

ANALYSIS OF THE EIGHT BEST-MEASURED SUPERALLOWED FERMI β -DECAYS

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ABSTRACT

The eight best-measured superallowed $0^+ - 0^+$ beta transitions are studied. The end-point total energies, W_0 , and the half-lives, t_0 , are determined from a new compilation of available experimental data. In order to verify the predictions of the conserved vector current theory, the $f't$ -values are analyzed in conjunction with both microscopic and phenomenological approaches for the isospin impurity corrections δ_C . A good internal consistency of the Ft -values is found overcoming some difficulties of a previous study of Wilkinson. The effective vector coupling constant is determined to be $G_V' = (1.41208 \pm 0.00032) \times 10^{-49}$ in cgs units. An upper limit of order 10^{-3} is fixed on the strength f_S of the induced scalar interaction. In addition, a value of the Fierz interference term $b_F = (1.1 \pm 1.3) \times 10^{-3}$, is determined.

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En este trabajo se estudian las ocho transiciones beta superpermitidas $0^+ - 0^+$ de las cuales se han publicado los mejores datos experimentales. De la compilación de los mismos se determinan los valores de las energías máximas W_0 y de las vidas medias t_0 . Se analizan los valores $f't$ utilizando conjuntamente las aproximaciones microscópica y fenomenológica para las correcciones por impurezas del isoespín δ_c , con el fin de verificar las predicciones de la teoría de la conservación de la corriente vectorial. Salvando algunas dificultades halladas en un estudio previo realizado por Wilkinson, se encuentra una buena consistencia para los valores $f't$. Se calcula la constante efectiva del acoplamiento vectorial obteniéndose el valor $G_V^1 = (1.41208 \pm 0.00032) \times 10^{-49}$ en el sistema de unidades cgs. En lo referente a la magnitud f_S de la interacción escalar inducida se propone un límite superior del orden de 10^{-3} . Además, se obtiene un valor para el término de interferencia de Fierz, $b_F = (1.1 \pm 1.3) \times 10^{-3}$, el cual aventaja a la mejor cota existente, calculada por Hardy y Towner.

1. INTRODUCTION

The polar vector component of the weak current is composed of three different terms, which correspond to the vector, the induced weak-magnetic and the induced scalar interaction characterized by the coupling constants f_V , f_M and f_S , respectively. The conserved vector current (CVC) theory gives precise values for these coupling constants, i.e., $f_V = 1$, $f_M = 3.706/2M = 1.008 \times 10^{-3}$ (M being the nucleon mass in units of mc^2) and $f_S = 0$. Considerable effort has been devoted to verify experimentally these theoretical predictions (Ref. 1 and the more recent Refs. 2-15). Experimental information about the coupling constants f_V and f_S can be obtained from the study of ft -values of $0^+ - 0^+$ superallowed β -decays⁽²⁻¹²⁾. Therefore a good knowledge of such a kind of transitions becomes very important.

Let us summarize the equations which relate the quantities relevant to superallowed Fermi transitions. The ft -values obey⁽¹⁰⁾

$$f't = \frac{K}{G_B^1{}^2 ({}^V F_{000}^0)^2} = \frac{K}{2G_V^1{}^2 (1 - \delta_c)} \quad (1)$$

where K is a combination of physical constants (Ref. 4) which reduces to $K = 1.230618 \times 10^{-94} \text{ erg}^2 \text{ cm}^6 \text{ s}$. The quantities appearing on the l.h.s. of (1) are the corrected integrated statistical rate function

$$f' = \overline{f C_\beta(W)} [1 + \overline{\delta_R(W)}] \quad (2)$$

and the partial half-life

$$t = \frac{t_0}{BR} \left(1 + \frac{\varepsilon}{\beta^+} \right), \quad (3)$$

where f is the integrated statistical rate function, $\overline{C_\beta(W)}$ is the shape factor including second order corrections averaged over the energies W of the positron spectrum as indicated by the bar, $\overline{\delta_R(W)}$ is the "outer" model-independent radiative corrections of order α , $Z\alpha^2$ and $Z^2\alpha^3$ (α being the fine structure constant and Z the nuclear charge of the daughter nucleus), t_0 is the half-life of the initial state, BR stands for the branching of the superallowed β -transition, and ε/β^+ indicates the electron-capture-to-positron-decay ratio.

Besides, in (1) use was made of the relation between the form factor coefficients (FFC) and the nuclear matrix elements (NME). Thus for the Fermi FFC V_{F000}^0 was adopted, setting $f_S = 0$, the usual expression

$$(V_{F000}^0)^2 = f_V^2 (M_{000}^0)^2 = 2f_V^2 (1 - \delta_c), \quad (4)$$

where δ_c accounts for the mismatch of the nuclear states involved in the transition due to isospin impurities. Finally, we have the dressed coupling constant of the polar vector current G_V' given by

$$G_V' = f_V G_\beta' = f_V G_\beta (1 + \Delta_R)^{1/2}, \quad (5)$$

where G_β is the bare weak coupling constant for nuclear β -decay and Δ_R is the "inner" model-dependent radiative correction.

It is usual to define the "corrected effective" Ft -values:

$$Ft = f't(1 - \delta_c) = \frac{K}{2G_V'^2}. \quad (6)$$

If CVC theory holds, i.e., $f_V = 1$, the Ft -values should be constant from nucleus to nucleus.

Two different ways of tackling the problem of the evaluation of the nuclear-model dependent isospin impurity correction δ_c can be found in the recent literature. One is a microscopic approach, where the charge-dependent effects are taken into account by direct computation. Actually, for the microscopic calculation, δ_c is splitted into two parts: the one-body δ_{c1} and the two-body δ_{c2} . Several evaluations of δ_{c1} have been reported⁽¹⁶⁻²⁰⁾ while for δ_{c2} the only values available are those obtained

by Towner and Hardy⁽²¹⁾. The second procedure is a phenomenological approach to δ_c suggested by Wilkinson^(2,22).

