THE ZERO-POINT TERM IN CAVITY RADIATION. II

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ABSTRACT

Following the same line of approach established in a previous work[6] we derive, using heuristic arguments, Planck's distribution law for cavity radiation employing classical fluctuation theory for the fluctuating zero-point field with a power spectrum proportional to the cube of the frequency. To do this we make no use of the interaction between particles and the cavity walls.

RESUMEN

Continuando con la misma línea de razonamiento establecida en un trabajo previo[6] derivamos, usando argumentos heurísticos, la ley de distribución de Planck para radiación de cavidad, empleando la teoría clásica de fluctuaciones para el campo de punto cero fluctuante con un espectro proporcional al cubo de la frecuencia. Para ello no consideramos la interacción entre las partículas y las paredes de la cavidad.

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I. INTRODUCTION

It has been shown recently[1] that the motion of classical charged particles in a random electromagnetic field has properties commonly associated with a quantum behaviour. This kind of motion depends crucially on the use of a radiation spectrum which at zero temperature, is proportional to ω^3. It must be emphasized that, unlike some points of view on the vacuum field of quantum electrodynamics, this is a real field[2] arising from the incoherent superposition of the radiation emitted by all the charges in the universe. In this case we can analyze some phenomena arising from the electromagnetic field fluctuations which subsist even at $T=0$. Bearing this in mind, Einstein's work in the early years of this century, concerning the analysis of the equilibrium conditions for the radiation inside a cavity (blackbody radiation), may be focussed from a new point of view. Some years ago, in a particularly interesting and suggestive work, Boyer[3] analyzed Einstein and Hopf's 1909 work[4]. In that paper Boyer took into account both the fluctuations of the background field and the particle collisions against the cavity walls to derive Planck's distribution law. In such derivation, the role played by the cavity walls is fundamental for the equilibrium to be established since, when the particle hits the wall, it dissipates the absorbed energy from the zero-point field. The proposed theory seems to predict that, in the absence of collisions, the particles would increase their energy indefinitely[5]. However, given the independence of the distribution law on the cavity size, in a recent work[6], hereafter to be referred to as I, some doubts have been raised about the need of introducing cavity walls in order to reach equilibrium. In I, we started with the Einstein and Stern's 1913 analysis[7] and were able to derive Planck's law using only the hypothesis of the existence of the ω^3 spectrum for the background field. This paper is devoted to a discussion of some further arguments in that direction.

In the following section we derive Planck's distribution law using fluctuation theory. In section III we discuss the third law of thermodynamics. In the next section we discuss the discrete energy levels, and in section V we finish with some remarks.

II. DERIVATION OF PLANCK'S LAW USING THE THEORY OF FLUCTUATIONS

As it was pointed out above, when analyzing in I Boyer's derivation[3] of Planck's law, some questions arised on the true relevance of the particle collisions against the cavity walls as a dissipation mechanism by means of which it would be possible to reach an equilibrium state. This was mainly due to two reasons: first, in such derivation the cavity size does not appear anywhere, and second, Boyer supposes that momentum exchange during collisions does not depend on temperature. As it will be shown here, it is possible to derive Planck's law not only avoiding the use of the cavity walls, but also without using the particular model implied in the above mentioned derivation: a dipole oscillating harmonically. This also eliminates the tiresome calculations of R and $\langle \Delta^2 \rangle$ *. In order to do this, we shall use both the existence of the zero-point energy fluctuations with a spectrum proportional to ω^3 and the results of the theory of fluctuations, which can be found in any text book[8]. It is important to recall that fluctuation theory date back to the works of Einstein[9] and Gibbs[10]. There exist other attempts in this direction[11], but our approach has the advantages of simplicity and brevity.

Let us consider radiation in thermal equilibrium. For this analysis it is sufficient to know the statistical properties of the radiation field, i.e., of $C(k,\lambda)$ in the equation for the electric field $\vec{E}$, namely

$$\vec{E} = \mathrm{Re} \sum_{\lambda=1}^{2} \int d^3k \hat{\varepsilon}(k,\lambda)\, C(k,\lambda) e^{i(\bar{k}\cdot\bar{r}-\omega t)} \quad . \tag{1}$$

Since Maxwell's equations do not couple the amplitudes of the fields, we can consider them as being statistically independent. Furthermore, if we take into account that $C(k,\lambda)$ is a random variable formed by the incoherent superposition of many waves produced by all charges in the universe, use can be made of the central limit theorem of probability theory to argue that the distribution of $C(k,\lambda)$ is a gaussian one[12]. As a consequence, the canonical field variables q and p are also gaussian.

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* where $\langle \Delta^2 \rangle$ are the dipole momentum fluctuations due to the field fluctuations and R is the coefficient in the dissipative force.

