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AN SU(3) SCHEME FOR LEPTONS. II. NONET CASE AND EXTENSION TO QUARKS

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ABSTRACT

A systematic analysis of the lepton mass spectrum is presented in the framework of a spontaneous symmetry breaking, with leptons and Higgs particles associated with nonets of SU(3). We find solutions with all known leptons in the same representation, whose most remarkable feature is the presence of a neutral lepton at about the τ lepton mass (1.8 GeV). Other leptons are also predicted, one charged and two additional neutrals, which can have a mass large enough to prevent their observation at present available accelerator energies. If ν_e and ν_μ are both massive then ν_τ should be and admixture of them. ν-oscillations can be accounted in this model even in the case of massless ν_e and ν_μ. A direct extension to quarks is proposed.

RESUMEN

Se presenta un análisis del espectro de masas leptónico en el contexto de una ruptura espontánea de simetría, con los leptones y las partículas de Higgs asociados con nonetes de SU(3). Encontramos soluciones con todos los leptones conocidos en la misma representación, cuya característica más notable es la presencia de un leptón neutro de masa próxima a la del τ (1.8GeV). También se predice la existencia de otros leptones, uno cargado y dos neutros adicionales, que pueden tener masa suficientemente grande como para imposibilitar su observación con las energías disponibles actualmente. Si ν_e y ν_μ tienen ambos masa, entonces ν_τ podría ser una mezcla de ellos. En este modelo pueden obtenerse oscilaciones de neutrinos aun en el caso en que ν_e y ν_μ tienen masa 0. Se propone una extensión directa para quarks.

I. INTRODUCTION

In a previous work[1] we found that the most naive assignation for the leptons in SU(3), namely that they form an octet of SU(3), is incompatible with their observed mass spectrum. In this work we present a mathematically direct, but conceptually important, modification of that scheme. Specifically we consider a nonet assignation for the leptons.

As is well known from the SU(3) work of the 1960's in connection with the hadrons[2], the fact that baryons came in octets and decuplets, but not in singlets, is deeply related with the quark substructure of the baryons. On the other hand, mesons, which we conceive at present as formed mainly of a quark and an antiquark, came in nonets, i.e., in octets and singlets with a strong mixing.

The result that we present here, which shows that leptons might form a nonet but not an octet, has therefore potentially far-reaching consequences for our understanding of the leptonic world.

The presentation is organized as follows. We give in the next section the most general mass term for a nonet of fermions that stems from a nonet of Higgs. Then we spontaneously break the symmetry so that the neutral Higgs acquire nonzero vacuum expectation values. In section III we analyse the constraints imposed by the observed lepton spectrum. We remark that this analysis is more general than previous works[3], because of one or more of the following reasons: i) only one neutral octet state is required to be massless; ii) masses of fermions may be positive or negative; iii) one single fermion might appear more than once in the same representation. In section IV we make an extension of these ideas to quarks. We make some final remarks in section V.

II. THE MASS SPECTRUM

Within the framework of spontaneously broken symmetries and the Higgs mechanism[4], the fermions in the theory acquire nonzero masses because some scalar particles have nonzero vacuum expectation values. The fermion masses stem from the Yukawa couplings of these fermions to the scalar Higgs bosons.

In SU(3) the most general coupling that preserves the renormalizability of the theory is[5,6]:

$$L_I = \mathrm{Tr}\left(g_A \bar{\psi}\,[\phi,\psi] + g_s \bar{\psi}\,\{\phi,\psi\}\right) + g_1 \mathrm{Tr}(\bar{\psi}\,\psi)\,\mathrm{Tr}\phi +$$

$$+ g_2\left[\mathrm{Tr}\bar{\psi}\,\mathrm{Tr}(\phi\psi) + \mathrm{Tr}(\bar{\psi}\phi)\mathrm{Tr}\psi\right] + g_3 \mathrm{Tr}\bar{\psi}\,\mathrm{Tr}\phi\mathrm{Tr}\psi \quad , \qquad (1)$$

with $\psi = \sum_{i=0}^{8} \psi_i \lambda_i$ and $\phi = \sum_{i=0}^{8} \phi_i \lambda_i$,

where $\lambda_{1,\ldots,8}$ are the Gell-Mann matrices, and $\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbb{1}$.