The most up-to-date surveys of the eight best-measured $0^+ - 0^+$ superallowed transitions are those published by Vonach *et al.*⁽⁵⁾, Towner and Hardy⁽⁶⁾ and Wilkinson, Gallmann and Alburger⁽⁷⁾. It should be pointed out that different criteria have been adopted in Refs. 5-7 to select the end-point total energy W_0 and the half-life t_0 corresponding to each decay from available data. The differences have been described in detail in a previous paper⁽¹⁰⁾. In short, we can mention that the technique used in Ref. 7 was mainly based on the most recent and precise measurements, whilst the procedures used in Refs. 5 and 6 were not so restrictive.

The authors of papers Refs. 5-7 have also calculated $f't$ -values using their own recommended values of W_0 and t_0 . The results of the analysis of the internal consistency of those sets of $f't$ -values in conjunction with the current knowledge of δ_c are summarized in Table 3 of Ref. 10. The sets of Ft -values calculated using the sets of $f't$ -values report in Refs. 5 and 6 and the isospin impurity corrections, with δ_{c1} estimated adopting harmonic oscillator wave functions, are in fair agreement with the CVC theory. However, since after publication of Refs. 5 and 6 new precise measurements became available those surveys of data require to be completed. On the other hand, no set of Ft -values calculated using the set of $f't$ -values published in Ref. 7 and any of the available estimations for δ_{c1} agrees with the CVC theory. In all these cases the Ft -values increase noticeably with Z and, moreover, the χ^2/ν of the fits are much more larger than unity. On the basis of these latter results one might be tempted to think of that the more recent data of W_0 and t_0 have destroyed the consistency shown by the $f't$ -values derived in the earlier compilations 5, 6 and 22. Therefore a new complete survey of data becomes necessary in order to clear up the present situation.

In view of the current state of this matter the aim of the present work is two-fold. On the one hand, we investigated whether a consistent set of $f't$ -values is obtained when one takes into account all statistically acceptable data. For this purpose we performed new compilations of experimental data of the eight best-measured transitions in order to select the W_0 and t_0 appropriate for each decay. The criteria adopted and the results obtained are reported in section 2. Subsequently,

in section 3, we study the internal consistency of the $f't$ -values. On the other hand, we complete a study of the influence of the relativistic FFC on the determination of the strength f_g reported in a letter⁽¹²⁾. This part of the study is provided in section 4, where we also set out a new limit on the Fierz interference term. Finally, section 5 is devoted to a summary.

2. DETERMINATION OF W_0 AND t_0 FOR EACH DECAY

To build up the survey of W_0 and t_0 reported in the present work we followed the method recommended by the Particle Data Group⁽²³⁾ and summarized by Vonach *et al.*⁽⁵⁾. The use of such a procedure yielded the results for the decay energies and the total half-lives listed in Tables I and II, respectively, where references to the data actually used in the final averages are given in parentheses.

We present some ideograms⁽²³⁾ to illustrate the consistency of the data in the most controvertible cases. Thus the results for the total half-lives of the decays of ^{38m}K , ^{42}Sc and ^{50}Mn are plotted in Fig. 1. This figure only includes the data used in the final averages listed in Table II. One can observe that the data are mainly splitted into two groups. We should point out that Wilkinson *et al.*⁽⁷⁾ have only accepted for their survey the data being under the highest peak. However, following the statistical procedure adopted in the present work, one should take into account all the plotted data for the evaluation of the weighted averages. It is important to note that the distribution of the data corresponding to the examples selected for Fig. 1 leads to external errors larger than internal ones.

3. ANALYSIS OF EXPERIMENTAL DATA

The average decay energies, W_0 , and the average total half-lives, t_0 , quoted in Tables I and II were used to calculate a set of $f't$ -values. We computed the corrected integrated statistical rate function, $f = \tilde{f} \overline{C_\beta(W)}$, according to the method out-lined in Ref. 21. The "outer" radiative corrections $\overline{\delta_R(W)}$ were taken from Ref. 3. The results for f' are listed in Table III. The partial half-lives t , given by Ref. 3, were determined

TABLE I
WEIGHTED AVERAGES OF DECAY ENERGIES OF SUPERALLOWED β -EMITTERS

Decaying nucleus	E_0 (keV)	References [*]
^{14}O	1809.18 \pm 0.34	(Bu 61; To 61 ^a ; Ba 62 ^a ; Ro 70; Cl 73; Wh 77; Vo 77)
$^{26\text{m}}\text{Al}$	3210.62 \pm 0.38	(Fr 62; Sp 64 ^b ; De 69; Ha 74a; Fr 75a; Vo 77)
^{34}Cl	4469.94 \pm 0.34	(Gr 69; Ha 74a; Fr 75a; Vo 77; Ba 77)
$^{38\text{m}}\text{K}$	5020.74 \pm 0.83	(Sq 75; Ja 78)
^{42}Sc	5401.64 \pm 0.39	(Ha 74a; Vo 77)
^{46}V	6028.60 \pm 0.56	(Sq 76; Da 76 ^c ; Vo 77)
^{50}Mn	6609.90 \pm 0.39	(Ha 74a; Fr 75a; Vo 77)
^{54}Co	7219.52 \pm 0.56	(Ha 74a; Ho 74; Vo 77)

^a The values attributed to these references appear as corrected by Ry 65.

^b The value was derived from ground-state E_0 using 228.44 \pm 0.15 keV (De 69) for the excited state.

^c This reference replaces Ha 74a.

^{*} References are listed at final

TABLE II
WEIGHTED AVERAGES OF HALF-LIVES OF SUPERALLOWED β -EMITTERS

Decaying nucleus	t_o (ms)	References*
^{14}O	70605. $\pm 18.$	(Cl 73; Az 74; Wi 78; Be 78)
$^{26\text{m}}\text{Al}$	6344.1 ± 2.7	(Fr 69; Ha 72; Az 75; Al 77)
^{34}Cl	1525.4 ± 1.0	(Ry 73; Wi 76)
$^{38\text{m}}\text{K}$	923.7 ± 1.3	(Ha 72; Sq 75; Th 78; Wi 78 ^a)
^{42}Sc	681.9 ± 1.0	(Fr 65; Ha 72; Wi 76)
^{46}V	422.34 ± 0.20	(Ha 74 b; Ba 77; Al 77)
^{50}Mn	283.00 ± 0.35	(Ha 74b; Fr 75b; Wi 76)
^{54}Co	193.23 ± 0.14	(Ha 74b; Ho 74; Al 77)

* For the references see Table I.

^a From this reference we took the Brookhaven recommended value $t_o = 921.86 \pm 0.56$ ms.