We shall also assume that the fluctuations of the zero-point field are independent of the fluctuations of the thermal field.

According to the well-known result of the theory of fluctuations (see appendix I)

$$\frac{d\langle U\rangle}{d\beta} = -\sigma^2 \quad , \tag{2}$$

where $\beta^{-1} = kT$, $\langle U\rangle$ is the mean energy and σ^2 the dispersion:

$$\sigma^2 = \left(\langle U^2\rangle - \langle U\rangle^2 \right) \quad . \tag{3}$$

We can rewrite the dispersion σ^2 in terms of $\langle U\rangle$, using the fact that we are dealing with a gaussian distribution for the field canonical variables, and thus obtain from Eq. (2) a differential equation for $\langle U\rangle$. The procedure may be outlined as follows:

Equation (3) in terms of the field variables q and p takes the form

$$\sigma^2 = \left(\langle \frac{1}{4}(p^2 + \omega^2q^2)^2 \rangle - \langle \frac{1}{2}(p^2 + \omega^2q^2)\rangle^2 \right) \quad . \tag{4}$$

For a gaussian distribution,

$$\langle p^4\rangle = 3\langle p^2\rangle^2 \qquad \langle q^4\rangle = 3\langle q^2\rangle^2 \quad , \tag{5}$$

but on the other hand, since we are dealing with the oscillators of the electromagnetic field,

$$\langle p^2\rangle = \omega^2\langle q^2\rangle \quad . \tag{6}$$

Also, since q and p are independent variables

$$\langle p^2q^2\rangle = \langle p^2\rangle\langle q^2\rangle \quad . \tag{7}$$

Substituting Eqs. (5) to (7) into Eq. (4), we have

$$\sigma^2 = \langle \frac{1}{2}(p^2 + \omega^2q^2)\rangle^2$$

$$= \langle U\rangle^2 \quad . \tag{8}$$

The above result allows to transform Eq. (2) into the differential equation

$$-\frac{d\langle U\rangle}{d\beta} = \langle U\rangle^2 \quad , \tag{9}$$

whose solution is

$$\langle U\rangle = \frac{1}{\beta} \quad , \tag{10}$$

i.e., the Rayleigh-Jeans law.

Analyzing this situation in a similar way as that we did in I, we can see that the left-hand side of Eq. (2) is temperature-dependent. If one accepts the existence of an energy $T = 0$, then the non-thermal component involved in is eliminated by deriving it with respect to β. Nevertheless, the right-hand side of Eq. (2) still contains the part of $\langle U\rangle$ independent of temperature. It would seem then, that Eq. (2) is inconsistent. However, on the basis of the generality under which it was derived, we could expect its validity would remain for thermal fluctuations (see appendix I), i.e.,

$$-\frac{d\langle U\rangle}{d\beta} = \sigma_T^2 \quad , \tag{11}$$

where σ_T^2 is

$$\sigma_T^2 = \sigma^2 - \sigma_{T=0}^2 \quad , \tag{12}$$

since we have assumed that the zero-point field fluctuations and those corresponding to the thermal field are independent. Thus, using the result obtained in Eq. (8) and the one for the zero-point fluctuations, one obtains

$$\sigma_T^2 = \langle U\rangle^2 - \langle U\rangle_{T=0}^2 \quad ; \tag{13}$$

and, using this dispersion in Eq. (11), one gets that

$$-\frac{d\langle U\rangle}{d\beta} = \langle U\rangle^2 - \langle U\rangle_{T=0}^2 \quad , \tag{14}$$

which can be readily integrated, and since

$$\lim_{\beta\to\infty} \langle U \rangle = \langle U \rangle_{T=0} \tag{15}$$

then, the solution is

$$\langle U \rangle = \frac{2\langle U \rangle_{T=0}}{e^{2\beta\langle U \rangle_{T=0}} - 1} + \langle U \rangle_{T=0} \quad . \tag{16}$$

Since Wien's law establishes that

$$\langle U \rangle = \omega\psi\left(\frac{\omega}{T}\right) \quad . \tag{17}$$

Then,

$$\langle U \rangle_{T=0} = \text{constant } \omega \quad , \tag{18}$$

must be satisfied. This is in full agreement with the assumed spectrum. In the constant in Eq. (18) is taken to be

$$\text{constant} = \frac{1}{2}\hbar \quad , \tag{19}$$

then Eq. (16) is, in fact, Planck's distribution law. As it can be seen, the constant fixes the fluctuations scale at zero temperature.

It is important to point out that in the foregoing derivation no use has been made of the cavity containing the radiation.

