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CENTER -OF-MASS EFFECTS IN THE $^{12}C(p,t)^{10}C$ REACTION AT 80 MeV

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ABSTRACT

Angular distributions for the $^{12}C(p,t)^{10}C$ reaction for $E_p = 80$ MeV are analyzed within the framework of the zero-range distorted-wave Born approximation (DWBA). An approximation is formulated to account for center-of-mass (CM) effects in the $^{10}C+2n$ form factor. The CM approximation can be implemented by altering the input to DWBA codes such as DWUCK or MERCURY. Form factor CM corrections lead to significant improvements in the shape of the calculated $^{12}C(p,t)$ angular distribution for the ground and excited state transitions. In addition, improved normalizations are obtained for heavier mass target reactions such as $^{40}Ca(t,p)$ and $^{208}Pb(p,t)$.

RESUMEN

Se analizan las distribuciones angulares para la reacción $^{12}C(p,t)^{10}C$ para E_p=80 MeV en el contexto de la aproximación de onda distorsionada de Born en el rango cero (DWBA). Se formula una aproximación que considera los efectos del centro de masa (CM) para el cálculo del factor de forma del $^{10}C+2n$. La aproximación de CM puede llevarse a cabo alterando la información de entrada de DWBA usando, por ejemplo, DWUCK o MERCURY. Las correcciones del factor de forma por el CM mejoran significativamente la forma de la distribución angular para el fondo y las transiciones de estados excitados, calculadas para $^{12}C(p,t)$. También se obtienen mejores normalizaciones para reacciones con blancos más pesados, tales como $^{40}Ca(t,p)$ y $^{208}Pb(p,t)$.

I. INTRODUCTION

In a recent paper, Shepard et al.[1] have shown that single-step zero-range distorted-wave Born approximation (ZRDWBA) calculations failed to adequately reproduce the $^{12}C(p,t)^{10}C$ data at 80 MeV proton energy. The ZRDWBA calculations of Ref. 1 led Shepard et al. to conclude that the discrepancies between experiment and DWBA results are not wholly attributable to an improper treatment of finite-range effects and that it is possible two-step mechanisms could account for the discrepancies. This interpretation should not be evoked until all single-step issues have been addressed. One major unresolved issue is the proper treatment of the center-of-mass (CM) motion in the $^{10}C + 2n$ two-nucleon form factor.

The importance of CM effects in the zero-range form factor for two-neutron stripping was noted by Pinkston[2] and Pinkston and Iano[3]. Feng et al.[4] also observe improved DWBA results with the proper treatment of the nuclear structure.

The implementation of the aforementioned CM correction approaches requires the modification of DWBA codes. It would be highly desirable to have a CM correction method, even if it is only an approximation, which could be implemented within the framework of existing codes by only altering the code input. Such a model would permit estimates of CM effects without significant effort.

The present paper will attempt to accomplish this task. Specifically, our approximation will be compared with a more rigorous method[2] of obtaining the core plus two-nucleon form factor. In addition, the present approach will be applied to the $^{10}C + 2n$ form factor in order to determine the effect of CM corrections on the $^{12}C(p,t)^{10}C$ reaction. Heavier systems, $^{40}Ca(p,t)$ and $^{208}Pb(p,t)$, will also be considered. Thus, it is the intent of this paper to resolve the CM issue in the $^{12}C(p,t)$ and other reactions. Moreover, the resolution will be provided within the frame-work of an easily applied CM correction approach.

II. FORMALISM

The DWBA differential cross section may be written as[5]

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} |T_{fi}|^2 \quad , \tag{1}$$

where i labels the entrance channel, f labels the exit channel, μ is the reduced mass, and $k = \mu v/\hbar$, where v is the particle velocity. T is the transition matrix element which is defined as

$$T_{fi} = \int d\vec{r}_i \int d\vec{r}_f \; \chi_f^{(-)*}(\vec{k}_f,\vec{r}_f) \langle B,b|V|A,a\rangle \times \chi_i^{(+)}(\vec{k}_i,\vec{r}_i) \quad . \tag{2}$$

The wave functions $\chi_i^{(+)}$ and $\chi_f^{(-)}$ describe the relative motion of the clusters in the entrance and exit channels. The χ's are distorted waves arising from the Coulomb and nuclear interactions which are generated from optical potentials that describe appropriate elastic scattering data. The overlap $\langle B,b|V|A,a\rangle$ is the transfer form factor representing the core plus transferred cluster wave function. The coordinates $\vec{r}_i$ and $\vec{r}_f$ are the entrance (B+b) and exit (A+a) channel coordinates.

