THE EINSTEIN AND HOPF WORK REVISITED

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ABSTRACT

The Einstein and Hopf's work is discussed and, taking into account the radiated energy by the dipole during the fluctuations and the zero point field, we deduce Planck's radiation law.

RESUMEN

Se discute el trabajo de Einstein y Hopf y, tomando en cuenta la energía radiada por el dipolo durante las fluctuaciones y el campo de punto cero, se deduce la ley de radiación de Planck.

1. INTRODUCTION

After Planck's proposal of his spectral distribution law for the radiation in a cavity, Einstein, among others, began studying both the consequences and the physical basis of the law. In 1909, using his famous "gedankenexperiment" of the Brownian mirror[1] within an ideal gas in equilibrium with electromagnetic radiation at temperature T, Einstein tried to establish, in a rigorous way, the corpuscular nature of radiation by means of which he showed the incompatibility between Planck's law and the electromagnetic theory. A year later, in colaboration with Hopf[2] he showed that the classical laws lead necessarily to Rayleigh's radiation law. At this stage, the problem seemed to be solved and the discussion came to an end. Nevertheless half a century later, in 1969, Boyer[3] analyzed once again this problem under new perspectives[4]: He supposes that even at $T=0$ there exists a classical random electromagnetic radiation field (zero point field or background field) which has a Lorentz invariant spectrum[5] $\rho(\omega,T=0)\sim\omega^3$; the fluctuations scale is fixed in such a way that the energy per normal mode is $\frac{1}{2}\hbar\omega$. In this way, basing his approach on Einstein and Hopf's ideas, Boyer was able to deduce Planck's complete distribution law (including the zero point term). However, this derivation is crucially dependent on the interactions between the dipole and the walls of the cavity, which Boyer considered independent of T. In the present paper an alternative derivation, where the central hypothesis is that the radiation exchange between the dipole and vacuum field does not depend on temperature, is presented.

2. A MODIFICATION OF EINSTEIN AND HOPF'S WORK FROM THE POINT OF VIEW OF STOCHASTIC ELECTRODYNAMICS

In order to bring about our discussion it is necessary to present briefly both the fundamental ideas of Einstein and Hopf's[2] work and the modification suggested by Boyer[3].

Einstein and Hopf studied the conditions under which there exists equilibrium between an oscillating dipole within an ideal mono-atomic gas and a radiation field at temperature T. They proposed as a characteristic of equilibrium the condition

$$\langle p^2(t+\delta t)\rangle = \langle p^2(t)\rangle \quad , \tag{1}$$

where

$$p(t+\delta t) = p(t) + \Delta - Rp(t)\delta t \quad . \tag{2}$$

Here Δ is the fluctuating impulse given by the field to the dipole during δt and Rp is a resistive force caused by the anisotropy of the field due to the motion of the dipole.

Einstein and Hopf assumed that Δ and p are not correlated, that $\langle\Delta\rangle = 0$ and that δt is very small, so Eqs. (1) and (2) led to the relationship

$$\langle \Delta^2\rangle = 2R\langle p^2\rangle\delta t \quad . \tag{3}$$

The values for $\langle\Delta^2\rangle$ and R, obtained using classical electromagnetism, are

$$\langle \Delta^2\rangle = \frac{8}{15}\frac{\pi^4 e^2 c}{m\omega^2}\rho^2(\omega,T)\delta t \tag{4}$$

and

$$R = \frac{4}{15}\frac{\pi^2 e^2}{m^2c^2}\left(\rho(\omega,T) - \frac{1}{3}\omega\frac{\partial\rho(\omega,T)}{\partial\omega}\right) \quad . \tag{5}$$

They also obtained $\langle p^2\rangle$ by means of the equipartition theorem, which they considered firmly established by experiment for the translational motion of the dipole. With this result and using Eqs. (4) and (5) into Eq. (3) they arrive to the differential equation

$$\frac{\pi^2c^3}{3kT}\frac{\rho^2(\omega,T)}{\omega^2} = \rho(\omega,T) - \frac{1}{3}\omega\frac{\partial\rho(\omega,T)}{\partial\omega} \quad , \tag{6}$$

whose solution is the Rayleigh's distribution law.

