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S-WAVE ASYMPTOTIC NORMALIZATION CONSTANTS FOR THE ^3H AND ^3H_E SYSTEMS

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ABSTRACT

Trinucleon wave functions, based on the Lane-Robson R-matrix formalism, have been previously shown to provide representative binding energies and rms radii for the ^3H and ^3He systems. Herein, calculations of asymptotic normalization constants are found to be within 15% of Faddeev, forward dispersion, and one-boson-expansion predictions. The model results indicate that the calculated wave functions yield representative A=3 properties and may be useful in studies of two-nucleon transfer reaction calculations involving ^3H or ^3He.

RESUMEN

Ha sido demostrado anteriormente que las funciones de onda de un sistema de 3 nucleones, basadas en el formalismo de la matrix R de Lane Robson, proporcionan valores representativos para las energías de ligadura y radios r m s del ^3H y el ^3He. En este trabajo se calculan las constantes de normalización asintóticas y se encuentra que no difieren en más de 15% de las predicciones de los métodos de Faddeev, dispersión hacia adelante, y desarrollo de un bosón. Los resultados del modelo indican que las funciones de onda calculadas dan propiedades representativas para A = 3, y pueden ser de utilidad en el cálculo de reacciones de transferencia de dos nucleones que involucren ^3H ó ^3He.

I. INTRODUCTION

The calculation of transfer reaction cross sections within a distorted-wave Born approximation format is often limited by nuclear structure information[1-15]. Some of the most studied processes are (p,t) transfer reactions which require both core plus two-neutron and triton wave functions as input. As an example, reactions involving ^{16}O[1-10], ^{40}Ca[4,9-13], and ^{208}Pb[4,9,13-15] targets have been analyzed by numerous authors, but normalization and shape difficulties between calculations and data persist. The aforementioned (p,t) transfer reactions require triton wave function input, and a study of trinucleon wave functions is a vital step in resolving (p,t) uncertainties.

For an A=3 wave function to be useful, it should provide a good representation of trinucleon bound state parameters such as the binding energy and rms charge radius[16]. These quantities provide useful constraints on the A=3 wave function, but additional constraints are needed to further restrict the trinucleon wave function. Additional constraints are provided by the S-wave asymptotic normalization constants for the ^3H and ^3He systems ($C_0(^3H)$ and $C_0(^3He)$) which have a status similar to that of the binding energy or rms charge radius[17-19]. Since the asymptotic normalization is a fundamental quantity, it provides an additional constraint on trinucleon wave functions. Thus, a consideration of $C_0(^3H)$ and $C_0(^3He)$ is of interest in assessments of trinucleon wave functions.

A consideration of trinucleon asymptotic normalization constants is justified because these quantities have provided much useful information. For example, considerations of the asymptotic normalization constant have led to an improved phase shift analysis of $p+^3He$ scattering[20] as well as an improved understanding of nuclear structure effects on (d,t) analyzing powers[21]. In a similar fashion, asymptotic normalization studies have the potential to analyze the effects of the details of the nuclear structure of transferred particles or clusters in reactions such as (t,p), (t,d), or (^{3}He,d) reactions.

The purpose of this paper is to determine the accuracy of detailed A=3 wave functions derived from a harmonic oscillator basis[16,22-26]. These wave functions have been previously shown to yield reasonable binding energies and rms radii, and further investigation into their applicability for use in transfer reactions is warranted.

The trinucleon wave functions will be assessed by considering the asymptotic normalization constants $C_0(^3H)$ and $C_0(^3He)$. The asymptotic normalization constraints when viewed in light of binding and rms radii results will provide insight into the applicability of harmonic oscillator trinucleon wave functions in (p,t) transfer reaction calculations.

II. THEORY AND FORMULATION

The model for the A=3 bound states was outlined in Refs. (25) and (26), and represents an application of the Lane-Robson R-matrix methodology[27]. The R-matrix equations can be written in the form

$$\sum_{\lambda'} \left[\langle \lambda | H - E | \lambda' \rangle + \sum_c \gamma_{\lambda c} (b_{\lambda' c} - b_c) \gamma_{\lambda' c} \right] A_{\lambda'} = 0 , \qquad (1)$$

where H is the A=3 hamiltonian[25] and $\gamma_{\lambda c}$ and $b_{\lambda c}$ are the reduced widths[28] and logarithmic derivatives associated with the expansion states $|\lambda\rangle$[25]. The expansion states are introduced in order to describe the nuclear wave function within the interaction region, $r_c \leq a_c$, in all channels. The quantities b_c are related to the radial wave functions $U_c(r_c)$ in the physical channels(c) by

$$b_c = \left(\frac{r_c}{U_c} \frac{dU_c}{dr_c} \right)_{r_c = a_c} , \qquad (2)$$

where r_c is the coordinate between the separating clusters and a_c is the channel radius. These quantities provide the connection between the interaction region and the various two-body break-up channels. The A_λ are expansion amplitudes which are to be determined by the solution of Eq. (1).