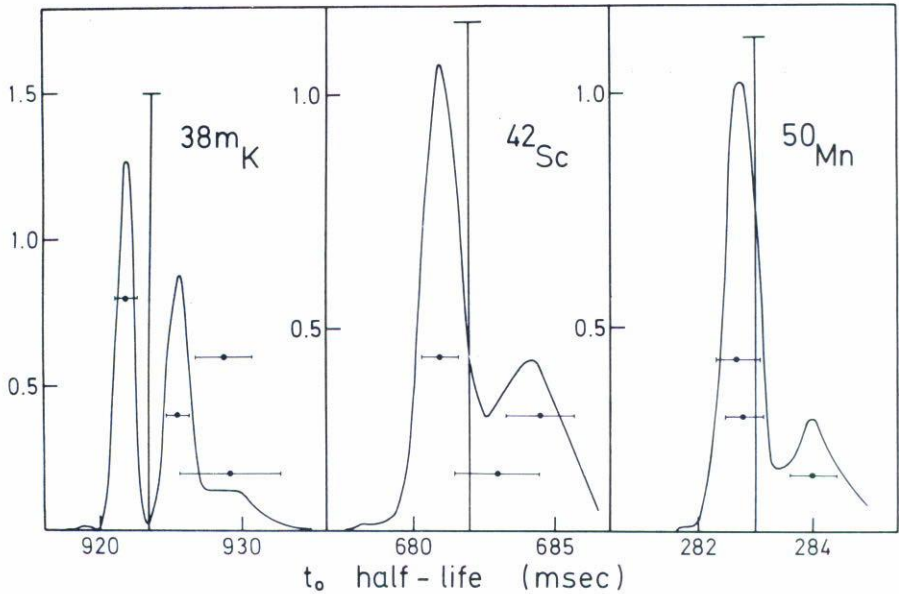


Fig. 1. Ideograms corresponding to the half-lives t_0 of the decays of ^{38m}K , ^{42}Sc and ^{50}Mn . This plot contains only the consistent data used for the evaluation of the averages. The experimental results appear bottom-to-top in the same chronological order as in Table II. The position of the averages is indicated by the vertical line and the horizontal line atop represents the error in the averages.

from the total half-lives using the branching ratios and the electron capture fractions adopted in Ref. 3. The results for t together with the corresponding $f't$ -values are included in Table III.

The internal consistency of the present set of $f't$ -values was studied in conjunction with both microscopic and phenomenological approaches for δ_c mentioned in the introduction.

TABLE III

THE $f't$ VALUES, AVERAGE ISOSPIN CORRECTION δ_c AND Ft VALUES OF SUPERALLOWED β -TRANSITIONS. THE FIGURES IN PARENTHESES IN THE f' , t AND $f't$ COLUMNS ARE THE RESPECTIVE ERRORS IN PERCENT.

Nucleus	f'	t (ms)	$f't$ [s]	$(1-\delta_c)\%$	$Ft(\delta_c)$ [s]
^{14}O	$43.443 \pm 0.035 (0.081)$	$71145 \pm 20 (0.028)$	$3090.8 \pm 2.7 (0.086)$	99.91 ± 0.03	3088.0 ± 2.8
^{26m}Al	$485.94 \pm 0.29 (0.060)$	$6349.4 \pm 2.7 (0.043)$	$3085.4 \pm 2.3 (0.074)$	99.81 ± 0.05	3079.5 ± 2.8
^{34}Cl	$2030.3 \pm 0.7 (0.034)$	$1526.6 \pm 1.0 (0.066)$	$3099.5 \pm 2.3 (0.074)$	99.58 ± 0.13	3086.5 ± 4.6
^{38m}K	$3350.6 \pm 2.6 (0.078)$	$924.5 \pm 1.3 (0.141)$	$3097.6 \pm 5.0 (0.161)$	99.62 ± 0.09	3085.8 ± 5.7
^{42}Sc	$4543.2 \pm 1.5 (0.033)$	$682.6 \pm 1.0 (0.146)$	$3101.2 \pm 4.7 (0.150)$	99.56 ± 0.11	3087.6 ± 5.7
^{46}V	$7333.5 \pm 3.2 (0.044)$	$422.76 \pm 0.20 (0.047)$	$3100.3 \pm 2.0 (0.064)$	99.61 ± 0.09	3088.2 ± 3.4
^{50}Mn	$10936.5 \pm 3.0 (0.027)$	$283.29 \pm 0.35 (0.124)$	$3098.2 \pm 3.9 (0.127)$	99.57 ± 0.09	3084.9 ± 4.8
^{54}Co	$16053.0 \pm 6.3 (0.039)$	$193.44 \pm 0.14 (0.072)$	$3105.3 \pm 2.5 (0.082)$	99.52 ± 0.10	3090.4 ± 4.0

3.1. Consistency analysis of Ft-values

Several sets of Ft-values were calculated using the ft-values listed in Table III and the various microscopic estimations for δ_c quoted in Table I of Ref. 10. Each set of Ft-values was fitted to the expression

$$Ft = -\frac{K}{2G_V^2} [1 + a|Z|] = (Ft)_{z=0} [1 + a|Z|] \quad (7)$$

in order to detect any lingering dependence on Z. For reference the case with $\delta_c = 0$ was also considered. These fits, as well as any other reported in the present work, were based on a plus-and-minus-one-standard-deviation analysis performed with the aid of the MINUITS code⁽²⁴⁾. It was found that the inclusion of δ_c improves the consistency of the data with respect to the set with $\delta_c = 0$ only if one adopts the δ_{c1} calculated using harmonic oscillator wave functions⁽¹⁶⁻¹⁹⁾, which are quoted in Table IV, confirming a finding of the previous work showed in Ref. 10. These results, together with the weighted averages Ft, are listed in Table V. An inspection of this table indicates that, although in none case the slope a is definitely larger than $\pm 2\alpha^2$ *, actually, only the set obtained using δ_{c1} reported in Ref. 16. is strictly consistent with a = 0. Fig. 2 illustrates the fit for this set. In addition, we can mention that, in this case the values $\overline{Ft} = 3084.3 \pm 1.1$ sec and $(Ft)_{z=0} = 3082.4 \pm 2.9$ sec are in excellent agreement.