On the other hand, Boyer took into account the existence of the zero point field in his analysis. The spectrum cannot contribute to a resistive force depending on the velocity due to the Lorentz invariance; hence the energy withdrawn from the background field by the dipole is not balanced by the work done by a dissipative force similar to Rp. Therefore, the energy of the particle should increase indefinitely, not allowing in this way to reach equilibrium. Nevertheless, Boyer correctly concluded that, in an equilibrium situation, there must exist a mechanism by means of which the absorbed energy of the background field may be dissipated. He proposed that the dissipation is due to the collisions of the dipole against the cavity walls. He proposed, instead of Eq. (2), the following:

$$p(t+\delta t) = p(t) + \Delta - Rp(t)\delta t + J \quad , \tag{7}$$

where J is the impulse communicated by collisions to the wall during the time interval δt. Boyer argues that the averages involving p, Δ and J could satisfy the conditions

$$\langle p\Delta\rangle = \langle p\Delta\rangle_o = 0 \quad , \tag{8}$$

$$\langle \Delta J\rangle = \langle \Delta J\rangle_o = 0 \quad , \tag{9}$$

$$\langle pJ\rangle_o < 0 \quad , \tag{10}$$

$$R\delta t \ll 1 \quad , \tag{11}$$

$$\langle J^2\rangle \ll \langle \Delta^2\rangle_o \quad , \tag{12}$$

$$\langle pJ\rangle = \langle pJ\rangle_o \quad . \tag{13}$$

On squaring (7) and averaging, he obtained from Eqs. (8), (9) and (12), for $T=0$, the following expression:

$$2\langle pJ\rangle_o = -\langle \Delta^2\rangle_o \quad . \tag{14}$$

With these hypothesis he derives instead of Eq. (3) the equation

$$\langle \Delta^2 \rangle = 2 R m k T \delta t + \langle \Delta^2 \rangle_\circ \quad . \tag{15}$$

Considering the values of $\langle \Delta^2 \rangle$ and R derived by Einstein and Hopf and using for $\langle \Delta^2 \rangle_\circ$ the value of $\langle \Delta^2 \rangle$ with $\rho(\omega, T=0) = \frac{\hbar\omega^3}{2\pi^2 c^3}$, he obtained

$$\rho - \frac{1}{3}\,\omega\,\frac{\partial\rho}{\partial\omega} = \frac{\pi^2 c^3}{3kT\omega^2}\left(\rho^2 - \frac{\hbar^2\omega^4}{4\pi^4 c^6}\right) \quad , \tag{16}$$

whose solution is

$$\rho(\omega,T) = \frac{\hbar\omega^3}{2\pi^2 c^3}\coth\frac{\hbar\omega}{2kT} \quad , \tag{17}$$

which is Planck's distribution law including the zero point term.

Although the preceding analysis is quite interesting, some of the hypothesis on which it is based are not justified as it has been discussed previously[6]. Such hypothesis are Eqs. (8), (9) and (14).

The conditions established by Eqs. (8) and (14) are inconsistent: let K being the average momentum exchange with the cavity wall during a collision. Since the collision itself is an inelastic one, and despite this the particle does not stick to the wall, the value for K should lie between $-\sqrt{\langle p^2 \rangle}$ and $2\sqrt{\langle p^2 \rangle}$, and if γ represents the number of collisions during the time interval δt, therefore $\langle J^2 \rangle \approx \gamma K^2$. Using a similar argument, one can arrive to $\langle pJ \rangle \approx -\gamma K^2$. Since γ is proportional to $V^{1/3}$, where V is the volume of the cavity, one could choose a volume such that either (8) or (14) might be satisfied, but not both. In addition, since γ is a temperature-dependent variable also, Eq. (13) could not be fulfilled. This latter discussion together with the fact concerning the independence of both the kinetic energy distribution and the spectral distribution upon cavity size shows the uncertain role played by the walls of the cavity in the Einstein and Hopf's work.

