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AN SU(3) SCHEME FOR LEPTONS. I. THE OCTET CASE

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ABSTRACT

A systematic analysis of the lepton mass spectrum is carried out assuming that the leptons belong to an SU(3) octet, and that the (spontaneous) symmetry breaking is provided by a nonet of Higgs particles. We conclude that the octet is not compatible with the present experimental data for the lepton masses.

RESUMEN

Se desarrolla un análisis sistemático del espectro de masas leptónico, suponiendo que los leptones pertenecen a un octete de SU(3), y que la ruptura espontánea de simetría se debe a un nonete de Higgs. Concluimos que el octete no es compatible con los datos experimentales actuales de las masas de los leptones.

I. INTRODUCTION

Ever since the discovery of the μ meson(1) and its identification with a heavy electron(2), the question of the elementarity and mass spectrum of the leptons has been a central problem in the physics of the weakly interacting particles. The discoveries of the neutrinos associated to the electron and the muon(3), and more recently the τ lepton(4) and its not yet well confirmed neutrino(5), have made it even more urgent to solve this problem.

Several models have been proposed in the last few decades, in which the leptons are considered as composite objects of more fundamental particles(6). Perhaps, the most notable models are those of Strasbourg(7), on the phenomenological side, and those within the spirit of technicolor(8). However, these ideas have to answer, as Lipkin remarked recently(9), the very small anomalous magnetic moment, $g-2\approx 0$, of the charged leptons(10). Taking into account this observation one is led to the conclusion that the lepton mass spectrum must be generated through a mechanism which should be equivalent to the spontaneous symmetry breaking; the small g-2 value would be then a consequence of the residual interaction of the leptons with much heavier particles. Still, this leaves unanswered the problem of the lepton mass spectrum.

One exception is the very phenomenological approach of Bjorken(11), in which a logaritmic fit is proposed, this unfortunately seems not to be a good fit to the charged lepton spectrum, because the predicted value of the next charged lepton seems to be ruled out by experiment. In this work we analyse the possibility that the observed leptons are in a SU(3) multiplet. If this SU(3) is a subsymmetry of a more general one, then the more general symmetry should be broken in such a form that the SU(3) symmetry breaking pattern is mantained. Here we consider an octet of SU(3) because this representation has the smallest dimensionality consistent with three charged and three neutral leptons.

II. THE MASS SPECTRUM OF THE OCTET

We assume the generators oriented so that a Gell-Mann-Nishijima relation is valid for the charge, the weak isospin, and the weak hypercharge[12]:

$$Q = I_3 + \frac{1}{2} Y \quad . \tag{1}$$

The most general renormalizable lagrangian that conserves the electromagnetic current, specified by the above relation, in which the symmetry is spontaneusly broken, is[13]

$$L = \mathrm{Tr}\left[\overline{\psi}_i \not{\partial} \psi + \partial\phi\partial\phi + g_A \overline{\psi}[\phi,\psi] + g_s \overline{\psi}\,\{\phi,\psi\}\right] + g_1 \mathrm{Tr}(\overline{\psi}\psi)\mathrm{Tr}\phi + V(\phi) \quad , \tag{2}$$

with

$$\psi = \sum_{i=1}^{8} \psi_i \lambda_i \quad ,$$

$$\phi = \sum_{i=0}^{8} \phi_i \lambda_i \quad ,$$

$\lambda_{i,\ldots,8}$ are the Gell-Mann matrices and $\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbb{1}$. The ψ field is an octet of fermions and the ϕ is a nonet of scalar particles. The ϕ could be also in a 10, a $\overline{10}$ or a 27 representation of SU(3), but in this work we consider the simplest choice, which is the octet plus singlet.

The symmetry is spontaneusly broken by requiring that some of the scalars acquire non-zero expectation values in the vacuum. The conservation of charge constraints $\langle\phi_i\rangle_0 = 0$ for $i = 1,2,4,5$. Therefore we are left with $\langle\phi_i\rangle_0 = m_i$ for $i = 3,6,7,8,0$. The mass terms associated with g_A and g_s have many non-diagonal components, and those associated with g_1 give an overall mass to the ψ's .