However, since there is no theoretical reason to prefer the estimates for δ_{c1} of Ref. 16 to the other ones, to improve the overall theoretical consistency we evaluated a set of Ft-values using a set of averages $\overline{\delta_c}$ calculated following a procedure already applied by Wilkinson⁽²⁾ and Vonach et al.⁽⁵⁾, but treating the uncertainties in $\overline{\delta_c}$ in a different way. Thus, we evaluated a straight average $\overline{\delta_{c1}}$ for each nuclide taking into account the estimates for δ_{c1} published in Refs. 16-19 and adopting an uncertainty $\Delta\overline{\delta_{c1}}$ equal to the standard deviation. In addition, in view of the features mentioned by Towner and Hardy⁽²¹⁾ we set $\Delta\delta_{c2} = 0.5 \delta_{c2}$ for all nuclei. The partial and final results are listed in Table IV, where $\Delta\overline{\delta_c}$ was determined adding quadratically $\Delta\overline{\delta_{c1}}$ and $\Delta\delta_{c2}$. Although in the present work and in Ref. 5 some different values were used the average

* A reliable limit on the slope fixed by the uncertainty in δ_R can be taken as $\pm 2Z\alpha^2$ (e.g. see Wilkinson⁽²⁵⁾).

TABLE IV

ISOSPIN IMPURITY CORRECTIONS ACCORDING TO VARIOUS ESTIMATIONS

Nucleus	δ_{c1} (%)				$\bar{\delta}_{c1} + \Delta\bar{\delta}_{c1}$ (%)	δ_{c2} (%)	$\bar{\delta}_c$ (%)	
	[16] ^a	[17]	[18]	[19]			Present work	[5] ^b
¹⁴ O	0.056	0.028	0.04	0.05	0.044±0.012	0.05	0.09±0.03	0.10±0.14
^{26m} Al	0.152	0.083	0.11	0.13	0.119±0.029	0.07	0.19±0.05	0.19±0.14
³⁴ Cl	0.226	0.134	0.18	0.20	0.185±0.039	0.23	0.42±0.13	0.42±0.14
^{38m} K	0.265	0.180	0.21	0.24	0.224±0.037	0.16	0.38±0.09	0.39±0.14
⁴² Sc	0.394	0.227	0.25	0.37	0.310±0.084	0.13	0.44±0.11	0.43±0.18
⁴⁶ V	0.448	0.264	0.29	0.40	0.351±0.088	0.04	0.39±0.09	0.39±0.18
⁵⁰ Mn	0.505	0.314	0.33	0.43	0.395±0.090	0.03	0.43±0.09	0.42±0.18
⁵⁴ Co	0.563	0.337	0.38	0.47	0.438±0.100	0.04	0.48±0.10	0.49±0.18

^a Here we listed the results as they appear in the Ref. 4.

^b It should be pointed out that Vonach et al.⁽⁵⁾ have computed the average δ_{c1} selecting from the work by Fayans⁽¹⁷⁾ the corrections obtained using a pure shell model, while for our calculations of $\bar{\delta}_{c1}$ we used from Ref. 17 the complete estimates including the interaction between quasiparticles. These latter values have been also adopted in the most recent work by Wilkinson et al.⁽⁷⁾. Moreover, the authors of Ref. 5. have not included the estimations of Ref. 19 in their average.

TABLE V
 RESULTS OF THE FITS OF THE Ft VALUES TO THE FORMULAE $Ft = (Ft)_{z=0} [1+a|z|]$
 TOGETHER WITH THE CORRESPONDING χ^2/ν

$\delta_c = \delta_{c1} + \delta_{c2}$		$(Ft)_{z=0}$ [s]	a [$\times 10^{-4}$]	χ^2/ν	\overline{Ft} [s]	$\Delta(\overline{Ft})$ [s]		χ^2/ν
δ_{c1}	δ_{c2}					int	ext	
0	0	3081.1 \pm 2.9	2.88 \pm 0.51	2.38	3096.9	1.0	2.5	6.65
[16]	[21]	3082.4 \pm 2.9	0.33 \pm 0.51	1.46	3084.3	1.0	1.1	1.32
[17]		3080.7 \pm 2.9	1.41 \pm 0.51	1.56	3088.4	1.0	1.5	2.44
[18]		3080.1 \pm 2.9	1.33 \pm 0.51	1.64	3087.4	1.0	1.5	2.39
[19]		3081.6 \pm 2.9	0.73 \pm 0.51	1.52	3085.6	1.0	1.2	1.60
$\overline{\delta}_{c1}$		3082.9 \pm 3.5	0.60 \pm 0.65	1.12	3085.9	1.3	1.4	1.09

values $\bar{\delta}_c$ are practically equal (see Table IV). However our errors $\Delta\bar{\delta}_c$ are smaller than those assigned in Ref. 5.

The new set of Ft -values is listed in Table III. This set when fitted to Ref. 7 is compatible with a zero slope (see Fig. 2). All the results of such an adjustment, as well as the \bar{Ft} -value, are quoted in Table V. Hereafter this set will be called as the $Ft(\bar{\delta}_c)$ one.