The only difference with the lagrangian considered in Ref. 1 is the inclusion of the singlet ψ_0, which results in two new terms, with coupling constants g_2 and g_3. g_2 appears in the singlet and the mixing terms between the singlet and the neutral components of the octet; and g_3 contributes only to the singlet component.

The mass matrix is of the form

$$M = \begin{pmatrix} M_+ & 0 & 0 \\ 0 & M_- & 0 \\ 0 & 0 & M_0 \end{pmatrix} \qquad (3)$$

with

$$M_+ = \begin{pmatrix} \mu + \frac{1}{\sqrt{3}} g_s m_8 + g_A m_3 & (g_s - g_A) m^*_{67} \\ (g_s - g_A) m_{67} & \mu - \frac{1}{2\sqrt{3}} (g_s - 3g_A) m_8 + \frac{1}{2} (g_s + g_A) m_3 \end{pmatrix} ,$$

$$M_- = M_+ (g_A \leftrightarrow -g_A) ,$$

$$M_0 = \begin{pmatrix} \mu + \frac{1}{\sqrt{3}} g_s m_8 & \frac{1}{\sqrt{3}} g_s m_3 & -\frac{1}{2} (g_s - g_A) m^*_{67} & -\frac{1}{\sqrt{2}} (g_s + g_A) m_{67} & \sqrt{\frac{2}{3}} (g_s + g_2) m_3 \\ - & \mu - \frac{1}{\sqrt{3}} g_s m_8 & -\frac{1}{\sqrt{6}} (g_s + 3g_A) m^*_{67} & -\frac{1}{\sqrt{6}} (g_s - 3g_A) m_{67} & \sqrt{\frac{2}{3}} (g_s + 3g_2) m_8 \\ & & \mu - \frac{1}{2} (g_s + g_A) m_3 - \frac{1}{2\sqrt{3}} (g_s - 3g_A) m_8 & 0 & \frac{2}{\sqrt{3}} (g_s + 3g_2) m_{67} \\ & & & \mu - \frac{1}{2} (g_s - g_A) m_3 - \frac{1}{2\sqrt{3}} (g_s + 3g_A) m_8 & \frac{2}{\sqrt{3}} (g_s + 3g_2) m^*_{67} \\ & & & & \mu + \sqrt{6} (2g_2 + 3g_3) m_0 \end{pmatrix}$$

where $\mu = \sqrt{\frac{2}{3}} (g_s + 3g_1) m_0$ and $m_{67} = \frac{1}{2} (m_6 + i m_7)$. (4)

The order adopted for rows and columns is given by

$$M_+ : \frac{1}{\sqrt{2}} (\psi_1 + i\psi_2) ; \frac{1}{\sqrt{2}} (\psi_4 + i\psi_5) ,$$

$$M_- : \frac{1}{\sqrt{2}} (\psi_1 - i\psi_2) ; \frac{1}{\sqrt{2}} (\psi_4 - i\psi_5) ,$$

$$M_0 : \psi_3 ; \psi_8 ; \frac{1}{\sqrt{2}} (\psi_6 + i\psi_7) ; \frac{1}{\sqrt{2}} (\psi_6 - i\psi_7) ; \psi_0 .$$

The diagonalization of the neutral fermion matrix is more difficult than in the octet model. Again, we have showed with the aid of REDUCE[7], a symbolic manipulation program, that the m_{67} contribution can be included in m_3 and m_8 using the parameters

$$a = \frac{1}{2} g_s \left(m_3 + \frac{1}{\sqrt{3}} m_8 \right) \quad ,$$
$$b = g_s \left[\left(\frac{\sqrt{3}\, m_8 - m_3}{2} \right)^2 + 4 \mid m_{67} \mid^2 \right]^{1/2} . \tag{5}$$