The most direct choice, $m_6 = m_7 = 0$, leads immediately to a Gell-Mann-Okubo like mass relation with first order electromagnetic correction included[6]. A more general symmetry breaking pattern follows from considering both m_6 and m_7 non-zero. This in turn implies a mixing among the equal charge leptons. The explicit mass formulae, which have been verified with the aid of an algebraic manipulation program, are

$$M_+ = \begin{pmatrix} \mu + \frac{1}{\sqrt{3}}\; g_s m_8 + g_A m_3 & (g_s - g_A)\; m_{67}^* \\ (g_s - g_A)\; m_{67} & \mu - \frac{1}{2\sqrt{3}}\,(g_s - 3g_A) m_8 + \frac{1}{2}\,(g_s + g_A) m_3 \end{pmatrix} \quad ,$$

$$M_- = M_+(g_A \to -g_A) \quad ,$$

$$M_0 = \begin{pmatrix} \mu + \frac{1}{\sqrt{3}} g_s m_8 & \frac{1}{\sqrt{3}} g_s m_3 & -\frac{1}{\sqrt{2}} (g_s - g_A) m_{67}^* & -\frac{1}{\sqrt{2}} (g_s + g_A) m_{67} \\ & \mu - \frac{1}{\sqrt{3}} g_s m_8 & -\frac{1}{\sqrt{6}} (g_s + 3g_A) m_{67}^* & -\frac{1}{\sqrt{6}} (g_s - 3g_A) m_{67} \\ & & \mu - \frac{1}{2}(g_s + g_A) m_3 - \frac{1}{2\sqrt{3}}(g_s - 3g_A) m_8 & 0 \\ & & & \mu - \frac{1}{2}(g_s - g_A) m_3 - \frac{1}{2\sqrt{3}}(g_s + 3g_A) m_8 \end{pmatrix} ,$$

with

$$\mu = \sqrt{\frac{2}{3}}\ (g_s + 3g_1) m_0 \tag{3}$$

and

$$m_{67} = \frac{1}{2}\ (m_6 + i\ m_7) \quad .$$

The order of rows and columns in the matrices is:

for M_+ : $\frac{1}{\sqrt{2}} (\psi_1 + i\psi_2)$, $\frac{1}{\sqrt{2}} (\psi_4 + i\psi_5)$.

for M_- : $\frac{1}{\sqrt{2}} (\psi_1 - i\psi_2)$, $\frac{1}{\sqrt{2}} (\psi_4 - i\psi_5)$

for M_0 : ψ_3, ψ_8, $\frac{1}{\sqrt{2}} (\psi_6 + i\psi_7)$, $\frac{1}{\sqrt{2}} (\psi_6 - i\psi_7)$.

One can straightforwardly diagonalize M_+ and M_-. For M_0 it is necessary to compute its characteristic polinomial an obtain its roots. After some algebraic manipulations we find the non trivial result that m_3, m_8 and m_{67} only appear in the following combinations:

$$a = \frac{1}{2}\ g_s \left(m_3 + \frac{1}{\sqrt{3}}\ m_8 \right) \quad , \tag{4}$$

and

$$b = g_s \left[\left(\frac{\sqrt{3}\ m_8 - m_3}{2} \right)^2 + 4\ |m_{67}|^2 \right]^{1/2} , \tag{5}$$

and therefore we can reabsorbe m_{67} in these parameters. This means that one can choose any values for m_6 and m_7, in particular the simplest possible: $m_6 = m_7 = 0$, if one is interested only in the fermion spectrum. With these parameters, and defining $\Gamma = g_A/g_s$, we get the following diagonalized spectrum, in units of g_s:

$$m_{k^+} = m^+_{45} = \mu + \frac{1}{2}(1+3\Gamma)\ a - \frac{1}{2}(1-\Gamma)\ b, \tag{6}$$

$$m_{\pi^+} = m^+_{12} = \mu + \frac{1}{2}(1+3\Gamma)\ a + \frac{1}{2}(1-\Gamma)\ b, \tag{7}$$