The form factor contains all the nuclear structure information. For two-nucleon (p,t) transfer reactions the identifications

$$B = A + x$$

and (3)

$$a = b + x$$

are made in order to facilitate description of the B(b,a)A reaction. The quantity x in Eq. (3) is the transferred cluster. Specifically, the following identifications can be made for the $^{12}C(p,t)^{10}C$ reaction

$$B = {}^{12}C, \quad b = p, \quad a = t, \quad A = {}^{10}C \quad \text{and} \quad x = 2n \quad . \tag{4}$$

Following integration over all target and projectile coordinates the form factor becomes

$$\langle B,b|V|A,a\rangle = \psi_B^*(\vec{r}_{Ax}) \; V(\vec{r}_{bx}) \; \psi_a(\vec{r}_{bx}) \quad , \tag{5}$$

where V is the effective interaction between b and x, and the ψ's are the bound state wave functions which describe the motion of the transferred

cluster x relative to A and b. For convenience, the spectroscopic strengths have been absorbed into ψ. The bound state wave functions are usually approximated by shell-model eigenfunctions which do not properly account for center-of-mass effects.

Since the calculation of the form factor and T matrix requires considerable computer time, it is desirable in (p,t) reactions studies to introduce the zero-range (ZR) approximation by replacing the radial dependence of V by a delta function

$$V(\vec{r}_{bx})\ \psi_a(\vec{r}_{bx}) = D_o\ \delta(\vec{r}_{bx}) \quad , \tag{6}$$

where

$$D_o = \int d\vec{r}_{bx}\ V(\vec{r}_{bx})\ \psi_a(\vec{r}_{bx}) \quad . \tag{7}$$

The introduction of the delta function reduces the T matrix to the three dimensional integral

$$T_{fi} = D_o \int d\vec{r}\ X_f^{(-)*}(A\vec{r}/B)\ \psi_B^*(\vec{r})\ X_i^{(+)}(\vec{r}) \quad . \tag{8}$$

Following Ref. 1, $D_o^2 = 25 \times 10^4$ MeV^2 fm^3. However, the center-of-mass difficulty of the $^{10}C + 2n$ form factor (ψ_B) remains.

In order to remove the CM difficulty, we introduce a repulsive component into the core plus two-nucleon potential which is used to determine ψ_B. The core + 2n wave function is generated from the potential U:

$$U = V_0\ f(r,r_0,a_0) + V_1\ f(r,r_1,a_1) + V_{Coulomb} + \frac{\lambda V_0}{45.2}\frac{1}{r}\frac{df(r,r_{s.o.},a_{s.o.})}{dr}\ L\cdot S \quad . \tag{9}$$

This only requires the addition of a single bound state potential card to the input for DWBA codes such as DWUCK[6] or MERCURY[7]. In Eq. (9), V_0 is the central potential, V_1 is the repulsive component potential, λ is the Thomas spin-orbit term, a_i is the diffuseness parameter, r_i is the radius parameter, and $V_{Coulomb}$ is the Coulomb interaction between the core and transferred cluster. The factor $f(r,r_i,a_i)$ has the Woods-Saxon form

$$f(r,r_i,a_i) = \left[1 + \exp\left(\frac{r - r_i A^{1/3}}{a_i}\right)\right]^{-1} , \qquad (10)$$

where A is the core mass. The values of V_1, r_1 and a_1 are chosen to be

$$\begin{aligned} V_1 &= -5V_0 \; ; \; V_0 < 0 \quad , \\ r_1 &= 0.44 r_0 \quad , \\ a_1 &= 0.31 a_0 \quad . \end{aligned} \qquad (11)$$

The use of a repulsive potential component leads to a core + 2n form factor which peaks at a slightly greater radial location than the usual shell-model form factor. Physically, the use of a repulsive potential arises from a consideration of the Schrödinger equation which is used to generate the form factor ψ:

$$(T + U)\psi = E\psi \quad , \qquad (12)$$

where T is the kinetic energy of the nucleon-nucleus form factor, U is the nucleon-nucleus potential, and E is the binding energy of the nucleon-nucleus system. According to Pinkston[2,3], the shell-model wave function is too small in the nuclear surface and exterior regions. Changes to the wave function can be affected by altering either U or E. Since E is an experimentally determined value and U is usually a function of adjusted parameters, it is logical to alter U. The alteration to U must be such that it reproduces the center-of-mass corrected wave function, i.e., the surface exterior wave function must be larger than the shell-model wave function[2-4]. Previously, it has been noted that the surface region and a fermi or so beyond dominate the (p,t) differential cross section[8].