III. THE RELATIONSHIP BETWEEN THE BACKGROUND FIELD AND THE THIRD LAW OF THERMODYNAMICS

The analysis performed in the preceeding section shows that it is possible to obtain Planck's distribution law by applying classical fluctuation theory to the zero-point fluctuating field. With the same ideas it is possible to show that the zero-point field is in agreement with the third law of thermodynamics. For this purpose let us again begin with the hypothesis of the existence of a fluctuating zero-point energy. Since at $T=0$ this would be the only available energy, it follows then that

$$\lim_{T\to 0} F = \langle U\rangle_{T=0} \quad , \tag{20}$$

where F is the Helmholtz free energy. On the other hand, we know from thermodynamics that if U is the internal energy

$$-\frac{\partial \beta F}{\partial \beta} = U \quad , \tag{21}$$

so identifying $\langle U\rangle$ with U and substituting the value of $\langle U\rangle$ given in Eq. (16) and the condition expressed in Eq. (20) into Eq. (21), one obtains

$$F = \frac{1}{\beta}\, \ell n\,(2 \text{ senh } \beta \langle U\rangle_{T=0}) \quad . \tag{22}$$

Since

$$S = -\frac{\partial F}{\partial T} \quad , \tag{23}$$

we obtain

$$S = k\,(\beta \langle U\rangle - \ell n\, |2 \text{ senh } \beta \langle U\rangle_{T=0}|) \quad , \tag{24}$$

which satisfies the condition

$$\lim_{T\to 0} S\,(U) = 0 \tag{25}$$

that is in agreement with the third law of thermodynamics.

It must be emphasized that in order to obtain the above result, use has been made, of the central limit theorem. Given the generality of these assumptions, we can be confident on the generality of the result expressed by Eq. (25). From this point of view, the mere existence of a fluctuating zero-point field leads to a result consistent with the third law of thermodynamics without using quantum mechanical arguments.

On the other hand, we must note that the third law of thermodynamics implies that, if u is the energy density, then

$$\lim_{T\to 0} \frac{\partial u}{\partial T} = 0 \quad . \tag{26}$$

As it was shown in I, this leads to the conclusion that the arbitrary function in Wien's law tends to be a constant when $T = 0$. As this in turn implies the existence of the zero-point energy, we could then use this close relationship to begin a course on Quantum Mechanics in the way desired by Planck: starting from thermodynamics.

IV. DISCRETE ENERGY LEVELS

A question which may arise is: How do discrete energy levels appear? To analyze this question, let us consider the following:

Since we know that

$$-\beta F = \ell n\, Z \quad , \tag{27}$$

where Z is the partition function, and using for F the value given by Eq. (22), one obtains

$$\begin{aligned} Z &= \frac{1}{2\ \mathrm{senh}\ \beta \langle U \rangle_{T=0}} \\ &= \frac{e^{-\beta \langle U \rangle_{T=0}}}{1 - e^{-2\beta \langle U \rangle_{T=0}}} \quad , \end{aligned} \tag{28}$$

but, if $a > 0$ and $x < 0$,

$$\sum_{\ell=0}^{\infty} a^{\ell x} = \frac{1}{1 - a^x} \tag{29}$$

it is satisfied. Thus, Eq. (28) is transformed into

$$\begin{aligned} Z &= e^{-\beta U_{T=0}} \sum_{\ell=0}^{\infty} e^{-2\beta\ell \langle U \rangle_{T=0}} \\ &= \sum_{\ell=0}^{\infty} e^{-2\beta \langle U \rangle_{T=0}\ (\ell+\frac{1}{2})} \quad . \end{aligned} \tag{30}$$

We must recall that if we use the traditional expression

$$Z = \int e^{-\beta U} \, dU \quad , \tag{31}$$

we obtain Eq. (10) rather than Eq. (16). Then in order to be consistent with the result obtained for Z, we must write

$$Z = \int f(U) \, e^{-\beta U} \, dU \quad , \tag{32}$$

where F(U) represents a density of states with energy U. Comparing Eqs. (11) and (30), it results evident that

$$f(U) = \sum_{\ell=0}^{\infty} \delta(U - 2\langle U\rangle_{T=0} \, (\ell + \tfrac{1}{2})) \quad , \tag{33}$$

that is, the density of states is equal to zero for any value which is not in accordance with the discrete levels given in equation

$$U = 2\langle U\rangle_{T=0} \, (\ell + \tfrac{1}{2}) \quad . \tag{34}$$

V. CONCLUSIONS

We have seen how some of the ideas analyzed by Einstein around 1913 can be modified in order to obtain Planck's distribution law. To do this, it is enough to take into account the existence of the fluctuating zero-point field. Such a field, unlike that of quantum electrodynamics as usually interpreted, is a real field which represents the incoherent sum of the radiation produced by all charges in the universe and subsisting even at $T=0$. Despite the idea of the zero-point energy dates back to the works of Einstein-Stern[7] and Nernst[13] where they analyzed Planck's second theory[14], such an idea was abandoned until Braffort et al.[15] and Marshall[16] provided it with new perspectives in the field presently known as Stochastic Electrodynamics. Both in paper I and in the present one we have seen how we can obtain Planck's distribution law from the existence of a fluctuating zero-point field and the laws of Classical Physics. This conclusion shows that this approach deserves a rigorous study and, from our point of view, a deeper understanding of the new theory: the Stochastic Electrodynamics.