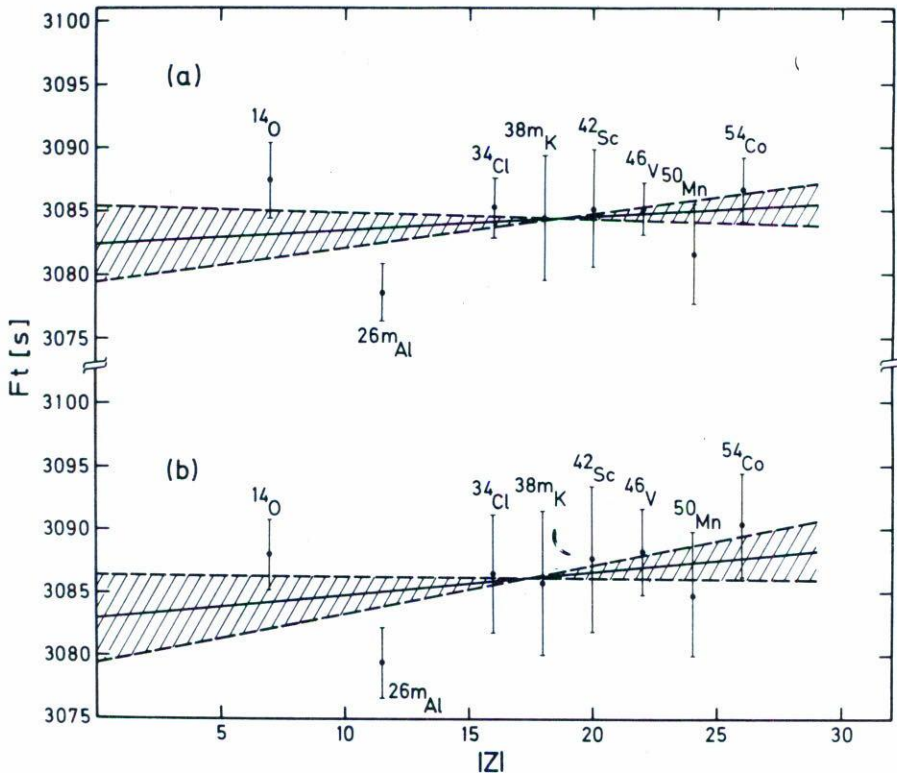


Fig. 2. The Ft -values of the superallowed Fermi β -transitions as a function of the charge $|Z|$ of the daughter nucleus together with the results of the fits according to the expression $Ft = (Ft)_{z=0} [1 + a|Z|]$. The solid line shows the best adjustment corresponding to the mean of $(Ft)_{z=0}$ and a , whereas the dashed area represents the uncertainties obtained in a plus-and-minus-one-standard-deviation fit. In part (a) the Ft -values were evaluated using δ_{c1} reported by Ref. 16, while the Ft -values calculated adopting the averages $\bar{\delta}_{c1}$ performed in the present work are shown in part (b).

3.2 Phenomenological approach to $f't$.

Some years ago Wilkinson^(2,22) proposed that the isospin correction can be expressed as $\delta_c = kZ^{1.86}$, where k is a free parameter (for more details the reader is referred to the original papers^(2,22)). Adopting such a phenomenological approximation for δ_c the Ec. (1) leads to

$$f't = \frac{K}{2G_V^2} (1 + \delta_c) = (f't)_{z=0} [1 + kZ^{1.86}], \quad (8)$$

where $(f't)_{z=0}$ is equal to $(Ft)_{z=0}$ and it can be considered appropriate to the free nucleon and therefore suitable to derive G_V' . The fit of the set of $f't$ -values listed in Table III to the quasi-parabolic law given by Ec. (8) yielded:

$$(f't)_{z=0} = 3087.1 \pm 2.0 \text{ sec}, \quad k = (1.37 \pm 0.24) \times 10^{-5}; \quad \chi^2/\nu = 2.35. \quad (9)$$

This relatively large chi-square, similar to that obtained fitting the Ft -values to Ec. (7) setting $\delta_c = 0$ (cf. Table V), can be attributed to the fact that, although the simple expression $kZ^{1.86}$ follows roughly the trend of δ_{c1} it cannot reproduce the shell effects due to two-body charge-dependent potential accounted for by δ_{c2} . Therefore the authors of paper 22 suggested that the goodness of adjustment can be improved correcting the $f't$ -values by the departure of the individual δ_{c2} values from $\bar{\delta}_{c2}$ fitting the experimental data according to the expression

$$f't [1 - (\delta_{c2} - \bar{\delta}_{c2})] = \frac{K}{2G_V'^2 [1 - (\delta_{c1} + \bar{\delta}_{c2})]} = (f't)_{z=0} [1 + kZ^{1.86}]. \quad (10)$$

In this case the computational procedure yielded:

$$(f't)_{z=0} = 3086.1 \pm 2.0 \text{ sec}, \quad k = (1.54 \pm 0.24) \times 10^{-5}; \quad \chi^2/\nu = 1.37. \quad (11)$$

This chi-square is smaller than that of the fit to Ref. 8.

3.3 Choice of a consistent set of $f't$ -values.

It was shown that the set $Ft(\bar{\delta}_c)$, which was obtained using for δ_c the average of microscopic calculations, fulfils satisfactorily the theoretical expectations. On the other hand, the phenomenological approach for δ_c provides an acceptable description of the experimental $f't$ -values

only when combined with the microscopic estimations of δ_{c2} . In view of these results we feel that at the moment the best way to gain information on fundamental interactions is to analyze the set $Ft(\bar{\delta}_c)$.

4. IS AND FUNDAMENTAL S INTERACTION

4.1 Induced scalar interaction.

Following the procedure reported in Ref. 8 it is straightforward to demonstrate that the set $Ft(\bar{\delta}_c)$ does not indicate any sizeable mesonic exchange contribution comparable to the estimation of Blin-Stoyle *et al.*⁽²⁶⁾. Therefore, in this section we report directly the results of our search for an upper limit on the coupling constant f_s corresponding to the IS interaction.

In deriving Ec. (1) the following relation was used for the β -decay matrix element M_β :

$$|M_\beta|^2 = M_0^2(1,1) + m_0^2(1,1) - 2 \frac{\mu_1 \gamma_1}{W} M_0(1,1)m_0(1,1) = (V_{F_{000}}^0)^2 C_\beta(W). \quad (12)$$

Here $M_0(1,1)$ and $m_0(1,1)$ are linear combinations of FFC of the type $V_{F_{000}}^N(1,m,n,\sigma)$ and $V_{F_{011}}^N(1,m,n,\sigma)$, the corresponding expressions can be obtained from the general formulas (A.10) and (A.11) of the appendix of Behrens and Bühring⁽²⁷⁾. The quantity μ_1 is a special Coulomb function defined in Behrens and Jänecke⁽²⁸⁾ and $\gamma_1^2 = 1 - (\alpha Z)^2$.

