with the exception of the ratio B(E2; $2_2^+ \to 2_1^+$)/B(E2; $2_2^+ \to 0_1^+$) for $^{186}_{80}Hg_{106}$. Since the amount of mixing between states depends upon the location of the energy levels before they are mixed as well as the mixing parameters,

α and β, improving the fit to the energy levels should improve the agreement of the calculated B(E2) values with experiment. Also, this experimental result may be in error.

These calculations were performed using an excited configuration built on three identical proton bosons. A more realistic calculation should make a distinction between the two proton-boson holes and the excited proton-boson particle. However, if one considers only symmetric combinations of these different proton bosons, then the calculation can still be performed treating them as identical. The argument is the same as that given for performing an IBM-1 calculation, where neutron bosons and proton bosons are treated as identical[13].

Even though the spectrum for $N_\pi = 3$ is shifted up in energy by 4 MeV with respect to the $N_\pi = 1$ spectrum, sufficient energy is gained through the attractive $\kappa\, Q_\pi \cdot Q_\nu$ interaction due to the increase in the number of active protons to overcome most of this shift. This effect is not present in the heavy Hg isotopes, since the number of active neutron bosons is small. The $N_\pi = 3$ spectrum lies very high in energy for these isotopes[43].

One might expect that states formed by exciting two proton bosons into the next major shell are also important. The fundamental question is whether or not the $\kappa\, Q_\pi \cdot Q_\nu$ interaction increases sufficiently in strength as the number of proton bosons is increased so as to overcome the excitation energy of each proton boson across the shell gap. By empirically studying the energy of the 2p-2h and 4p-4h states in heavy nuclei, one finds that the 4p-4h excited state lies significantly higher than the 2p-2h state[23]. Hence, only the 2p-2h excitation (i.e., the one proton boson excitation) should make an important contribution to the excited-state configuration.

To summarize, Duval and Barrett[23] have developed an expanded IBM-2 technique, in which different configurations are described in terms of separate IBM-2 calculations, the results of which are then mixed using the appropriate IBM-2 mixing Hamiltonian. In the case of the Hg isotopes the vibrational and rotational characters of the energy levels arise naturally in their approach due to the U(5) symmetry of the $N_\pi = 1$ configuration and the SU(3) symmetry of the $N_\pi = 3$ configuration. More extensive calculations are now being carried out for the entire chain of

Hg isotopes[24]. It should be emphasized that the Duval-Barrett technique is quite general and that it has also been applied to other nuclei, such as the platinum and molybdenum isotopes, which also exhibit the features of the mixing of two distinctly different configurations[44].

IV. MICROSCOPIC THEORY OF THE IBM

As originally formulated by Arima and Iachello[6], the IBM was a purely empirical model with no direct connection to the underlying microscopic fermion structure. Since the tremendous success of both the IBM-1 and IBM-2 in describing the properties of even-even medium-to-heavy mass nuclei[8,13,21], considerable progress has been made in understanding the structure of the IBM in terms of the nuclear shell-model theory.

Like the nuclear shell model, the IBM is first and foremost a wave function truncation procedure. It reflects the standard problem which exists in most of physics, namely that one cannnot solve the dynamics of the problem, e.g., the Schrödinger equation, in the full space spanned by all the variables, since this space is too large or even infinite. Consequently, one is forced to work in a truncated model space, so as to have a finite and tractable number of degrees of freedrom which can be treated numerically. The nuclear shell model is itself such a truncation scheme; for example, the low-lying spectrum of ^{18}O is usually described as two neutrons interacting in the restricted model space of ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$). However, if one works in a truncated model space instead of the full space, one must then use effective operators appropriate for this model space instead of the full space operators; otherwise incorrect results will be obtained. We will return to this point latter.

In the case of medium-to-heavy mass nuclei, which have several valence nucleons outside the nearest closed proton and neutron shells, even the usual shell-model truncation, consisting of the assumption that these valence protons and neutrons interact only in the lowest-lying, open major shells appropriate for each, is intractable. Such calculations often involve millions or billions of states of each possible angular momentum. Consequently, even a more restrictive truncation of the full space is necessary in order to make microscopic calculations feasible for such nuclei. The boson degrees of freedom assumed in the IBM appear to be a reasonable truncation scheme for describing these nuclei

as demonstrated by the success of the phenomenological IBM exhibited in Section III. The remaining question is to establish the link between these boson degrees of freedom and the corresponding fermion degrees of freedom.