The desired effect of approximating the tail of the center-of-mass corrected form factor can be obtained by adding a repulsive term to the form factor potential U. The repulsive potential forces the wave function outside the attractive potential well and increases the magnitude of the surface region wave function. The strength of repulsive potential is chosen to reproduce the center-of-mass corrected form factor. The reader should note that V_0 is determined just as in standard DWBA codes[6,7]. Hence, the simulation of the center-of-mass corrected form fac-

tor is obtained by adding a repulsive component to the form factor calculation with a strength $-5V_0$ and a range of $0.44\ r_0$ (see Eq. (11)). The net result is a form factor which, over the dominant (p,t) spatial region, provides a good approximation to Pinkston's approach. The application of Eq. (11) to the $^{16}O + 2n$ form factor will be discussed and compared with Pinkston's approach in the next section.

III. FORM FACTOR COMPARISON ($^{16}O + 2n$)

The approach of Eq. (11) is applied to the $^{16}O + 2n$ form factor, and compared with Pinkston's form factor[2] in Fig. 1. The bound state parameters are taken from Ref. 2. Both calculations of Fig. 1 assume the two neutrons are in $1d_{5/2}$ orbits coupled to form a 0^+ state in ^{18}O.

The agreement between model and Pinkston's form factors would initially appear to be quite poor. However, the approaches need only agree in the vicinity of the nuclear surface region[4] and possibly a fermi or two beyond[8], because these regions dominate the (p,t) cross section. The equivalence of the two form factor approaches in the vicinity of the nuclear surface region can be demonstrated by integrating the form factors over the region

$$r_{surface} \leq r \leq r_{surface} + 2 \text{ fm} \quad . \tag{13}$$

The resulting overlaps o(c.f. Eq. (8)),

$$o = \int_{r=r_{surface}}^{r_{surface}+2\text{ fm}} \int_{\theta=0°}^{\pi} \int_{\phi=0°}^{2\pi} X_f^{(-)*}\, F\, X_i^{(+)} \times r^2 \sin(\theta)\, dr\, d\theta\, d\phi \ , \tag{14}$$

are essentially independent of the input CM corrected form factor, i.e., Pinkston's form factor approach or the approach utilized herein. The nuclear surface occurs at about a radius $r_{surface}$:

$$r_{surface} = c\, A^{1/3} \quad , \tag{15}$$

where $c = 1.1$ fm[9]. For the $^{16}O + 2n$ system, Eq. (12) places the surface at about 2.9 fm. The approximation model of Eq. (11) is well within 10%