Since the work by Damgaard⁽¹⁶⁾ it is currently assumed that the contribution of all the relativistic FFC $V_{F_{011}}^N(1,m,n,\sigma)$ to $C_\beta(W)$ can be neglected. For instance, see the explicit formula for $C_\beta(W)$ given by Ec. (6) of Behrens and Bühring⁽²⁹⁾. In the framework of such an approximation a procedure has been developed to determine an experimental value for the strength f_s ⁽⁸⁻¹¹⁾. The method was based on the fact that the effect of the IS term contributing to $V_{F_{000}}^0$ is much more important than the corresponding contributions carried by terms stemming from the remaining FFC of the type $V_{F_{000}}^N(1,m,n,\sigma)$. However, since the neglected $V_{F_{011}}^N(1,m,n,\sigma)$ depend on the IS interaction, we analyzed in Ref. 12 the question of whether or not there is a relevant contribution of such relativistic FFC to terms proportional to f_s . In that letter⁽¹²⁾ it is shown that, as far as IS contribution is concerned, the effect of relativistic FFC is so strong

that cancels exactly the corresponding contribution carried by the Fermi FFC in $M_0(1,1)$. So, this result invalidates the procedure for determining f_S used in the previous works^(8,11). In the appendix we complete the short demonstration published in Ref. 12* and show that the main contribution of the IS interaction is given by

$$Ft = f' t(1 - \delta_c) = \frac{K}{2G_V'^2} [1 + 2(f_S/f_V)\gamma_1 <1/W>] . \quad (13)$$

Assuming that G_V' is not renormalized by the nuclear β -decay process f_S/f_V becomes proportional to the slope of a plot of Ft -values versus $\gamma_1 <1/W>$. Therefore, we fitted the experimental $Ft(\bar{\delta}_c)$ -values to Eq. (13) using the values of $\gamma_1 <1/W>$ quoted in Table I of Ref. 12. The results are listed in Table VI. The value $f_S/f_V = (-0.82 \pm 2.1) \times 10^{-3}$ is in agreement with the CVC theory. This result has been already published in Ref. 12. It is the object of this paper to present a more complete search for the strength f_S .

In order to complete the study we analyzed what happens with the limit on f_S/f_V when one uses the other approaches for δ_c mentioned in section 3. Thus, on the one hand, we fitted to (13) the Ft -values calculated using δ_{c1} of Ref. 16, which carry the error in $f' t$ only. The results are included in Table VI. On the other hand, f_S/f_V was determined using the two phenomenological approaches for δ_c described in section 3. For such a purpose the $f' t$ -values were fitted according to

$$f' t = \frac{K}{2G_V'^2} [1 + kZ^{1.86} + 2(f_S/f_V)\gamma_1 <1/W>] \quad (14)$$

and

$$f' t [1 - (\delta_{c2} - \bar{\delta}_{c2})] = \frac{K}{2G_V'^2} [1 + kZ^{1.86} + 2(f_S/f_V)\gamma_1 <1/W>] . \quad (15)$$

Both, the fits to (14) and (15), were performed adopting two different choices for the parameter k : (i) it was fixed at the value singled out in section 3 and (ii) it was left free. The results are quoted in Table VI. It is to be noted that when k is variable the error in the strength f_S/f_V is significantly larger than any other, while δ_c is taken without uncertainty

* We should point out that in Ref. 12 there are several misprints.

the error in f_S/f_V is only slightly smaller than that obtained from the fit of the set $Ft(\bar{\delta}_c)$. A glance at Table VI indicates that excluding the results for k free, there is no significant quantitative difference between the remaining limits on f_S/f_V . However, from the qualitative point of view we feel that the more reliable determination is that based on the $Ft(\bar{\delta}_c)$ -values due to the arguments presented in section 3. Since it is always of great interest to set out the coupling constant of any interaction with the highest possible precision, we also determined f_S/f_V from $Ft(\bar{\delta}_c)$ using an improved method.

Let us point out that a fit to Ec. (13) requires a simultaneous determination of $K/2G_V'^2$ and f_S/f_V and in such a procedure the error in f_S/f_V is enlarged by the uncertainty in $K/2G_V'^2$. Thus it turns out to be clear that to diminish the error in the IS coupling constant, one should use a method in which the experimental data were expressed as a function of f_S/f_V only. A simple way to eliminate such an unwanted effect is to study ratios of Ft -values, since then the factor $K/2G_V'^2$ cancels out⁽¹¹⁾. Following this idea, one can normalize the Ft -values to the $Ft(^{14}O)$ -value. Using Ec. (13) it is straightforward to demonstrate that such ratios obey the following expression:

$$\frac{Ft(Z)}{Ft(^{14}O)} = 1 - 2(f_S/f_V) [\langle v_1/W \rangle(^{14}O) - \langle v_1/W \rangle(Z)] . \quad (16)$$

In this case it is possible to take advantage of the comparatively large difference between the value of $\langle \gamma_1/W \rangle$ for the decay from ^{14}O and those values corresponding to other transitions⁽¹²⁾. So, the ratios $Ft(Z)/Ft(^{14}O)$ evaluated from the set $Ft(\bar{\delta}_c)$ were fitted to (16). The results are listed in Table VI and, in addition, to illustrate the goodness of the adjustment they are displayed in Fig. 3. Looking at Table VI one can observe that the new value of f_S/f_V is also in agreement with the CVC theory and the error of which, 1.3×10^{-3} , is about 40% smaller than the error 2.1×10^{-3} obtained from the fit to (13) and it is also smaller than the error 1.7×10^{-3} got from the fits of sets where $\bar{\delta}_c$ were considered without uncertainty.

TABLE VI
RESULTS FOR THE IS COUPLING CONSTANT f_S/f_V

Fit to Eq.	δ_c	$k[x10^{-5}]$	$K/2G_V^2[s]$	$f_S/f_V[x10^{-3}]$	$ f_S/f_V [x10^{-3}]$	χ^2/ν
(13)	$\bar{\delta}_c$		3087.1 \pm 3.7	-0.8 \pm 2.1	\leq 2.9	1.24
(13)	$\delta_{c1}[16]+\delta_{c2}[21]$		3084.6 \pm 2.7	-0.3 \pm 1.7	\leq 2.0	1.54
(14)	$KZ^{1.86}$	1.37	3087.2 \pm 2.7	-0.1 \pm 1.7	\leq 1.8	2.35
		1.32 \pm 0.54	3088.0 \pm 9.4	-0.4 \pm 3.9	\leq 4.3	2.82
(15)	$\delta_{c1}+\bar{\delta}_{c2}=KZ^{1.86}$	1.54	3084.6 \pm 2.7	1.0 \pm 1.7	\leq 2.7	1.31
		1.83 \pm 0.54	3080.0 \pm 9.4	2.8 \pm 3.9	\leq 6.9	1.37
(16)	$\bar{\delta}_c$			1.1 \pm 1.3	\leq 2.4	0.79