The bosons of the IBM are clearly more complicated in structure than a simple pair of identical nucleons coupled to J = 0 or J = 2. We consider only pairs of identical nucleons, because in heavier nuclei the protons and neutrons, in general, occupy different major shells, so that it would not be possible to form a pair with J = 0 and positive parity from a valence proton and a valence neutron. The IBM bosons already contain some of the collectivity of the interacting fermions. This effect is similar to the manner in which the nuclear reaction matrix already contains the influence of the strong, short-range repulsion in the nucleon-nucleon potential. Thus, the IBM bosons represent correlated pairs of identical nucleons, but what kind of correlated pairs?

The recent work of Talmi[11] indicates that, at least for the J = 0 bosons, these correlated pairs are of the type described by the generalized seniority approach[45]. This approach has the advantages (1) of strictly conserving the number of nucleons, and hence the number of correlated pairs or "bosons", (2) of treating the J = 0 and J = 2 pairs in the same manner, and (3) of being amenable to calculation with a wide class of effective interactions, instead of only the pairing interaction, as in the case of BCS theory.

In the generalized seniority approach[45], one constructs the creation operator of a correlated J = 0 pair for several j-orbits in the same major shell in the form

$$S^{\dagger} = \sum_{j} \alpha_j S_j^{\dagger} , \tag{22}$$

where

$$S_j^{\dagger} = \frac{1}{2} \sum_{m} (-1)^{j-m} a_{j,m}^{\dagger} a_{j,-m}^{\dagger} \tag{23}$$

and the α_j are weighting factors for each j orbit.

An example of such a correlated pair would be two identical

nucleons in the 82 to 126 major shell. This shell contains six j-orbits, so we can construct six $J = 0$, $T = 1$ states. If we diagonalize the 6×6 matrix representing the interaction between these two nucleons in this major shell for any appropriate effective interaction, we find that one of the eigenstates lies considerably lower in energy than the other five. This lowest-lying collective $J^\pi = 0^+$ state is the $J = 0$ correlated pair defined in Eq.(22).

If all the α_j are equal in Eq.(22), then $S^\dagger$ is one component of a total quasi-spin operator, and the seniority scheme is easily extended to several j orbits. In real nuclei, of course, the α_j will be unequal, and we will lose the appealing SU(2) properties of the quasi-spin operator. In spite of this, Talmi has shown that some important properties of the seniority scheme survive this generalization.

In particular, Talmi[11,45] asked under what conditions will a "condensate" of pairs created by (22) be an eigenstate of a shell-model Hamiltonian H_F, containing only single-nucleon energies and two-body effective interactions. In other words, what are the conditions on H_F, so that

$$H_F (S^\dagger)^n \mid 0 \rangle_F = E_n (S^\dagger)^n \mid 0 \rangle_F \quad , \tag{24}$$

where $H_F \mid 0 \rangle_F = 0$ and $\mid 0 \rangle_F$ is the fermion vacuum state. Talmi[45] showed that (24) holds for <u>any</u> value of n, provided that

$$H_F S^\dagger \mid 0 \rangle_F = V_o S^\dagger \mid 0 \rangle_F \quad , \tag{25}$$

$$\left[[H_F, S^\dagger] , S^\dagger \right] \mid 0 \rangle_F = \Delta (S^\dagger)^2 \mid 0 \rangle_F \tag{26}$$

and

$$E_n = nV_o + \frac{n(n-1)}{2} \Delta \quad , \tag{27}$$

where V_o and Δ are simply c-numbers. The trends regarding binding energies and separation energies predicted by Eq.(27) agree very well with the experimental trends. Hence, a correlated pair of the form (22) is a likely candidate for a $J = 0$ boson, since it leads to a condensate,

i.e., $(S^\dagger)^n \mid 0\rangle_F$, which yields results in agreement with experiment for nearly spherical nuclei. Unfortunately, if we calculate the commutator of $S^\dagger$ with $(S^\dagger)^\dagger$, we find that it is not equal to one but is

$$[(S^\dagger)^\dagger, S^\dagger] = \sum_j \frac{1}{2}(2j+1)\alpha_j^2 - \sum_j \alpha_j^2 \sum_m a_{jm}^\dagger a_{jm} \tag{28}$$