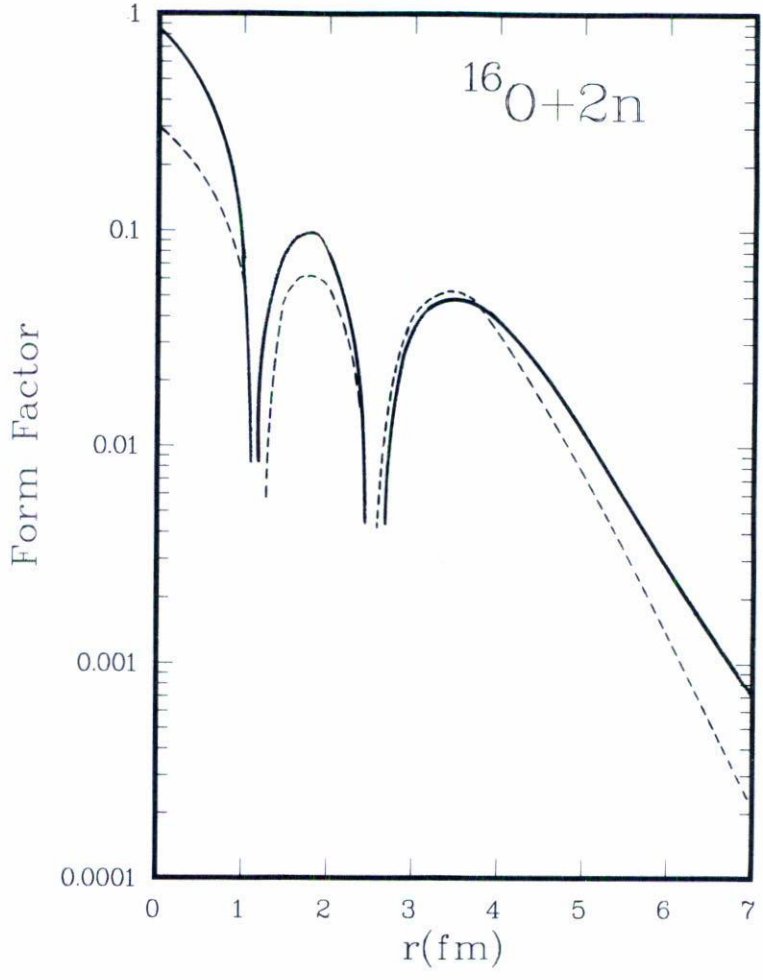

Fig. 1 Zero-range two-particle form factors for the $1d^2_{5/2}$ $J = 0$ state in ^{18}O. The solid curve is the form factor obtained in Ref. 2, and the dashed curve is the approximation of Eq. (11).

of Pinkston's form factor in the vicinity of the ^{18}O surface, and provides an adequate approximation to the more rigorous approach of Refs. 2 and 3. Our approximation method, may now be confidently applied to the ^{12}C(p,t) reaction to investigate the impact of ^{10}C + 2n CM form factor corrections. This will be addressed in the next section of this paper.

IV. $^{10}C + 2n$ FORM FACTOR

Pinkston and Iano[3] find that CM corrected form factor overlaps relevant to (t,p) reactions on ^{16}O, ^{40}Ca and ^{90}Zr result in amplitude enhancements of roughly 40%, 20% and 10%, respectively. Model form factor overlaps have been calculated to further verify the adequacy of the approximation of Eq. (11). The results of these calculations are summarized in Table I and are within 10% of the results of Pinkston and Iano. These results suggest Eq. (11) is providing a realistic solution to the center-of-mass form factor problem.

TABLE I

Nucleus	Enhancements due to center-of-mass correction	
	Pinkston and Iano (Ref. 3)	This work
^{16}O	1.4	1.30
^{40}Ca	1.2	1.10
^{90}Zr	1.1	1.05

Table I Form factor amplitude enhancements for (p,t) transfer reactions (A = 16 - 90).

Using the approach of Eq. (11), we calculate the $^{10}C + 2n$ form factor and summarize the results in Fig. 2. The $^{12}C(^{10}C + 2n)$ surface occurs at a radius of about 2.5 fm (Eq. (13)). The CM form factor and shell-model form factor have somewhat different shapes between 2.5 and 3.0 fm, which may influence the cross section shape. Beyond the nuclear surface region, the CM form factor is about 40% larger than the shell-model result. This is consistent with the results of Ref. 3. The effect of the altered surface shape and form factor enhancement will be discussed in the next section.