This is a consequence of the Gourdin theorem[8] that we commented in I. Using the mnemonic notation of Fig. 1 this result implies that the 5 x 5 neutral fermion matrix is broken in two submatrices of 2 x 2 and 3 x 3 corresponding to the state K^0, $\overline{K}^0$ and π, η, η'. The K^0, $\overline{K}^0$ matrix is easily diagonalized, and we have:

a) For the diagonalized part:

$$m_{k+} = \mu + \frac{1}{2}(1+3\Gamma)a - \frac{1}{2}(1-\Gamma)b \ ,$$
$$m_{\pi+} = \mu + \frac{1}{2}(1+3\Gamma)a + \frac{1}{2}(1-\Gamma)b \ ,$$
$$m_{k-} = \mu + \frac{1}{2}(1-3\Gamma)a - \frac{1}{2}(1+\Gamma)b \ , \tag{6}$$
$$m_{\pi-} = \mu + \frac{1}{2}(1-3\Gamma)a + \frac{1}{2}(1+\Gamma)b \ ,$$
$$m_{k^0} = \mu - a + \Gamma b \ ,$$
$$m_{\overline{k}^0} = \mu - a - \Gamma b \ .$$

b) For the non-diagonalized submatrix, in order 3, 8, 0:

$$M_0' = \begin{pmatrix} \mu + \frac{1}{2}(a+b) & \frac{1}{2\sqrt{3}}(3a-b) & \frac{1}{\sqrt{3}}\gamma(3a-b) \\ \frac{1}{2\sqrt{3}}(3a-b) & \mu - \frac{1}{2}(a+b) & \gamma(a+b) \\ \frac{1}{\sqrt{3}}\gamma(3a-b) & \gamma(a+b) & \mu + \mu_0 \end{pmatrix} , \tag{7}$$

where

$$\Gamma = g_A/g_s \quad , \tag{8}$$
$$\gamma = \frac{1}{\sqrt{2}}(g_s + 3g_2) \tag{9}$$

and

$$\mu_0 = \sqrt{6}\ \ (2g_2+3g_3)\ m_0 \tag{10}$$

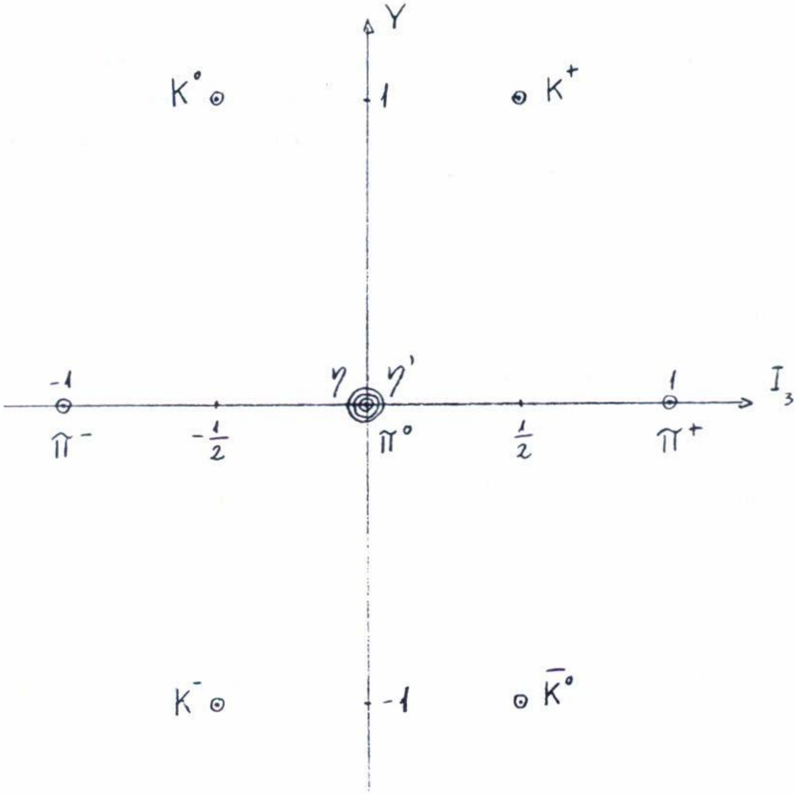

Fig. 1. Mnemonic of the classification for the nonet. The symbols π, K, η, η' are just to remind which states of the meson nonet are the corresponding states of the leptonic classification.

III. EXPERIMENTAL CONSTRAINTS.

We will now analyse the possibility of having in the same nonet all the known leptons. Their masses are(9):

$m_e \simeq 0.5$ MeV , $\qquad m_{\nu e} \simeq 0$ Mev ,

$m_\mu \simeq 105$ Mev , $\qquad m_{\nu\mu} \lesssim 0.5$ Mev,

$m_\tau \simeq 1800$ Mev , $\qquad m_{\nu_\tau} \lesssim 50$ Mev , besides, we know that no charged lepton has been observed up to about 18 GeV,

$$m_\lambda \gtrsim 15\ m_\tau \quad .$$

For the sake of mathematical simplicity, we will analyse the case in which the electron and muon masses can be neglected as compared to the τ mass,

$$m_e = m_\mu = m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \ ; \qquad (11)$$

the lifting of this condition for m_e and m_μ implies corrections to the spectra of all leptons other than 3,8 and 0.