So the $S^\dagger$ do not create real bosons. We could neglect the second term on the right hand side of Eq.(28), which is the commonly used quasi-boson approximation. This is, however, a good approximation only when the number of valence nucleons is small compared with $\sum_j (2j+1)$. We seek a result that will be true for any number of valence particles.

At this point we recall that the IBM corresponds to a wave function truncation and <u>not</u> to an operator expansion. Consequently, we are <u>not</u> interested in expressing fermion pair operators in terms of boson operators. What we want is a correspondence between the boson <u>states</u> and the fermion pair <u>states</u>. This is done by making a one-to-one mapping of the fermion pair states, $(S^\dagger)^n \mid 0\rangle_F$, onto the boson states $(s^\dagger)^n \mid 0\rangle_B$, where $\mid 0\rangle_B$ of the boson vacuum state[9,10,18,46]. For simplicity and convenience we have assumed that the states in both spaces are properly normalized.

Once such a direct correspondence has been established between the correlated fermion pair states and the boson states, one can determine the appropriate boson operators by constructing their "image" in the boson space[9,10]. That is, one requires that the matrix element of the corresponding boson operator in the boson space be the <u>same</u> as the matrix element of the fermion operator in the fermion space. For example, we would require that

$$\langle 0 \mid \left((S^\dagger)^\dagger\right)^n \mid H_F \mid (S^\dagger)^n \mid 0\rangle_F = nV_0 + \frac{n(n-1)}{2}\Delta \equiv \langle 0 \mid s^n H_B (s^\dagger)^n \mid 0\rangle_B \tag{29}$$

or that

$$H_B = V_0 s^\dagger s + \frac{1}{2}\Delta\,(s^\dagger)^2 s^2 \quad . \tag{30}$$

This is the appropriate effective Hamiltonian in the boson space. It should be noted that H_B as given by Eq.(30) does not reproduce all possible eigenstates of the fermion Hamiltonian but only the one corresponding to $(s^\dagger)^n \mid 0 \rangle_B$. Also, H_B is not an expansion in powers of $S^\dagger$ and $(S^\dagger)^\dagger$. Hence, once we have established the appropriate correspondence or mapping between the correlated fermion pair states and the boson states, we can determine the form of all the boson operators (i.e., effective operators) using the "imaging" technique.

So far we have discussed only the $J = 0$ correlated pair with generalized seniority $v = 0$ and the "condensate" of pairs formed from it. Let us now consider pair states with $J = 2$ which have generalized seniority $v = 2$. The operator which creates a correlated $J = 2$ pair can be written in the form

$$D_M^\dagger = \sum_{j \leq j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \sum_{mm'} (jmj'm' \mid jj'2M)\, a_{jm}^\dagger a_{j'm'}^\dagger \quad . \qquad (31)$$

As before we can ask under what conditions on the shell-model Hamiltonian H_F will the states $(S^\dagger)^{n-1}(D_M^\dagger) \mid 0 \rangle_F$ be eigenstates. Once again Talmi[11,45] has established that such conditions exist, that the eigenvalues obtained produce results in agreement with experiment and that the corresponding boson Hamiltonian can be constructed by the imaging procedure.

So far there has been an exact one-to-one correspondence between the results in the correlated fermion pair space and in the boson space. However, states constructed with two or more $D_M^\dagger$ operators may not be orthogonal to states with smaller numbers of $D_M^\dagger$ operators, e.g., the state $(D^\dagger \cdot D^\dagger) \mid 0 \rangle_F$ is, in general, not orthogonal to the $v = 0$ state $(S^\dagger)^2 \mid 0 \rangle_F$, where the dot represents a scalar product of the two $D^\dagger$. In order to make the correspondence between these higher $D_M^\dagger$ states and the boson states, we must project out of the fermion states those components which are states with lower generalized seniority. Hence, the state $(S^\dagger)^{n-2} D_M^\dagger D_M^\dagger \mid 0 \rangle_\perp$ corresponds to the boson state $(s^\dagger)^{n-2}(d^\dagger)^2 \mid 0 \rangle_B$, where $\rangle_\perp$ denotes that the part not orthogonal to $(S^\dagger)^n \mid 0 \rangle_F$ has been projected out.