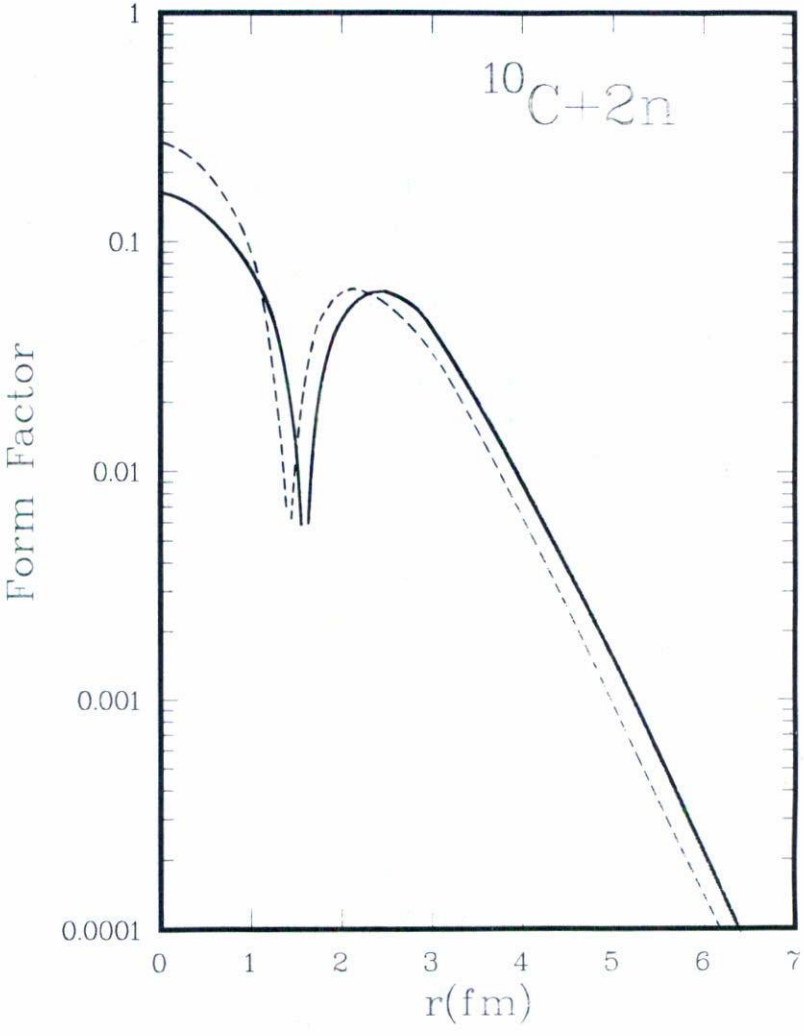

Fig. 2 Zero-range two-particle form factors for the $1p^2_{3/2}$ J = 0 state in ^{12}C. The solid curve is the center-of-mass corrected form factor and the dashed curve is the shell-model form factor.

V. DWBA CALCULATIONS

Zero-range distorted-wave Born approximation (ZRDWBA) calcula-

tions for the $^{12}C(p,t)$ reaction were performed within the framework of the computer code DWUCK[6]. The calculations assume the simultaneous pick-up of two $p_{3/2}$ neutrons coupled to $S \neq 0$ with zero relative angular momentum. The method of Bayman and Kallio[10] was used to compute the two-neutron form factor. Standard DWBA calculations were compared with those of Ref. 1 in order to verify the model parameters.

Optical parameters for the $p + {}^{12}C$ and $t + {}^{10}C$ channels, and the $n + {}^{10}C$ bound state are taken from Shepard et al.[1]. Following Shepard's analysis, the form factor overlaps were evaluated using the half-separation-energy ansatz for the single particle wave functions, together with shell-model spectroscopic amplitudes. Although this procedure neglects correlations between the transferred neutrons, its impact on the cross section is small and does not significantly alter the conclusions of this study (see Fig. 1, Ref. 4).

Shepard et al.[1] have noted that the $^{12}C(p,t)^{10}C(0^+,g.s.)$ transition is very sensitive to the triton optical potential. Thus, as noted earlier, care was taken to insure that our parameters matched those of Ref. 1. The uncorrected DWBA calculation (see Fig. 4, Ref. 1) for the 0^+ transition generally reproduces the data for $\theta_{c.m.} \leqslant 20°$, but the fit is poorer beyond this angle. The local minimum at about 22° and the primary minimum at about 50° are not reproduced by the uncorrected DWBA calculations.

The uncorrected DWBA calculations of Ref. 1 can be compared with a CM corrected calculation in Fig. 3. The CM corrections improve the calculated shape when compared with the uncorrected DWBA model. The result of Fig. 3 indicates that CM corrections can have a significant impact on DWBA calculations.

Poorer agreement between experiment and DWBA calculations is obtained for the 3353 keV (see Fig. 4) and 5280 keV (see Fig. 5) transitions. Both transitions involve 2^+ states with an angular momentum transfer of 2. The CM corrected DWBA calculations for both transitions are significantly better than standard DWBA calculations (see Fig. 4, Ref. 1 for comparison). As with the 0^+ transition, the CM effects are significant and lead to improved angular distributions.