Within this framework, the model is defined by choosing a form for the hamiltonian and a set of expansion states and cluster wave functions. The hamiltonian which includes the nucleon kinetic energies and two-body nuclear and Coulomb interactions is specified in Refs. 25, 26 . As expansion or basis states, we utilize a set of properly symmetrized translationally invariant harmonic oscillator eigenstates[29]. The use of harmonic oscillator states facilitates transformations required in (p,t) transfer reaction studies. The oscillator transforma-

tion properties endow the $|\lambda>$ states with a favored status when compared with non-oscillator basis states. Additional details of the A=3 formulation is provided in Ref. 25 .

In Ref. 16 , an effective interaction for oscillator basis states was determined for the two, three, and four nucleon systems. This interaction is based on the Sussex matrix elements[30] and is of the form

$$V^{\text{modified Sussex}} = C\, V^{\text{Sussex}} \quad , \tag{3}$$

where C is a strength parameter of order unity. Good fits to A=2, 3, 4, and 5 ground state binding energies and rms radii were obtained from the modified Sussex interaction (MSI) with the value C=1.168 for the b=1.60 fm Sussex matrix elements[16,24]. The MS interaction yielded ^{3}H and ^{3}He binding energies within 5% of experiment. The ^{3}H rms radius was within 2% of experiment and the ^{5}He rms radius was predicted to be within 8% of data[16]. Thus, the MSI provides representative binding energies and rms radii for the ^{5}H and ^{5}He systems. For completeness, we note that using a slightly stronger interaction (C=1.190) leads to binding energies in agreement with A=3 data[31]. However, we will use the MSI in calculations presented herein since it provides a consistent description of A=2, 3, 4, and 5 systems. This consistency is needed if systematic studies of (t,p) and other reactions such as (t,d) or (α,d) are to be performed.

For binding energy and rms radius calculations, the A=3 wave functions were restricted to a $4\hbar\omega$ model space. This truncation is severe but still provides a 63 component wave function, which is quite detailed. However, a comparison of our model with previous S-wave asymptotic normalization calculations[32] requires that the basis be truncated to only permit $^1S_0$ and $^3S_1 + {}^3D_1$ states in the $\vec{r}_{13}$ coordinate and S-states in the $\vec{r}_c$ coordinate (see Fig. 1, Ref. 25). In this manner, the detail provided by our A=3 wave functions is at least comparable to other treatments[25,26,32-35].

III. ASYMPTOTIC NORMALIZATION CONSTANTS

As noted in the introduction, the S-wave asymptotic norma-

lization constant C_0 is a fundamental quantity which provides an important constraint on the A=3 wave function. In this section, we report calculations of $C_0(^3H)$ and $C_0(^3He)$ and will compare these values with those recently obtained from Faddeev [32,33], forward dispersion [34], and one-boson-expansion [35] methods.

The A=3 wave functions are written in terms of the internal coordinates $\vec{r}_{12}$ and $\vec{r}_B$ (see Fig. 1, Ref. 25). For ^3H, $\vec{r}_{12}$ is the coordinate joining the two neutrons and $\vec{r}_B$ is the coordinate between the centers-of-mass of the dineutron and the proton. The $\vec{r}_{12}$ and $\vec{r}_B$ coordinates are chosen to facilitate antisymmetry requirements [25,26]. Using the formalism outlined in section II, the triton wave function is defined as

$$\Psi(\vec{r}_{12}, \vec{r}_B) = \sum_{\lambda} A_\lambda |\lambda(\vec{r}_{12}, \vec{r}_B)\rangle , \tag{4}$$

where λ is the number of basis states included in the model, and

$$|\lambda(\vec{r}_{12}, \vec{r}_B)\rangle = R_{N_{12}L_{12}}(r_{12})\, R_{N_BL_B}(r_B)\, Y_{L_{12}M_{12}}(\hat{r}_{12}) \cdot Y_{L_BM_B}(\hat{r}_B)\, X(S_{12}, S_3, S) . \tag{5}$$

In Eq. (5), $R_{NL}(r)$ are radial wave functions [36], $Y_{LM}(\hat{r})$ are spherical harmonics normalized over the unit sphere [37], and $X(S_{12}, S_3, S)$ are wave functions representing the spin coupling structure of the A=3 system. Additional details concerning the formalism of the A=3 wave functions and their coupling structures are discussed in Ref. 25,26.

As noted earlier, the $(\vec{r}_{12}, \vec{r}_B)$ coordinates were chosen to facilitate antisymmetry requirements. However, the formulas for C_0 [32] require the $(\vec{r}_{13}, \vec{r}_C)$ coordinates [25]. The desired transformation between the original expansion states $(\vec{r}_{12}, \vec{r}_B)$ and the recoupled states $(\vec{r}_{13}, \vec{r}_C)$ are developed in terms of standard angular momentum recoupling coefficients and unequal mass Moshinsky brackets [38-40].