Therefore, the mapping procedure of Ginocchio and Talmi[46]

and the generalized seniority method[45] permit us to establish a one-to one correspondence between correlated fermion pair states and states in the s and d boson space. Whether or not these particular correlated fermion pair states, as constructed in the above manner, actually correspond to the s and d boson states of the IBM must await future investigation.

So far we have discussed only configurations for identical nucleons interacting through the T = 1 component of the nucleon-nucleon potential, which conserves seniority. On the other hand, the T = 0 interaction, which we have not yet considered, strongly breaks seniority. We would expect that the important matrix elements of this T = 0 interaction are those taken between the proton bosons and the neutron bosons, which occupy different shells in heavy nuclei. There is evidence in light nuclei that the proton-neutron T = 0 interaction is responsible for producing the deformation observed for these nuclei[47]. In the boson language this seniority breaking manifests itself as a mixing of the s and d boson components of the wave functions. For example, in strongly deformed nuclei which show good rotational properties, the ground-state wave function is essentially an equal mixture of s and d bosons, while a nucleus which behaves like a spherical vibrator has a ground-state wave function which is essentially pure s boson in structure. The simplest interaction which breaks seniority strongly is the quadrupole-quadrupole interaction[11,48]. This fact makes reasonable the choice of a quadrupole-quadrupole interaction for the basic proton-boson-neutron-boson interaction in the IBM-2 [Eq.(14)]. Thus, even though the IBM-1 exhibits the beautiful symmetry properties of the U(6) algebra, it is the IBM-2 which appears to possess the most direct connection to the underlying microscopic fermion structure through the generalized seniority scheme and the quadrupole-quadrupole proton-neutron interaction.

The above discussion gives only the basic "flavor" of the arguments of Talmi[11], Otsuka et al.[9,10,18,49], Ginocchio[46,50] and others[8,13,21] in their efforts to establish the relationship between the IBM and the nuclear shell model. The interested reader is particularly referred to Refs. 8-11,13 and 21 for more details.

As pointed out earlier, once a direct relationship has been obtained between the wave functions in the fermion space and the s-d boson space, one can use the imaging technique to determine the structure and the N_π and N_ν dependence of the boson operator corresponding to any given fermion operator. This procedure has been described in detail by Otsuka[49] and by Otsuka, Arima and Iachello[10], who specially work out the form of the quadrupole operator in the s-d boson space and determine the N_π and N_ν dependence of κ, χ_π and χ_ν. The results of such calculations for the 50 to 82 major shell, based on the work of Otsuka[49], are shown in Fig.20. In these calculations the 50 to 82 major shell is treated as a single j shell of $j = \frac{31}{2}$. The prediction for ε is also illustrated. In the basic IBM, ε should be constant across a major shell, since it is simply the amount of energy required to change an s boson into a d boson, i.e., to change a $J = 0$ correlated fermion pair into a $J = 2$ correlated fermion pair. The generalized seniority arguments given above predict a constant value for this excitation energy across a major shell.

In Fig.20 one should particularly note that χ_ν changes sign at the middle of the shell and is of the same form for both positive and negative values. On the other hand, the values of χ_ν determined empirically for $N_\nu = 50$ to 82 are skewed with respect to the middle of the shell (see Fig.16), and the empirical values for $N_\nu = 82$ to 126 show an oscillation with a minimum at $N_\nu = 108$ in the case of the tungsten isotopes (Fig.11).

Duval and Barrett[51] have extended the imaging calculations for the boson quadrupole operator, so as to treat each major shell as if it consisted of two different j shells, instead of only a single j shell, as in the earlier calculations[10,20,49]. Typical results of their calculations are given in Fig.21. One observes that these calculations are capable of reproducing both qualitatively and quantitatively the empirical structure found for κ, χ_ν, and χ_π.