$$m_{k^-} = m^-_{45} = \mu + \frac{1}{2}(1-3\Gamma)\ a - \frac{1}{2}(1+\Gamma)\ b, \tag{8}$$

$$m_{\pi^-} = m^-_{12} = \mu + \frac{1}{2}(1-3\Gamma)\ a + \frac{1}{2}(1+\Gamma)\ b, \tag{9}$$

$$m_{k^0} = m^+_{67} = \mu - a + \Gamma b, \tag{10}$$

$$m_{\bar{k}^0} = m^-_{67} = \mu - a - \Gamma b, \tag{11}$$

$$m_{\pi^0} = m_3 = \mu + \left(a^2 + \frac{1}{3}\ b^2\right)^{1/2}, \tag{12}$$

$$m_{\eta} = m_8 = \mu - \left(a^2 + \frac{1}{3}\ b^2\right)^{1/2}, \tag{13}$$

where the notation implied on the left hand terms reminds the octet classification of the hadrons in the zero mixing limit (see Fig. 1).

III. THE EXPERIMENTAL CONSTRAINTS

We will now analyse the consequences of having in the same octet the electron ($m_e \simeq 0.5$ MeV), the muon ($m_\mu \simeq 105$ MeV), the tau ($m_\tau \simeq 1800$ MeV), and an extra charged lepton $\lambda (m_\lambda \gtrsim 15\ M_\tau)$.

Additionally, there must be the two well established neutrinos, ν_e and ν_μ, with $m_{\nu_e} = m_{\nu_\mu} = 0$. For mathematical simplicity, considering

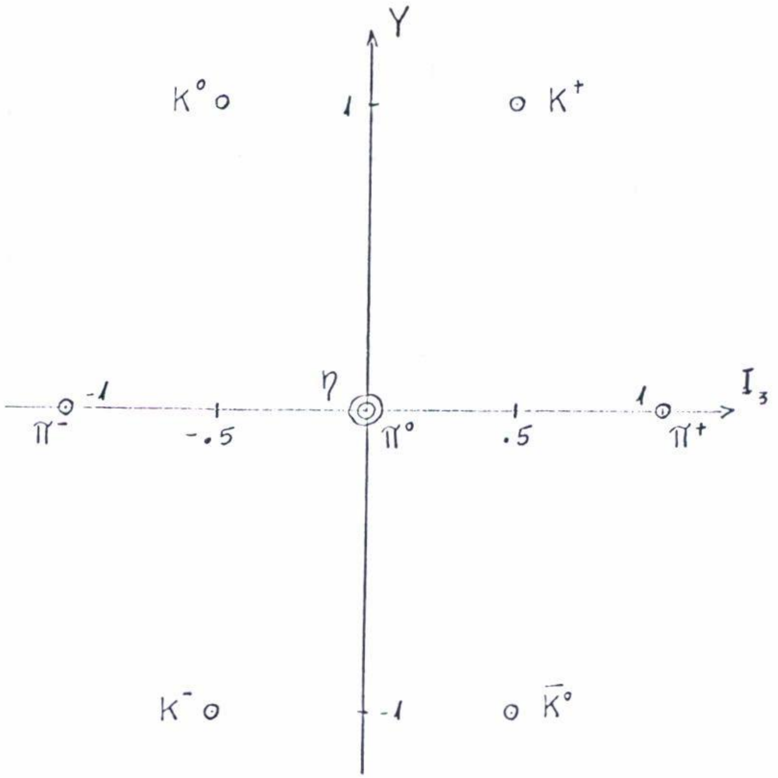

Fig. 1. Mnemonic of the SU(3) classification for the octet. The symbols π, K, η are just to remind which states of the meson SU(3) octet are the corresponding states of the leptonic classification.

that the masses of the electron and the muon are negligible compared with the tau mass, we will take $m_e = m_\mu = 0$.

To consider the actual masses implies corrections of the mass spectrum up to the order of the muon mass, which are not significant at this level.