Following Sasakawa et al. [32], the ^3H asymptotic normalization constant is defined by the relation

$$C_0(^3H) = \frac{1}{\sqrt{\beta}} \lim_{r_c \to \infty} \frac{\langle \phi_d(\vec{r}_{13}) | \Psi(\vec{r}_{13}, \vec{r}_c) \rangle}{r_c^{-1} \exp(-\beta r_c)} , \quad (6)$$

where ϕ_d is the deuteron wave function, Ψ is the triton wave function, and β is given by

$$\beta = \left[\frac{8M}{3\hbar^2}(|E| - |E_d|)\right]^{\frac{1}{2}} . \quad (7)$$

The overlap in Eq. (6) is only a function of r_c, and hence the limit only contains terms involving this coordinate. In Eq. (7), M is the reduced mass in the d + n channel, $\hbar$ is Planck's constant divided by 2π, $|E|$ is the model triton binding energy, and $|E_d|$ is the model deuteron binding energy. The ^{3}He asymptotic normalization constant $C_0(^3He)$ and wave function are defined in a similar manner. Details are provided in Refs. 25, 26 and 32.

The results of the R-matrix (RM) A=3 asymptotic normalization constant ($C_0(^3H)$ and $C_0(^3He)$) calculations are summarized and compared with other calculations [32-35] in Table I. Equation (6), which defines the ^{3}H asymptotic normalization constant, involves the product of a constant factor ($1/\sqrt{\beta}$) and a limit dependent on the deuteron-triton overlap and terms involving r_c. The constant factor can be compared with data since the binding energies are known. To date, sufficient data to properly define the second factor is not available. However, experimental data suggests $\beta(^3H) = 0.516$ and $\beta(^3He) = 0.483$. These results are in close agreement with the R-matrix values which are based on calculated binding energies[16]. However, Faddeev results[32] yield smaller β values. This is attributed to the triton binding energy which is underpredicted by the Reid soft-core/Faddeev model[32]. However, the agreement between the RM model and experimental β values does not imply that the R-matrix asymptotic normalization constants are correct.

The ^{3}H and ^{3}He asymptotic normalization constants are larger than their Faddeev counterparts[32,33]. However, the RM $C_0(^3H)$ value (1.882) lies between the one-boson-expansion (OBE) value (1.910) and the Faddeev values (1.706[32] and 1.776[33]). In view of the uncertainties involved, these results are quite close, and the RM value is within 11%

of the Faddeev[32] and within 2% of the OBE values.

TABLE I

Asymptotic normalization constants

$\beta(^3H)$ (fm^{-1})	$C_o(^3H)$	$\beta(^3He)$ (fm^{-1})	$C_o(^3He)$	Reference
0.421	1.706	0.388	1.765	32
0.390	1.776	—	—	33
—	—	(a)	1.84[b]	34
(a)	1.91[c]	—	—	35
0.516	1.882	0.486	1.948	This work

a) Not provided in the subject reference.
b) Derived from $C_o^2(^3He)$.
c) Derived from sum of S=0 and S=1 deuteron spin components.

As noted in Ref. 32, calculations of $C_o(^3He)$ are more difficult than $C_o(^3H)$ calculations because a detailed treatment of the Coulomb interaction is required. The RM $C_o(^3He)$ value is 13% greater than Faddeev estimate[32] and 8% greater than the forward dispersion estimate[34]. Although the RM estimate is larger than the other estimates, it is not significantly larger and all three calculations are in reasonable agreement.

In general, the results summarized in Table I support the RM ^3H and ^3He wave functions. Generally, harmonic oscillator calculations do not yield accurate results, but the present model has obtained reason-

able results with a careful choice of the model interaction[16]. Thus, $C_0(^3H)$ and $C_0(^3He)$ values add additional support for the use of R-matrix A=3 wave functions in (p,t) and other transfer reaction studies.

The calculated differences between $C_0(^3H)$ and $C_0(^3He)$ are small (5%) and close to the Faddeev differences (3-4%). Nevertheless, the differences are measurable in an accurate experiment. Within the scope of the RM model, the difference is due to the Coulomb interaction, but differences could also be attributed to charge symmetry violations of the nuclear force. Since papers concerning charge symmetry violations in few nucleon systems persist[22,41-44], measurements of $C_0(^3H)$ and $C_0(^3He)$ may also provide further clues to the violation of charge symmetry in the nuclear interaction.

IV. CONCLUSIONS

In addition to reasonably accurate A=3 binding energies and rms radii results, R-matrix calculations are shown to yield S-wave asymptotic normalization constant values which are within 15% of other calculated values. The results are important because they are based on an interaction which has been used to obtain reasonably accurate predictions of scattering and structure properties in the A=2, 3, 4 and 5 systems. This consistency and accuracy suggests that ^3H and ^3He wave functions derived from our R-matrix model will be useful in resolving shape and normalization difficulties in (p,t) and other few nucleon transfer reaction studies.

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