The treatment of a major shell as consisting of two different j shells has a direct connection with experimental observations[52]. For example, in the $N = 50$ to 82 major shell the five single-particle levels tend to cluster into two distinct groups.

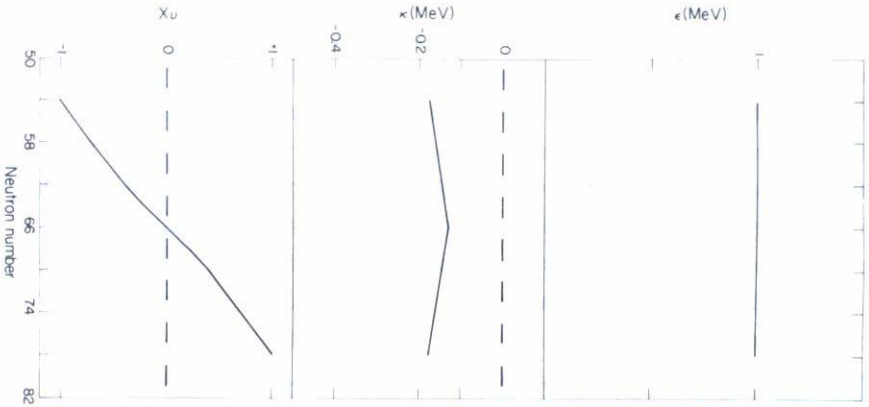

Fig.20 The dependence of ε, κ and χ_ν on N_ν as expected in a microscopic theory with degenerate single-particle orbitals (Ref.49).

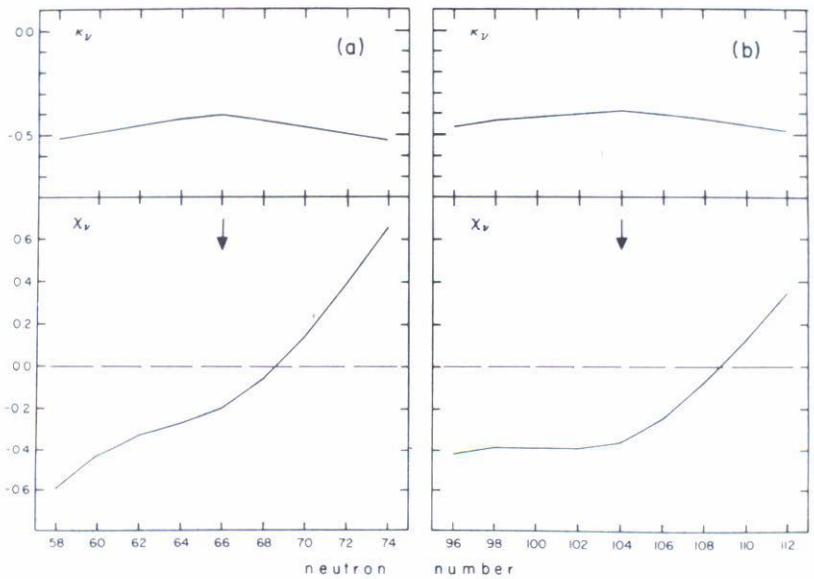

Fig.21 Two nondegenerate j-shell predictions for κ_ν and χ_ν in (a) the 50-82 shell and (b) the 82-126 shell. The units for κ_ν are arbitrary. The arrows indicate mid-shell (Ref.51)

The above-mentioned calculations clearly have predictive power for the values of the parameters κ, χ_ν and χ_π in mass regions where no empirical fits using the IBM-2 have been carried out so far. In particular, Duval and Barrett have used the values of κ and χ_ν predicted from their two j shell calculations for N = 82 to 126 to recalculate their earlier IBM-2 predictions for the properties of the tungsten isotopes[22]. Their new calculations[51] were in even better agreement with the experimental data than the original empirical fit.