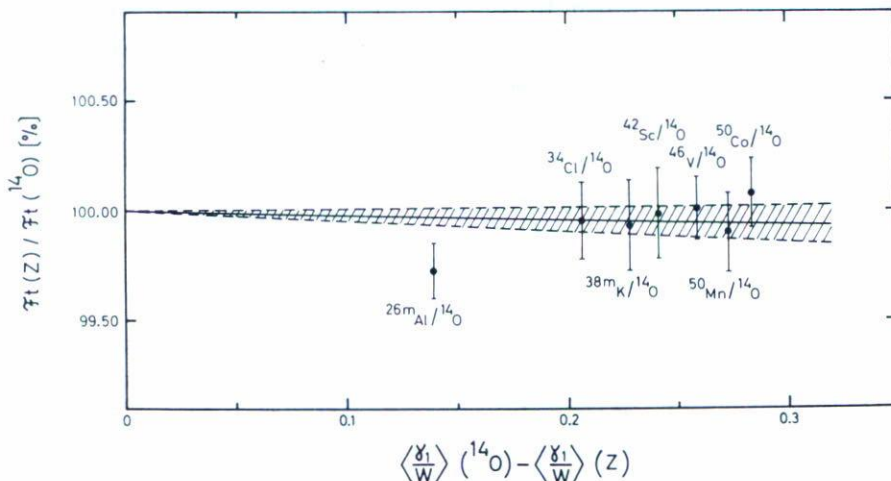


Fig. 3. The $Ft(\bar{\delta})$ -values of the superallowed Fermi β -transitions normalized to the ${}^cFt(\bar{\delta})$ of ${}^{14}\text{O}$ as a function of the difference $\langle \gamma_1/W \rangle (^{14}\text{O}) - \langle \gamma_1/W \rangle^c(Z)$. The solid line shows the best adjustment obtained fitting the ratios to the expression $Ft(Z)/Ft(^{14}\text{O}) = 1 - 2(f_S/f_V) [\langle \gamma_1/W \rangle (^{14}\text{O}) - \langle \gamma_1/W \rangle (Z)]$, the dashed area indicates the uncertainties corresponding to a plus-and-minus-one-standard-deviation analysis.

4.2 Fierz interference term.

Let us mention another aspect related to the matter tackled in the present work. The two-component (massless) neutrino theory requires that the Fierz interference term, b_F , should be exactly zero. In the case of pure Fermi transitions b_F is connected with the fundamental scalar (G_S) and vector (G_V) coupling constants only (see e.g., (V-26) in Ref. 1), and for $G_S \ll G_V$ one obtains $b_F = G_S/G_V$ in units of $m_0 c^2$. The pioneering attempts to set an upper limit on b_F are summarized in Ch. V of Ref. 1. It turns out that the analysis of $0^+ \rightarrow 0^+$ superallowed β -transitions provides a precise way of determining b_F . In a first approximation the fundamental

scalar interaction changes $C_\beta(W)$ into $C_\beta(W) |1 - 2\gamma_1 b_F/W|$, hence it is straightforward to demonstrate

$$Ft = \frac{K}{2G_V^2} = |1 + 2 b_F \gamma_1 <1/W>|. \quad (17)$$

This relation is equivalent to Ec. 13. This result shows that, in first approximation, the IS interaction changes the spectrum shape factor of a pure Fermi transition in the same way as the fundamental scalar interaction, against the previous assumptions^(8,11). Therefore, if one assumes that CVC theory holds, i.e. $f_S = 0$, the numerical results obtained in the previous section can be taken as limits on b_F .

5. SUMMARY AND CONCLUSION

In the present work a set of $f't$ -values was calculated using weighted averages of energy releases W_0 and partial half-lives t obtained adopting a standard technique for selecting information from a series of measured data⁽²³⁾.

It is shown that, if one combines the $f't$ -values reported in Table III with an average of the available microscopic estimations of δ_c , with δ_{c1} based on spherical harmonic oscillator wave functions⁽¹⁶⁻¹⁹⁾ one obtains a set $Ft(\delta_c)$ which is independent of Z . This means that, $Ft = \text{const.}$ and the vector coupling constant f_V should not be renormalized. It should be pointed out that at present the dispersion of the theoretical evaluations of δ_c are of the same magnitude as the experimental uncertainties in the $f't$ -values. See the corresponding relative errors in percent quoted in Table III.

On the other hand, it was found that the phenomenological approach $\delta_c = kZ^{1.86}$ provides a poorer description of the $f't$ -values than any microscopic one calculated with harmonic oscillator wave functions. In addition, the quality of the fit measured by $\chi^2/\nu = 2.35$ is equal to $\chi^2/\nu = 2.38$ obtained adopting $\delta_c = 0$. If one eliminates the fluctuations due to shell effects accounted for by δ_{c2} the resulting $\chi^2/\nu = 1.37$ is still larger than the value $\chi^2/\nu = 1.12$ corresponding to $\bar{\delta}_c$. Therefore in the light of the results of the present work it is possible to establish a preference for the microscopic method of calculating δ_c over the

phenomenological one.

The value of the dressed vector coupling constant derived from $Ft(\bar{\delta}_c)$, $G_V' = (1.41208 \pm 0.00032) \times 10^{-49}$ in cgs units, is near equal to the average of the values listed in Table A of the recent survey by Raman *et al.*⁽³⁰⁾. However, the error of our value is smaller than any quoted in that table.

A limit on the strength $|f_S/f_V| < 2.4 \times 10^{-3}$ is set, being of order $2/M$, but unfortunately larger than those obtained in previous works by means of a procedure which is no longer valid⁽⁸⁻¹¹⁾. Since the new upper limit on f_S is larger than $2/M$ we can only state that the result of the present work is in agreement with the CVC theory, but not that it supports strongly such a theory.