The normalization N for the $^{12}C(p,t)^{10}C$ reaction is essentially

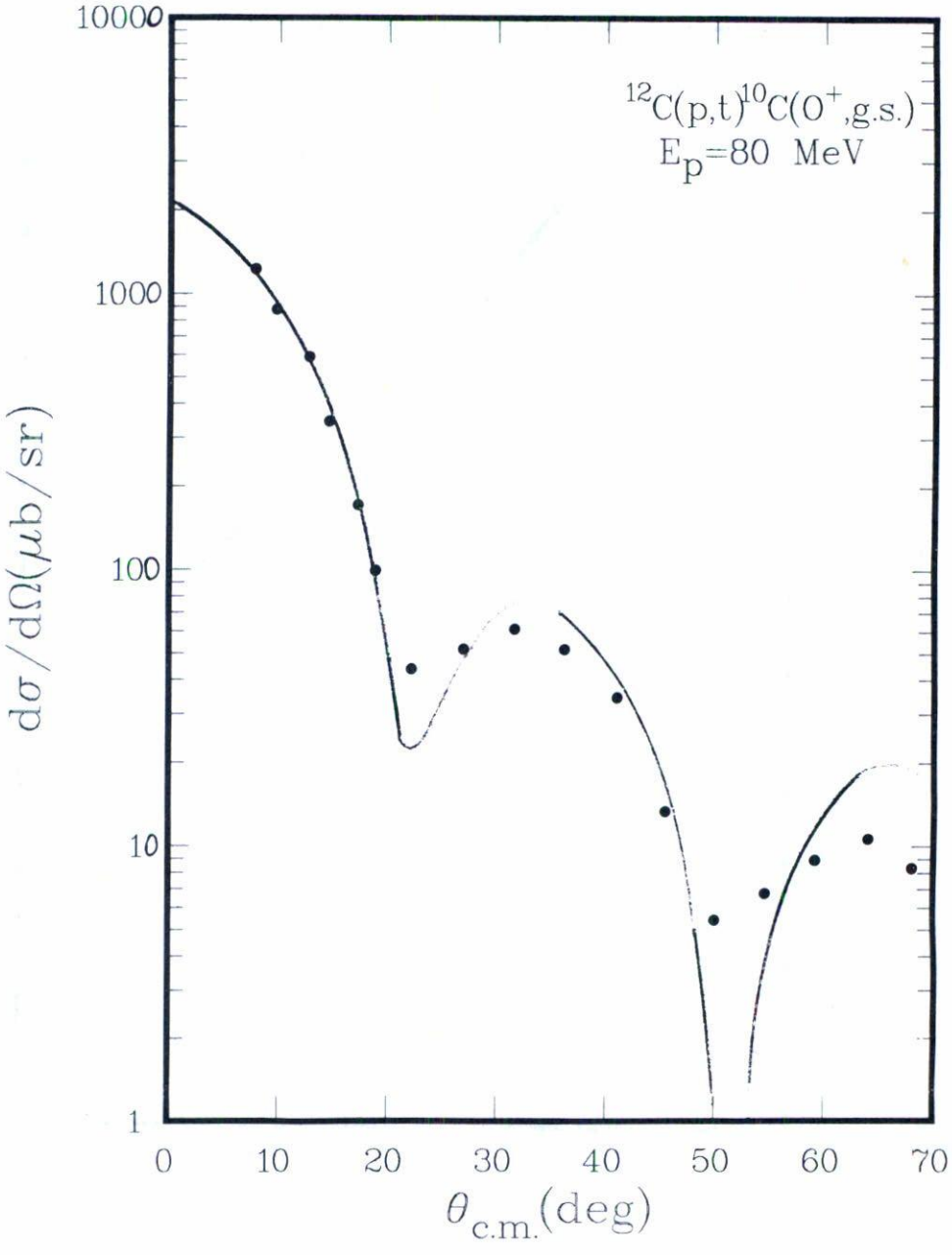

Fig. 3 Center-of-mass corrected DWBA calculations for the $^{12}C(p,t)^{10}C(0^+,g.s.)$ reaction at 80 MeV. The data is from Ref. 1. The normalization of the calculation is given in Table II.