Taking into account the above, and the fact that the states of the octet have four degress of freedom (as Dirac particles), whilst the neutrinos could have only two, our minimal requirement will be to have two charged massless states, and at least one neutral state of zero mass(14). Notice that these minimal requirements distinguish our approach from less systematic analysis(7).

We distinguish the following cases:

1) $m_{\pi+} = m_{k+} = 0$

From Eqs. (6) and (7) we obtain

$(\Gamma-1)b = 0$,

$2\mu + (3\Gamma+1)a = 0,$

therefore we have either $\Gamma = 1$ or $b = 0$.

1-a) $\Gamma = 1$

We get $\mu = -2a$

and so:

$m_{k-} = m_{\overline{k}^0} = -(3a+b),$

$m_{\pi-} = m_{k^0} = -(3a-b),$

$m_{\pi^0} = -2a + \left(a^2 + \frac{1}{3} b^2\right)^{1/2},$

$m_{\eta} = -2a - \left(a^2 + \frac{1}{3} b^2\right)^{1/2}.$

If one neutral is massless, we obtain

$a = \pm \frac{1}{3} b$.

Therefore at least another charged particle and another neutral particle will have a zero mass.

1-b) $b = 0$

Then

$m_{\pi-} = m_{k-} = -3\Gamma a,$

$m_{k^0} = m_{\overline{k}^0} = -\frac{3}{2}(1+\Gamma)a$,

$m_{\pi^0} = -\frac{3}{2}(1+\Gamma)a$,

$m_{\eta} = \frac{1}{2}(1-3\Gamma)a$.

The other two charged leptons have the same mass ($m_{\pi-} = m_{k-}$), and there also are three neutrals of equal mass ($m_{k^0} = m_{\overline{k}^0} = m_{\pi^0}$). If one neutral is massless we have:

i) $a = 0$

All the particles are massless.

ii) $\Gamma = -1$

Then three neutral particles have zero mass, and there is a heavy neutral at $\frac{2}{3}\, m_{\pi-}$.

iii) $\Gamma = \frac{1}{3}$

There is only one massless neutral particle. The other three neutrals have a mass equal to $2\, m_{\pi-}$.

In these cases the τ is doubly degenerate. If $m_\mu - m_e \neq 0$ this degeneracy is also broken, so one would expect another charged lepton at about the $m_\tau \pm m_\mu$ mass.

2) $m_{k+} = m_{k-} = 0$

From Eq. (6) and (8) we obtain:

$\Gamma(3a+b) = 0$,

$2\mu - (b-a) = 0$,

and we have either $\Gamma = 0$ or $b = -3a$.

2-a) $\Gamma = 0$

We have:

$$m_{\pi+} = m_{\pi-} = b \quad ,$$

$$m_{k^0} = m_{\overline{k}^0} = \frac{1}{2}(b-3a) \ ,$$

$$m_{\pi^0} = \frac{1}{2}(b-a) + \left(a^2 + \frac{1}{3}b^2\right)^{1/2} ,$$

$$m_\eta = \frac{1}{2}(b-a) - \left(a^2 + \frac{1}{3}b^2\right)^{1/2} .$$

The τ is degenerate in the massless muon approximation. Lifting this approximation implies two charged leptons at the masses of m_τ and $m_{\tau'} \simeq m_\tau \pm m_\mu$. π^0 and η cannot be massless. $b = -3a$ makes K^0 and $\overline{K^0}$ massless, with $|m_{\pi^0}|$ and $|m_\eta|$ greater than and of the order of m_τ.

2-b) $b = -3a$

So $\mu = -2a$,

$$m_{\pi+} = m_{\overline{k}^0} = -3(1+\Gamma)a \ ,$$

$$m_{\pi-} = m_{k^0} = -3(1-\Gamma)a \ ,$$

$$m_{\pi^0} = 0 \quad ,$$

$$m_\eta = -4a \ .$$

Here we see that $m_{k+} = m_{k-} = 0$ implies $m_{\pi^0} = 0$, but this neutral particle acquires mass when we remove that condition, so it is not suitable solution. Other massless neutral needs $\Gamma = \pm 1$, and therefore another massless charged lepton appears, or $a = 0$, which corresponds to a completely degenerate massless octet.