APPENDIX I

Here we recall the derivation of Eq. (2) in order to appreciate its generality. The procedure is as follows:

Let us consider a system with entropy S and energy U. The probability that such a system may be out of equilibrium, due to a fluctuation, can be determined from the entropy change that this would imply since the probability is proportional to the number of states compatible with the fluctuation.

According to the famous Boltzmann's equation,

$$S = k \,\ell n\, \Omega \quad , \tag{A1}$$

the probability that a fluctuation might occur is

$$\text{Prob} \;\alpha\; d\,\Omega = e^{\Delta S/k} \quad . \tag{A2}$$

To second order ΔS is given by

$$S(U) - S(U)_{eq} = \left(\frac{\partial S}{\partial U}\right)_{eq} \Delta U + \frac{1}{2}\left(\frac{\partial^2 S}{\partial U^2}\right)_{eq} (\Delta U)^2 \quad , \tag{A3}$$

but since, at equilibrium the entropy is a maximum

$$\left(\frac{\partial S}{\partial U}\right)_{eq} = 0 \qquad\qquad \left(\frac{\partial^2 S}{\partial U^2}\right) < 0 \tag{A4}$$

and therefore

$$\Delta S = \frac{1}{2}\left(\frac{\partial^2 S}{\partial U^2}\right)_{eq} (\Delta U)^2 \quad . \tag{A5}$$

With this, Eq. (A2) is transformed into the gaussian distribution

$$d\,\Omega = e^{\frac{1}{2k}\left(\frac{\partial^2 S}{\partial U^2}\right)_{eq} (\Delta U)^2} \quad , \tag{A6}$$

whose dispersion, given by

$$\sigma_U^2 = \frac{k}{\left(\frac{\partial^2 S}{\partial U^2}\right)_{eq}} \quad , \tag{A7}$$

gives, precisely, the energy fluctuations. If $U_{eq} = \langle U \rangle$, then, using

$$\frac{\partial S}{\partial \langle U \rangle} = \frac{1}{T} \qquad \text{A8}$$

we can express (A7) as

$$\sigma_U^2 = - \frac{d\langle U \rangle}{d\beta} \quad , \qquad \text{A9}$$

which is, in fact, Eq. (2).

A more general formulation of fluctuation theory can be found in Kestin's book[8].

We shall now analyze the consequences of the existence of the background field on the above deduction. As it was pointed out, this background field gives rise to a fluctuating zero-point energy depending only on frequency. Therefore, an energy-dependent variable already includes the zero-point energy; but, since this latter is a constant, its derivative with respect to U and the energy differences above considered are transformed, respectively, into

$$\frac{\partial}{\partial U} = \frac{\partial}{\partial U_T} \qquad \text{A10}$$

and

$$\Delta U = U - U_{eq} = \Delta U_T \qquad \text{A11}$$

unless changes in ω are considered. Taking into account these two latter equations, the expression given by Eq. (A6) is transformed into

$$d\,\Omega = e^{\frac{1}{2k}\left(\frac{\partial^2 S}{\partial U_T^2}\right)(\Delta U_T)^2} \quad , \qquad \text{A12}$$

where, instead of Eq. (A7), we obtain

$$\sigma_T^2 = k\left[\left(\frac{\partial^2 S}{\partial U_T^2}\right)\right]^{-1} \quad , \qquad \text{A13}$$

that is, an expression for the thermal fluctuations. If we use

$$\frac{\partial S}{\partial U_T} = \frac{1}{T} \quad . \tag{A14}$$

we will have

$$\sigma_T^2 = -\left(\frac{dU_T}{d\beta}\right)_{eq} \tag{A15}$$

or, if it is desired, using

$$\frac{dU_{T=0}}{d\beta} = 0 \tag{A16}$$

one obtains

$$\sigma_T^2 = -\left(\frac{dU}{d\beta}\right)_{eq} \quad . \tag{A17}$$

If, in addition, $U_{eq} = \langle U \rangle$ we obtain Eq. (11)

$$\sigma_T^2 = -\frac{d\langle U \rangle}{d\beta} \quad .$$

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