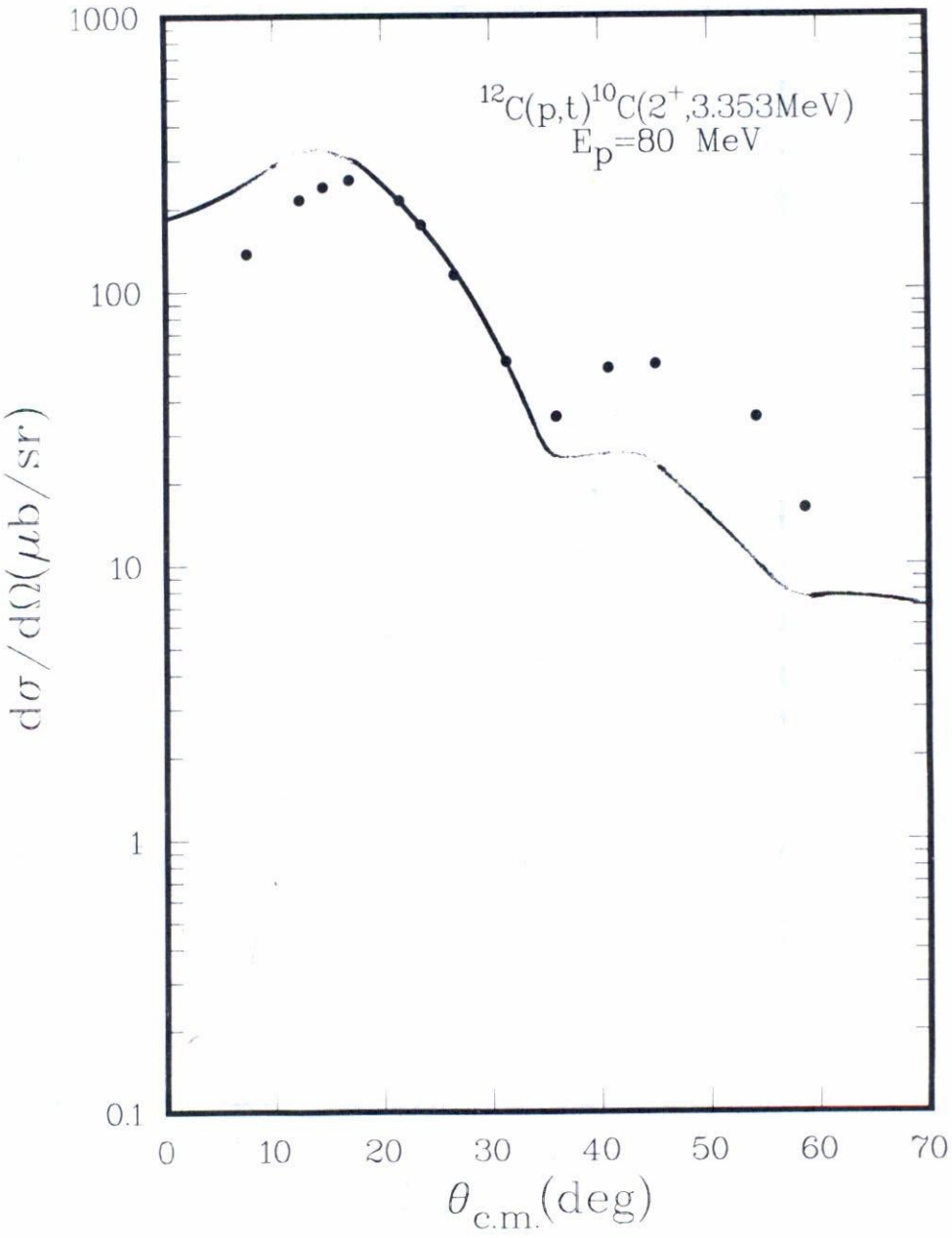

Fig. 4 Same as Fig. 3, except for the $^{12}C(p,t)^{10}C(2^+,3.353$ MeV) transition.