Boyer's conclusion about the need for a dissipative mechanism by means of which the absorbed energy of the background field may be eliminated so that the equilibrium may be reached, is quite important. If we look for another possible dissipation process, we realize, by analyzing Eq. (2), that the radiated energy that results from the dipole violent fluctuations has not been taken into account. This radiation produces a resistive force of the same order of magnitude as that of Rp and therefore it must be considered. As we shall see, this allows us to obtain Planck's law.

If the lost impulse due to the radiation emission during the fluctuations is represented by r, we can write, instead of Eq. (2),

$$p(t+\delta t) = p(t) + \Delta - Rp(t)\delta t - r \quad , \tag{18}$$

where r is given by

$$r = F_{rad}\,\delta t \quad , \tag{19}$$

and F_{rad} is the radiation damping force.

Considering that Δ and r are statistically independent in equilibrium, then

$$\langle \Delta r \rangle = 0 \quad . \tag{20}$$

Using Eq. (18) and Einstein and Hopf's hypothesis into Eq. (1) we obtain, instead of Eq. (3),

$$\langle \Delta^2 \rangle = 2Rk T\delta t - 2\langle pr \rangle \quad . \tag{21}$$

Now we need to know the value of $\langle pr \rangle$. From Eq. (19) we obtain

$$\langle pr \rangle = m P \delta t \quad , \tag{22}$$

where $P = \langle \frac{p}{m} F_{rad} \rangle = \frac{2e^2}{3m^2c^3} \langle \dot{p}^2 \rangle$ is the radiated power. Since P does not depend on velocity it can not be temperature dependent*, and its value, in terms of the energy absorbed from the background field, can be obtained in the following way. Using Eq. (1) into Eq. (18) for $T=0$ (in this case $R=0$), we get

$$\langle \Delta^2 \rangle_{T=0} = 2\langle pr \rangle \tag{23}$$

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* Also it should be noticed that if P is independent of reference system, it could not be temperature dependent.

and since the right term represents the radiated power, which is independent of T, then

$$\langle \Delta^2 \rangle_{T=0} = 2 P \delta t \quad . \tag{24}$$

Using this result into Eq. (21) we obtain

$$\langle \Delta^2 \rangle - \langle \Delta^2 \rangle_{T=0} = 2 m R k T \delta t \quad , \tag{25}$$

which is precisely Eq. (15) that led to Planck's law[3,6,7].

It should be noted that if we write

$$\langle \Delta^2 \rangle - \langle \Delta^2 \rangle_{T=0} = \langle \Delta^2 \rangle_T \quad , \tag{26}$$

where $\langle \Delta^2 \rangle_T$ are the thermal fluctuations*, then, from Eq. (25), we can conclude that the energy absorbed by the dipole from the thermal field is exactly that one dissipated by means of the velocity dependent force. Hence the radiation is the process by means of which the energy withdrawn from the background field is dissipated. Equation (26) is in a somewhat naive sense similar to a fluctuation-dissipation relationship fro $T = 0$.

In order to be certain on that the radiation damping is the dissipation mechanism it seems worthwhile to consider Eq. (18) from another point of view; Eq. (18) can be rewritten as

$$\frac{p(t+\delta t) - p(t)}{\delta t} = \frac{\Delta}{\delta t} - R p(t) - F_{rad} \quad . \tag{27}$$

Using Eq. (19) and since δt has been assumed to be very small, Eq. (27) can take the form

$$\dot{p}(t) = F(t) - R p(t) - \frac{2e^2}{3c^3} \dddot{p} \quad , \tag{28}$$

where $F_{rad} = \frac{2e^2}{3c^3} \dddot{p}$.

Since F(t) is the random force exerted by the field on the dipole, Eq. (28) and hence Eq. (18), represent the Abraham-Lorentz equation to which the systematic force $-R p(t)$, exerted by the radiation field

* It has been assumed that the thermal and vacuum interactions are statistically independent. See Refs. 6 and 7.

pressure on the dipole, has been added.

3. CONCLUSIONS

On the basis of Einstein and Hopf's work, we can conclude that they obtained Rayleigh's distribution law rather than to Planck's law because of the omission of two fundamental facts: The radiated energy during fluctuations and the existence of the background field.

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