All the conclusions obtained for the mass spectra in the octet case(1) for the peripherical leptons $(\pi^\pm, \kappa^\pm, \kappa^0, \overline{\kappa}^0)$ remain valid in the nonet case because, in this subspace, the two representations have the same mass structure, as was found in the last section. We summarize the results for this sector:

a) $m_{k^+} = m_{\pi^+} = 0$

we have:

$$\begin{cases} m_{k^-} = m_{\overline{k}^0} = -\ (3a+b) \\ m_{\pi^-} = m_{k^0} = -\ (3a-b) \end{cases} \quad , \quad \text{with } \Gamma = 1, \quad \mu = -2a \quad , \qquad (12)$$

or

$$\begin{cases} m_{\pi^-} = m_{k^-} = -3\Gamma a \\ m_{k^0} = m_{\overline{k}^0} = -\ \frac{3}{2}\ (1+\Gamma)a \end{cases} \quad , \qquad \text{with } b = 0 \ . \qquad (13)$$

b) $m_{k^+} = m_{k^-} = 0$

we have

$$\begin{cases} m_{\pi^+} = m_{\pi^-} = b \\ m_{k^0} = m_{\overline{k}^0} = \frac{1}{2}\ (b-3a) \end{cases} \quad , \qquad \text{with } \Gamma = 0 \ , \qquad (14)$$

or

$$\begin{cases} m_{\pi^+} = m_{\overline{k}^0} = -3(1+\Gamma)a \\ m_{\pi^-} = m_{k^0} = -3(1-\Gamma)a \end{cases} \quad , \quad \text{with } b = -3a, \quad \mu = -2a \quad . \qquad (15)$$

c) $m_{k+} = m_{\pi-} = 0$

we have

$$\begin{cases} m_{\pi+} = (1-\Gamma)b \\ m_{k^-} = -(1+\Gamma)b \\ m_{k^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} - \Gamma\right)b \\ m_{\overline{k}^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} + 3\Gamma\right)b \end{cases} \qquad \text{with } a = \frac{b}{3\Gamma}, \quad \mu = -\frac{1}{6\Gamma}(1+\Gamma^2)b \quad (16)$$

The remaining possibilities are given by

$$\begin{aligned} &d)\ m_{k-} = m_{\pi-} = 0, && \text{(solution (a) with } \Gamma \to -\Gamma), \\ &e)\ m_{\pi+} = m_{\pi-} = 0, && \text{(solution (b) with } b \to -b), \qquad (17) \\ &f)\ m_{k-} = m_{\pi+} = 0, && \text{(solution (c) with } \Gamma \to -\Gamma \text{ and } b \to -b) \end{aligned}$$

The possibility of having a massless neutral in this subspace is ruled out by the mentioned octet analysis(1), so the massless neutral leptons must be in the (π^0, η, η') subspace. Considering that it is well established that there are not two charged leptons with masses in the interval $(m_\tau - m_\mu) \lesssim m \lesssim (m_\tau + m_\mu)$, we conclude that the interesting possibilities are only:

$$\begin{aligned} 1)\quad & m_{k+} = m_{\pi+} = 0, \\ & m_{k^-} = m_{\overline{k}^0} = -(3a+b), \\ & m_{\pi^-} = m_{k^0} = -(3a-b), \qquad (18) \\ & \text{with } \Gamma = 1 \text{ and } \mu = -2a. \end{aligned}$$

$$\begin{aligned} 2)\quad & m_{k^+} = m_{k^-} = 0, \\ & m_{\pi^+} = m_{\overline{k}^0} = -3(1+\Gamma)a, \\ & m_{\pi^-} = m_{k^0} = -3(1-\Gamma)a, \qquad (19) \\ & \text{with } b = -3a \text{ and } \mu = -2a. \end{aligned}$$