Like the single j-shell calculations, the two j-shell calculations of Duval and Barrett also predict a constant value of ε across a major shell. On the other hand, the empirically determined value of ε changes dramatically from its near-closed-shell value to its center-of-the shell value. This change is necessary in the IBM to reproduce the large decrease in the $2_1^+ - 0_1^+$ splitting as the masses of the isotopes vary from their closed-shell values to their center-of-the-shell values. In the IBM-2, part of this decrease comes from the change in the number of d bosons in the 0_1^+ and 2_1^+ states as N_ν approaches its midshell value. Part of it also comes from the effect of states left out of the s-d boson model space, such as g and i bosons and s' and d' bosons. Sage and Barrett[53] have used ordinary second-order perturbation theory to estimate the renormalization effects on the IBM-2 parameters due to the g boson. In order to perform microscopic calculations they begin in the correlated fermion pair space and then project their results into the s-d boson space, a la Ginocchio and Talmi[46]. A typical diagram which they compute in the coupled fermion space is shown in Fig.22(a). Assuming a two-body quadrupole-quadrupole interaction between the correlated fermion pairs, they obtain a multipole-multipole result for the second-order process illustrated in Fig.22(b). In particular, after projection into the s-d boson space, they find the following expression for the monopole-monopole term in second-order perturbation theory, denoted by V_2^B (L = 0):

$$V_2^B(L=0) = \kappa^2 \sum{}' \; | \langle (J_\pi J_\nu)J \,||\, Q_\pi \cdot Q_\nu \,||\, (I_\pi I_\nu)J \rangle |^2 \frac{1}{\Delta E} \frac{2J+1}{(2J_\pi+1)(2J_\nu+1)} \; n_{J_\pi} n_{J_\nu} \; . \tag{32}$$

In Eq.(32) the angular momentum notation is given by Fig.22(b), ΔE is the difference between the unperturbed ground-state energy and the unperturbed intermediate-state energy and the prime on the summation indicates that the sum is restricted to those intermediate states with $I_{\pi(\nu)} = 4$, while $I_{\nu(\pi)} = 0$ or 2.

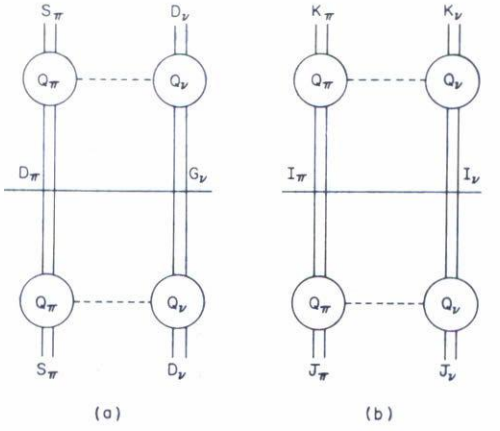

Fig.22 (a) Typical second-order perturbation theory diagram containing one correlated fermion pair coupled to $J = 4$ as an intermediate state. Each pair of vertical lines represents a correlated fermion pair.
(b) Typical second-order diagram showing the notation used in Eq.(32)

Comparing Eq.(32) with Eq.(14) for the IBM-2 Hamiltonian, we see that Eq.(32) corresponds to a renormalization of the terms depending on n_{d_π} and n_{d_ν} and, hence, is related to the change in ε as N_ν and N_π vary. Sage and Barrett[53] used the renormalization term in Eq.(32) to compute the $2_1^+ - 0_1^+$ splitting in the barium isotopes. Figure 23 shows the results of their calculations compared with the empirical results of Otsuka et al.[9] (lower curve) and the results obtained assuming a constant value of ε across the entire major shell (upper curve). From Fig.23 one immediately sees that a sizable amount of the change in the $2_1^+ - 0_1^+$ splitting arises from the change in $\langle n_d \rangle$ as N_ν varies (upper curve). But it is also true that the renormalization due to the g boson in second-order perturbation theory is: (1) of the correct sign and (2) tends to improve the agreement with the empirically determined value.