Since it is always of interest when an improved limit on absence of the Fierz interference term can be reached we can mention that, assuming $f_S = 0$, the analysis of the $Ft(\bar{\delta}_c)$ -values yielded a new value $b_F = (1.1 \pm 1.3) \times 10^{-3}$. This value supports the V-A interaction and it is smaller than the best previously existing limit, $b_F = (-0.5 \pm 3.0) \times 10^{-3}$, obtained by Hardy and Towner⁽³⁾. Actually the value of b_F reported in Ref. 3 is $(-1 \pm 6) \times 10^{-3}$, this is due to the fact that Hardy and Towner adopted a definition for b_F which is twice that quoted by Blin-Stoyle⁽¹⁾ and adopted in the present work.

As a final remark, we can point out that the inconsistencies arised from the analysis of the most recent experimental data performed by Wilkinson *et al.*⁽⁷⁾ can be overcome if one calculates the averages of W_0 and t_0 according the prescription recommended by the Particle Data Group⁽²³⁾. The resulting data are in agreement with the CVC theory and the V-A interaction. Thus we can state that, at present one has a coherent overall description of the best-measured $0^+ \rightarrow 0^+$ Fermi superallowed β -transitions.

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APPENDIX

Using the formulas listed in appendices A and B of Behrens et al. (14), if only the FFC carrying dominant IS terms are kept, one arrives at

$$M_0(1,1) = \frac{v_0}{F_{000}} - \frac{W_0 R}{3} \frac{v_0}{F_{011}} + \frac{\alpha |Z|}{3} \frac{v_0}{F_{011}} (1,1,1,1) , \quad (\text{A.1})$$

$$m_0(1,1) = - \frac{R}{3} \frac{v_0}{F_{011}} , \quad (\text{A.2})$$

where R is the nuclear radius of the daughter nucleus. The FFC are given in terms of NME $V_{KLS}^{MN}(1,m,n,\sigma)$ and coupling constants in Table 6 of Ref. 27. Using those expressions one obtains for β^+ -decays:

$$\frac{v_0}{F_{000}} = f_V \frac{v_0}{M_{000}} + \left(\frac{f_S}{R} \right) \left([W_0 R - \alpha |Z| U(r)] \beta T_{000} \right) , \quad (\text{A.3})$$

$$\begin{aligned} - \frac{v_0}{F_{011}}(1,m,n,\sigma) &= f_V \frac{v_0}{M_{011}}(1,m,n,\sigma) + \left(\frac{f_M}{R} \right) \left(\frac{r}{R} \right) I(r) [W_0 R - \alpha |Z| U(r)] \beta T_{011} \\ &- \left(\frac{f_S}{R} \right) \left([3I(r) + r I'(r)] \beta T_{000} \right) . \end{aligned} \quad (\text{A.4})$$

Here $U(r)$ is the potential of the nuclear charge distribution, $I(r) \equiv I(1,m,n,\sigma;r)$ is a function of the nuclear charge distribution defined in Table 1 of Ref. 27 and $I'(r) \equiv dI/dr$. In addition, β is the usual Dirac matrix and T_{KLS} are spherical tensor operators.

Using (A.3) and (A.4) and the trivial result $I(1,m,n,\sigma;r) \equiv 1$, the dependence on f_S of the linear combinations given by (A.1)-(A.2) can be reduced to

$$M_0(1,1) [f_S] = f_S \frac{\alpha |Z|}{R} \int [-U(r) + I(1,1,1,1;r) + \frac{r}{3} I'(1,1,1,1;r)] \beta T_{000} , \quad (\text{A.5})$$

$$m_0(1,1) [f_S] = - f_S \int \beta T_{000} . \quad (\text{A.6})$$

Assuming the usual uniform charge distribution according to the formulas quoted in Table 3 of Ref. 27 one obtains

$$U(r) = \begin{cases} \frac{3}{2} - \frac{1}{2} \left(\frac{r}{R}\right)^2, & 0 \leq r \leq R \\ \frac{R}{r}, & R \leq r \end{cases} \quad (\text{A.7a})$$

$$(\text{A.7b})$$

$$I(1,1,1,1;r) = \begin{cases} \frac{3}{2} - \frac{3}{10} \left(\frac{r}{R}\right)^2, & 0 \leq r \leq R \\ \frac{3}{2} \frac{R}{r} - \frac{3}{10} \left(\frac{R}{r}\right)^3, & R \leq r \end{cases} \quad (\text{A.8a})$$

$$(\text{A.8b})$$

$$r I'(1,1,1,1;r) = \begin{cases} -\frac{3}{5} \left(\frac{r}{R}\right)^2, & 0 \leq r \leq R \\ -\frac{3}{2} \left(\frac{R}{r}\right) + \frac{9}{10} \left(\frac{R}{r}\right)^3, & R \leq r \end{cases} \quad (\text{A.9a})$$

$$(\text{A.9b})$$

Using these results it can be readily demonstrated that the square bracket of (A.5) vanishes for the whole range $0 \leq r \leq \infty$.

However, in the approximation adopted the third term of Ec. (12), namely $M_0(1,1) m_0(1,1)$, gives a sizeable effect proportional to the strength f_S which allows to derive a meaningful information about the IS interaction from superallowed Fermi transitions⁽¹²⁾. Keeping the large components only, this means $\beta = -1$, we have

$$m_0(1,1) [f_S] = f_S M_{000}^V \quad (\text{A.10})$$

Therefore, instead of (12) we can write

$$M_0^2(1,1) + m_0^2(1,1) - 2 \frac{\mu_1 \gamma_1}{W} M_0(1,1) m_0(1,1) = f_V^2 (M_{000}^V)^2 C_\beta(W) \left[1 - 2(f_S/f_V) \frac{\mu_1 \gamma_1}{W} \right] \quad (\text{A.11})$$

where the main contribution of the IS interaction is separated explicitly. Consequently Ec. (1) can be rewritten

$$f' t = \frac{K}{G_V^2 (M_{000}^V)^2 [1 - 2(f_S/f_V) \mu_1 \gamma_1 \langle 1/W \rangle]}, \quad (\text{A.12})$$

where $\langle 1/W \rangle$ is the factor $1/W$ averaged over the positron spectrum.

Setting $\mu_1 = 1$ and since on dimensional grounds it might be expected

$|f_S| \leq 10^{-2}$ we can write

$$Ft = f' t (1 - \delta_C) = \frac{K}{2G_V^2} [1 + 2(f_S/f_V) \gamma_1 \langle 1/W \rangle]. \quad (\text{A.13})$$

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