3) $m_{k+} = m_{\pi-} = 0$

$$m_{\pi+} = (1-\Gamma)b$$

$$m_{k^-} = -(1+\Gamma)b \tag{20}$$

$$m_{k^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} - \Gamma\right)b$$

$$m_{\overline{k}^0} = -\frac{1}{2}\left(\frac{1}{\Gamma} + 3\Gamma\right)b$$

with $a = \frac{1}{3\Gamma}b$ and $\mu = -\frac{1}{6\Gamma}(1+3\Gamma^2)b.$

We analyse now the center of the nonet which corresponds to the SU(3) indices 3,8 and 0. These states are mixed according to Eq. (7) and must provide at least two massless neutrinos because, as was mentioned above, K^0 and $\overline{K}^0$ are necessarily massive. The charactheristic polynomial is

$$\det |M_0' - \lambda \mathbb{1}| = 0 \quad , \tag{21}$$

or

$$\xi^3 + 3p\,\xi^2 + 2q = 0 \quad , \tag{22}$$

where

$$\xi = \lambda - \mu - \frac{\mu_0}{3} \quad , \tag{23}$$

$$-p = (1+\gamma^2)t + \left(\frac{\mu_0}{3}\right)^2 \quad , \tag{24}$$

$$-q = \left(\frac{\mu_0}{3}\right)^3 + 2(\gamma^2-1)\left(\frac{\mu_0}{3}\right)t + 2\gamma^2 sa \quad ; \tag{25}$$

$$t = a^2 + \frac{1}{3}b^2 \geq 0 \tag{26}$$

and

$$s = a^2 - b^2 \quad . \tag{27}$$

The condition to have three real solutions, two of them equal, is that the discriminant:

$$D \equiv q^2 + p^3 = 0 \ , \tag{28}$$

with $q^2 \neq 0$. Since at least two neutrinos are massless one gets (for $q^{1/3} > 0$ (<0) upper (lower) signs):

$$\xi_1 = \xi_2 = \pm q^{1/3} = \pm\sqrt{-p} \quad ,$$

$$\xi_3 = \mp 2q^{1/3} = \mp 2\sqrt{-p} \quad , \tag{29}$$

or by Eq. (13)

$$\lambda_1 = \lambda_2 = 0 \ ,$$

$$\lambda_3 = 3\mu + \mu_0 \ . \tag{30}$$

Notice that if q=0 all roots are zero, but then p=0 and the form of p in Eq. (24) implies a=b= μ_0= 0 and therefore every mass in the multiplet is zero. From Eqs. (23), (29) and (30), we get

$$q = -\left(\mu + \frac{\mu_0}{3}\right)^3 \quad , \quad p = -\left(\mu + \frac{\mu_0}{3}\right)^2 \quad , \tag{31}$$

which using Eqs. (24) and (25) imply

$$4\gamma^2 = \frac{3\mu + 2\mu\mu_0}{t} - 1 \tag{32}$$

and using this,

$$\left[\frac{\mu^2+t}{2} - \mu\left(\frac{as}{t}\right)\right]\mu_0 = -\left[\mu^3 + \frac{t-3\mu^2}{2}\left(\frac{as}{t}\right)\right]. \tag{33}$$

The last two relations must be satisfied by the parameters a, b, μ, μ_0 and γ in order to have two massless neutral states in the center of the nonet. We should now check whether or not these conditions are compatible with those arising from the periphery of the nonet.

Condition 1, in Eq. (18), requires Γ = 1 and μ = -2a for arbitrary b. When μ = -2a is replaced in Eq. (33) we get

$$\mu_0 = 3a \quad , \tag{34}$$

that in turn gives

$$4\gamma^2 = -1 \quad , \tag{35}$$

when Eq. (32) is used. This last result means that CP is violated in the lepton sector and therefore solution 1 is not very attractive physically.