3) $m_{k+} = m_{\pi-} = 0$

From Eqs. (6) and (9), this implies:

$$a = \frac{1}{3\Gamma} b \quad , \qquad \mu = -\frac{1}{6\Gamma}(1+3\Gamma^2) \quad b,$$

$$m_{\pi+} = (1-\Gamma)\ b,$$

$$m_{k-} = -(1+\Gamma)b,$$

$$m_{k^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} - \Gamma\right) b,$$

$$m_{\overline{k}^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} + 3\Gamma\right) b,$$

$$m_{\pi^0} = -\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{\Gamma} + 3\Gamma\right) - \left(\frac{1}{\Gamma^2} + 3\right)^{1/2}\right] b \ ,$$

$$m_{\eta} = -\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{\Gamma} + 3\Gamma\right) + \left(\frac{1}{\Gamma^2} + 3\right)^{1/2}\right] b \ .$$

The condition of one massless neutral implies:

i) $b = 0$

All the particles are massless.

ii) $\Gamma = \pm 1$

This means that there is another massless neutral and an extra charged lepton of zero mass.

We finally note that the remaining possibilities:

4) $m_{k-} = m_{\pi-} = 0$,

$m_{\pi+} = m_{\pi-} = 0$,

$m_{k-} = m_{\pi+} = 0$,

are equivalent to (1), (2) and (3) respectively, with $\Gamma \to -\Gamma$ or $b \to -b$.

IV. CONCLUSIONS AND FINAL REMARKS

We conclude that there are no solutions within the scheme presented above, an SU(3) octet broken by a nonet of Higgs, in which there are at least one massless neutral state and two charged states with negligible masses compared to the tau mass. Although the analysis presented here has been carried out for a massless electron and muon, the condition that the muon acquieres mass was tested numerically with the result that the mass of the fourth charged lepton is shifted by an amount comparable to the muon mass. Our analysis shows that an octet model with nonet symmetry breaking can be dismissed. In particular the model by Fritzsch and Minkowski(14), in which ν_e and $\bar{\nu}_\mu$ are associated with a single member of the octet, can be ruled out if the Higgs particles are in an octet or a nonet.

We finally want to comment on the apparent irrelevance of m_6 and m_7 for the lepton spectrum. It seems that this result is a particular case of a more general theorem. Gourdin(15) has shown that if the nonsinglet part of the mass hamiltonian transforms like a component of the adjoint representation of the SU(4) algebra, then:

i) it may be associated to any direction of the bidimensional subspace of the Cartan algebra orthogonal to I_3 (or Q),

ii) the mass formula can be written like:

$$M = m_0 + m_1(\tilde{c} - \tilde{s}) + m_2\tilde{c} + m_3\,\Omega^{SYM}_{\tilde{c}-\tilde{s}} + m_4\,\Omega^{SYM}_{\tilde{c}} \;,$$

where $\tilde{s}$ and $\tilde{c}$ are SU(4) traceless operators associatet to strangeness and charm, and Ω^{SYM} are computed in the enveloping algebra as

$$\Omega^{SYM}_{\tilde{c}-\tilde{s}} = 2\,\bar{I}\,(\bar{I}+1)-2I(I+1),$$

$$\Omega^{SYM}_{\tilde{c}} = \frac{1}{2}\,X_4 - X_3 + \frac{2}{3}\,c^2 \quad ,$$

with $\bar{I}$ spin, the associated SU(2) subgroup of SU(4) commuting with isospin, and X_3 and X_4, SU(3) and SU(4) quadratic Casimir operators.

In our case, SU(3), $\tilde{c}$ and therefore $\Omega^{SYM}_{\tilde{c}}$ are absent, and the number of relevant "mass" parameters is reduced to m_0, a function of m_1 and m_2, and a function of m_3 and m_4. Therefore we should have three mass parameters in agreement with the result of Eqs. (6)....(13).

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