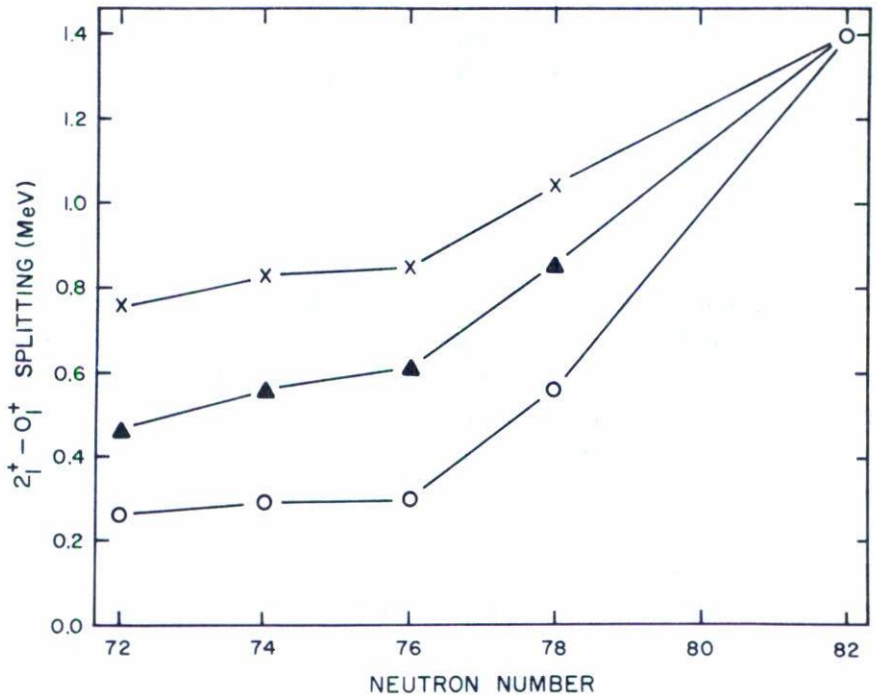

Fig.23 Calculated splittings between the first excited 2^+ state and the ground state for several Ba isotopes. The circles are the empirical values found by Otsuka et al. (Ref.9); the triangles are the results of Sage and Barrett (Ref.53); and the X are calculated values assuming a constant value for ϵ. (Ref.53).

Independent calculations by Otsuka[54] have shown that the inclusion of the g boson in second-order perturbation theory in the boson space improves the agreement between exact shell-model calculations and the same spectra calculated in the IBM. Much work still remains to be done regarding the importance of terms not present in the simple s-d boson IBM. Must g bosons be included in the IBM on an equal footing with the s-d bosons or can their influence be accurately treated using second-order perturbation theory? How important are i bosons and bosons of higher J? How significant are bosons of a different structure, i.e., collectivity, such as s' and d' bosons? Some work along this latter line

is already being investigated by van Isacker[55], who has expanded the basic IBM-1 to include a single s', d' or g boson.

Another area of considerable interest and activity is that of establishing the connection between the IBM and the collective models of Bohr and Mottelson[2]; however, this work will not be discussed in detail here. The research in this area is proceeding along two lines. The first consists of constructing the coherent or intrinsic state for the IBM and demonstrating that the classical limits, corresponding to the different geometrical models, can be projected from it[56-58]. In particular, Dieperink and Scholten[58] have shown that the three limiting cases that appear in the IBM correspond to different shape phases. The work carried out so far[57,58] seems to indicate that the IBM and the collective models are two related formulations of the same phenomena, in the same way that the Heisenberg picture and the Schrödinger picture, respectively, are two formulations of quantum mechanics.

The second line of research in this area is investigating the relationship between the IBM wave function (in the boson space) and the Bohr-Mottelson geometrical-model wave function[59,60]. Similar to the research with regard to the coherent state, this work also indicates that a relationship can be established between the two formulations, namely a one-to-one correspondence can be established between the wave functions in the two formulations, if a monopole vibration (i.e., an s boson) is added to the geometrical-model wave function. Much research is continuing in both of these areas.

To summarize this section, we have gained some insight into the structure of the IBM bosons through the generalized seniority scheme and the concept of correlated fermion pairs which are capable of forming a condensate. On the other hand, no general theory exists which establishes the exact relationship between states in the fermion shell-model space and states in the s-d boson space. Until we have such a general theory, we will not be able to say precisely when the assumptions of the IBM are valid for a particular nucleus and when these assumptions will break down. When we are able to calculate the IBM basis states and the IBM parameters using the fermion shell-model theory, then we will be able to predict when the IBM will break down due to loss of collectivity

in the basis states (i.e., in the structure of the bosons) and when other degrees of freedom (i.e., g and i bosons and/or s' and d' bosons) must be included in order to explain the nuclear structure observed experimentally.