Solution 2 in Eq. (29) also needs μ = -2a, but now b = -3a and Γ is arbitrary. These values for μ and b make the square brackets in Eq. (33)

equal to zero. Thus, μ_0 is not determined in the limit m_e and m_μ equal to zero. In order to give a physical meaning to this limit we have two possibilities. First one can define μ_0 through the expresions $\mu = -2a + \delta$ and $b = -3a + \varepsilon$, where δ and ε are fixed by the physical values of the electron and muon masses in the equivalent expresions to Eq. (29) for m_{k^+} and $m_{k^-} \neq 0$. In this case

$$\mu_0 = 3a$$

and

$$4\gamma^2 = -1 ,$$

which, as in Solution 1, implies a CP violating phase in the lagrangian. The other possibility is that the lagrangian in zero order of perturbation theory gives massless electron and muon. Then μ_0 is really arbitrary and γ^2 can be greater than zero. Because this last situation is the only one acceptable within this framework, it is convenient to study in greater detail the consequences of the vanishing of the expressions in squared brackets of Eq. (33). We have

$$\mu^2 + t - 2\mu \frac{as}{t} = 0 , \tag{36}$$

and

$$2\mu^3 - (3\mu^2 - t) \frac{as}{t} = 0. \tag{37}$$

Solving the first equation for μ gives

$$\mu = \frac{as}{t} \pm \sqrt{\left(\frac{as}{t}\right)^2 - t} , \tag{38}$$

and substitution of μ^2 from Eq. (36) into Eq. (37) until μ^2 does not appear implies

$$\mu \left[\left(\frac{as}{t}\right)^2 - t\right] = 0 . \tag{39}$$

The last two equations require any of the following configurations:

i) $b = 0$ with $\mu = a$,

ii) $b = -3a$ with $\mu = -2a$,

iii) $b = 3a$ with $\mu = -2a$;

i) leads to an incompatible spectra for the leptons. ii) and iii) are precisely the solutions b) and e) in Eq. (15) and (17).

The last solution 3) can be analysed along the same lines; μ_0 here is well determined for $\mu = -\frac{1+3\Gamma^2}{2}a$ and $b = 3\Gamma a$ (except for $\Gamma = \pm 1$, for which solutions 2 and 3 coincide), and use of these in Eqs. (32) and (33) gives

$$\mu_0 = 3\Gamma^2 a \tag{40}$$

and

$$4\gamma^2 = -\frac{1}{4}(3\Gamma^2 + 1) \quad . \tag{41}$$

The last expresion is negative and we obtain once more a CP violating theory.

So, we are lead to a very restricted mass spectra in the case 2 which corresponds to configurations (b) and (e). Electron and muon are massless and their physical masses should be accounted by higher orders of perturbation theory. Their neutrinos are prevented to acquire renormalized masses because of the V - A nature of their main interaction. A neutral partner of the τ lepton is predicted; the mass diference between τ and its neutral is of order m_μ. For the periphery leptons we have two parameters, a and Γ; the scale of a is set by the τ mass

$$|a| = \frac{m_\tau}{3(1-|\Gamma|)}$$

and the lower bound in the mass of the λ gives a bound for Γ:

$$.8 < |\Gamma| < 1.2 \quad .$$

The mass of the remaining neutral lepton is

$$m = -4(1+\gamma^2)a \tag{42}$$

and therefore we have a lower bound of

$$|m| > 4|a| \simeq 6\, m_\tau$$

for this lepton.

Summarizing, we obtain two solutions that in lowest order in perturbation theory correspond to zero mass for electron and muon; the SU(3) states to which these particles correspond are either K^+ and K^- in solution b, or π^+ and π^- in solution c. The other charged states are at 1.8 GeV, for the τ, and a new state, λ, that can easily be accomodated above 18 GeV. There are three massive neutral leptons; one is necessarily at about the mass of the τ plus or minus the μ mass. The other two neutrals are, one at the λ mass; the other has a lower bound of $\frac{2}{9}\,|m_\lambda| + \frac{1}{3}\,|m_\tau|$.

IV. EXTENSION TO QUARKS

It is clear that this set of ideas can be extended to the quark sector. The motivation for doing so is the evident paralelism of the quark and lepton spectra. This has been exploited in the past and is indeed one of the supports for the concepts of generations[10].

The extension of the scheme for quarks that we propose is a direct one; we simply assume that the quarks also form an octet or nonet. Because the current quark masses are not so well defined, the experimental constraints are not so strong and we cannot decide at present whether or not the octet is excluded.