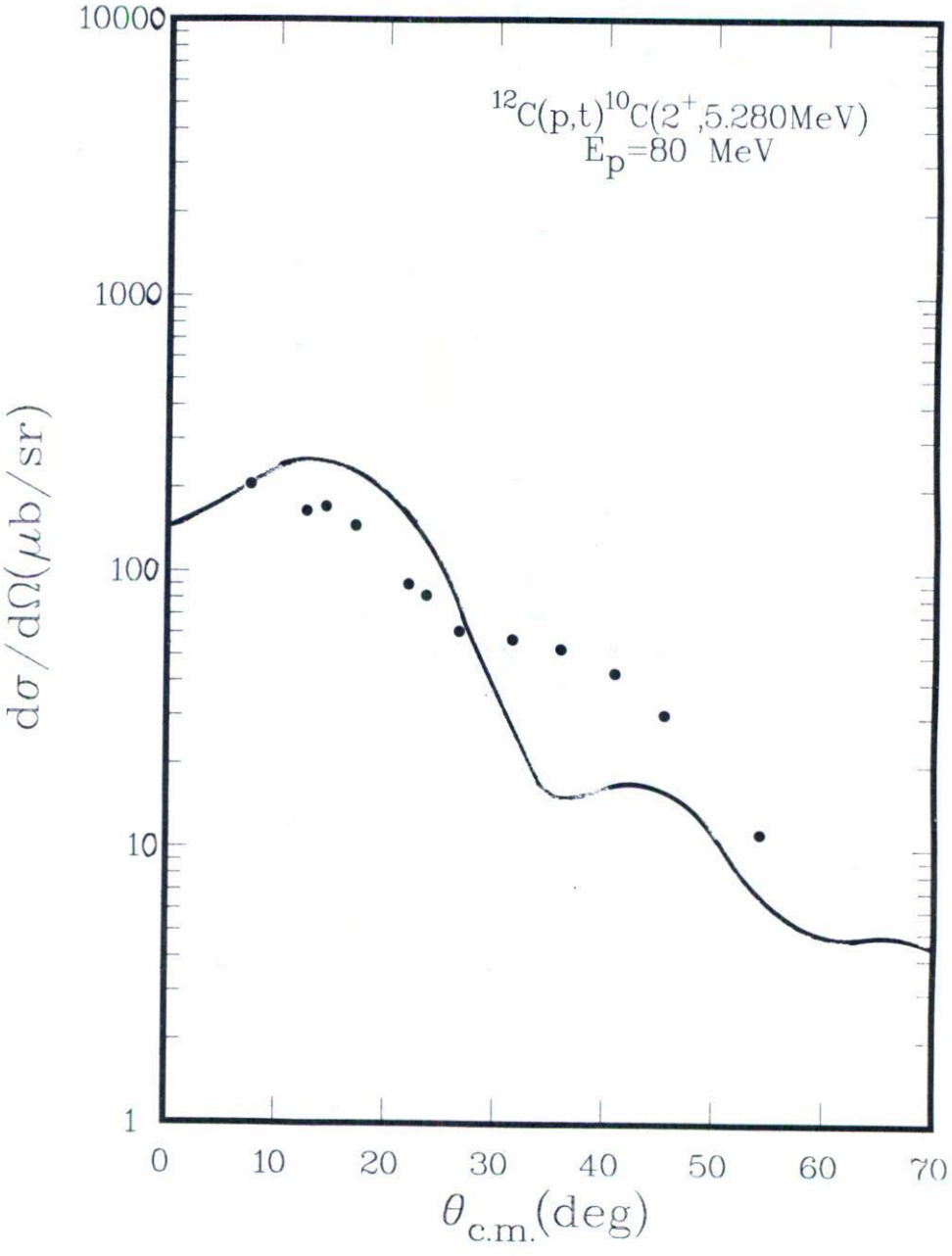

Fig. 5 Same as Fig. 3, except for the $^{12}C(p,t)^{10}C(2^+,5.280$ MeV) transition.

the factor required to bring the DWBA calculations into agreement with data. The DWBA cross section σ_{DWBA} is related to the experimental cross section by

$$\sigma_{exp} = N\,(S^{1/2})^2\,(D_o^{\,2}/10^4)\,\frac{\sigma_{DWBA}}{2L+1}\quad , \tag{16}$$

where $S^{1/2}$ is the two-neutron spectroscopic amplitude[11], D_o is the zero-range normalization factor and L is the angular momentum transfer. The DWBA calculations of Ref. 1 and those presented herein are of different shapes. This suggests that normalizations may be guided by personal bias. In order to avoid this difficulty, we present a modified normalization factor N':

$$N' = \frac{\sigma_{CMDWBA}(\theta_{c.m.} = 0°)}{\sigma_{DWBA}(\theta_{c.m.} = 0°)}\quad , \tag{17}$$

where σ_{CMDWBA} is the CM corrected DWBA cross section. The use of Eqs. (14) and (15) will permit the reader to compare our calculations with those of Ref. 1 in an unambiguous manner. The values of N' for the transitions of Figs. 3, 4 and 5 are summarized in Table II.

TABLE II

^{10}C Final state E_x (keV)	J^π	N[a]	N'[b]
0	0^+	0.863	2.95
3353	2^+	0.255	1.03
5280	2^+	0.255	1.14

a) Table I, Ref. 1.

b) Eq. (14).

Table II Normalizations for the $^{12}C(p,t)^{10}C$ reaction.

VI. EXTENSION OF CALCULATIONS TO HEAVIER NUCLEI

For completeness, calculations have been extended to the $^{40}Ca(t,p)$ and $^{208}Pb(p,t)$ reactions. CMDWBA calculations were compared with data [12-13] and standard DWBA calculations. The CM corrected DWBA calculations have shapes which are very similar to standard DWBA predictions[8], but normalizations have improved[8]. However, improvements are not as significant as in lighter target reactions because CM effects become less important as the target mass increases.

VII. CONCLUSIONS

Center-of-mass corrections to the $^{10}C + 2n$ form factor lead to improvements in the calculated $^{12}C(p,t)^{10}C$ angular distributions. The CM correction method used to calculate the $^{10}C + 2n$ form factor only approximates a more rigorous method by Pinkston, but can be easily input into existing DWBA codes. The CM form factor approximation permits a qualitative assessment of CM effects, and has the advantage that DWBA code modifications are not required. As expected center-of-mass corrections have been found to be most important for light systems (^{12}C and ^{16}O), but also improve the calculated cross section normalizations for transfer reactions involving heavier nuclei such as ^{40}Ca and ^{208}Pb.

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