V. DISCUSSION AND CONCLUSIONS

At the very least the IBM is an extremely useful model for describing the low-lying properties of even-even, medium-to-heavy mass nuclei. It is simple model both to employ and to understand and can be easily utilized to describe the properties of nuclei with many valence nucleons far from closed shells. These properties are often extremely difficult or impossible to calculate using earlier models[2]. But the IBM is more than just a model, because through the IBM we can now also understand the connection between the different geometrical models (or symmetry limits) as the number of valence nucleons changes and can describe the change between these limits in a smooth manner as a function of nucleon number.

Since the original IBM is a phenomenological model, it has its limitations; it does not work for all nuclei and for all excitation energies. Because of the previous successes of the IBM, some people believe that the IBM has a wider range of applicability than its basic assumptions permit. For example, the IBM is meant to describe the low-lying properties of medium-to-heavy mass nuclei, roughly up to 2 or 3 MeV in excitation energy, depending upon the nucleus. For higher excitation energies the correlated pairs would begin to be broken up. This does not mean, of course, that it is impossible to extend the IBM in some way to treat high spin states and backbending; in fact, some work is already being done along this line[61]. We have already seen in the case of the Hg isotopes that the basic IBM can be extended in a straightforward manner to describe configuration mixing in nuclei[23]. The IBM also is being applied to light nuclei (i.e., N and $Z < 50$), where the basic assumptions of the model are probably not valid. Again the basic IBM can most likely be extended to treat selected properties of some of these nuclei, probably through the use of the F-spin formalism[62].

Since the IBM represents a wave function truncation procedure, we have the possibility of understanding the connection between the boson

configurations and the underlying microscopic fermion structure. In Section IV we found that the generalized seniority scheme[45] plus the correspondence or mapping procedure of Ginocchio and Talmi[46] offers us the possibility of such an understanding. It was also observed in Section IV that the structure of the IBM-2 Hamiltonian appears to be simply related physically to the underlying fermion interactions. If in the long run this connection or one similar to it proves to be correct, then we will at last reach the goal of a unified theory of nuclear structure for all nuclei.

The IBM and the Bohr-Mottelson Model appear to be related[57-60], as we discussed in Section IV. They seem to be simply two representations of the same phenomena, as the Heisenberg picture and the Schrödinger picture are two equivalent representations of quantum mechanics. It is only a matter of which representation is more convenient or useful for describing and understanding any particular system. At the present time the IBM is the easier model to use in performing numerical calculations of nuclear properties for many medium-to-heavy mass nuclei.

The IBM is a rich and varied model, and many aspects of its development, application and interpretation have, unfortunately, been only briefly discussed or simply mentioned here. For more details the interested reader is referred to Refs. 8, 12-14, 21 and 49, which are more general in nature. Due to a limited scope for the present paper, we have not even touched on the tremendously exciting subject of the newly developed Interacting Boson-Fermion Model (IBFM) for describing the properties of odd-A nuclei[21,63]. The combination of the IBM and IBFM has led to the prediction of the possible existence of so-called supersymmetries in the spectral structure of certain neighboring even and odd mass nuclei[21,64].

In conclusion, the IBM is an exciting development for nuclear physics, not only because of the successes it has had so far in describing an explaining nuclear properties, but also because of the possibilities it offers for our future understanding of nuclear structure. It is an active, dynamic and growing field of research with much challenging and stimulating work remaining to be done.

ACKNOWLEDGEMENTS

It is a pleasure to thank A. Arima, A. E. L. Dieperink, P.D. Duval, F. Iachello, S. Pittel, K.A. Sage, I.Talmi and J. L. Wood for numerous helpful discussions and/or for reading and commenting on sections of the manuscript. I also thank R. Siemssen and F. Iachello for the hospitality provided to me at the Kernfysisch Versneller Instituut, Groningen, where part of this work was carried out.

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