However, two cualitative predictions naturally arise in this scheme. Namely, that there should be no-top quark of electric charge 2/3 and that there should be two -4/3 quarks. This is so because we already know that there are at least three -1/3 quarks -the d, s and b quarks- and the only place they can fit in either the octet or nonet, is in the K^0, $\overline{K}^0$, π , η ,η' sector. This means, of course, that the octet (nonet) must be displaced as in Fig. 2. This displacement leaves π^+ and K^+ with 2/3 charges and π^-, K^- with -4/3 charges. We can also say that a symmetry breaking of the type (a) should be preferred by the quarks because we know that the 2/3 charged quarks, the μ and c, are much lighter than the -4/3 partners in the supermultiplet.

V. FINAL REMARKS

In this work we have shown that the lepton spectrum is compatible with an SU(3) nonet scheme in which the symmetry is broken spontaneously "a la" Higgs. The most remarkable prediction is the existence of a neutral heavy lepton at about the τ mass. Another charged lepton, λ , is predicted and its mass can be accommodated above 26 GeV. Two other neutral leptons are expected, one near the λ mass and another with a lower bound $\left(\frac{2}{9}\, m_\lambda + \frac{1}{3}\, m_\tau\right)$.

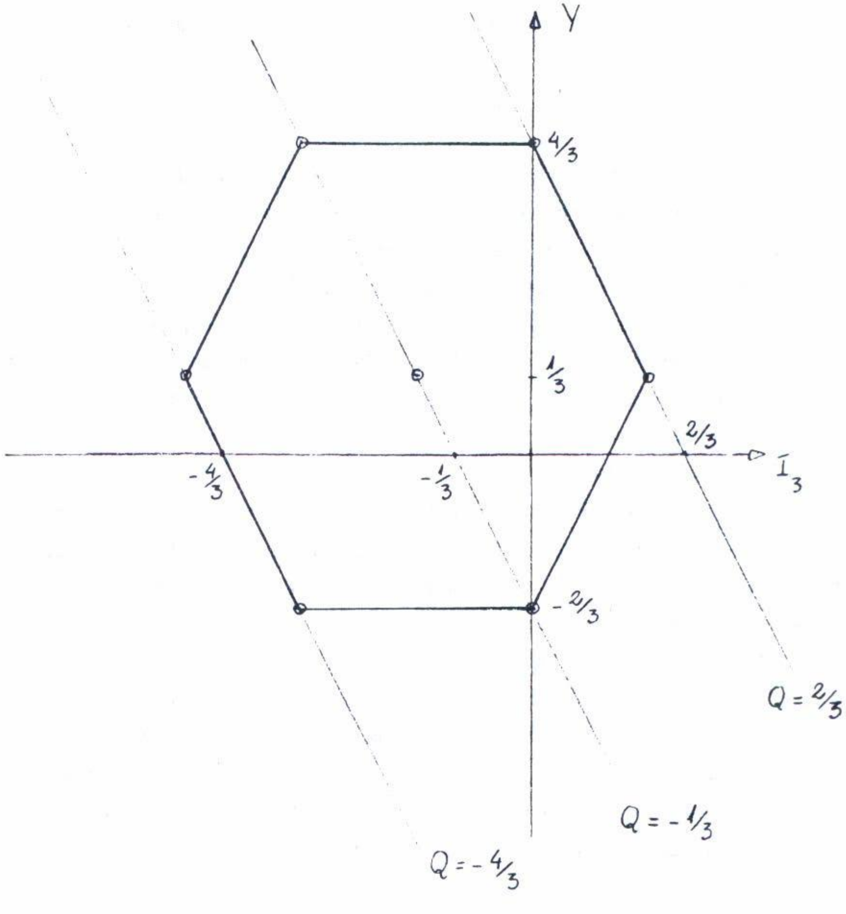

Fig. 2. An octet or nonet assignation for quarks, corresponding to Y = 4/3, 1/3, -2/3.

The extension of this model to quarks predicts that there should not be a top quark of charge 2/3, but there should be two -4/3 charge quarks. This extension is based on the similarity of the quark and lepton spectra, and must be taken with caution because it may prove impossible to make the theory renormalizable, due to the triangle anomalies that are not obviously canceled.

In a forthcoming work we will analyse the gauge boson sector of the theory. This is a real test for this scheme because it has to meet lots of constraints of theoretical and experimental character. In particular, it must explain e, μ and τ number conservation, suppression of neutral strangeness changing currents, absence of anomalies, and should explain why the neutral lepton at the tau mass has not been observed. It should, finally, give rise to adequate mass corrections for electron and muon.

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