A SU (3) THEORY FOR LEPTONS, QUARKS AND ELECTROWEAK INTERACTIONS

R. Montemayor

Departamento de Física
Facultad de Ciencias, U.N.A.M.
04510-México, D.F.

and

M. Moreno

Instituto de Física, U.N.A.M.
Apartado Postal 20-364, 01000-México, D.F.

ABSTRACT

The gauged version of the SU(3) model for vector interactions of leptons and quarks is developed. After reviewing the main features of the global-spontaneously broken SU(3) scheme for leptons, we conclude that the gauged version of the theory is incompatible with phenomenology. When the quark sector is included in the picture the problems of the gauge theory are overcome. This is so provided that the quark and lepton mass matrices cannot be simultaneously diagonalized. The scheme therefore presents a natural link of the electroweak angles, the existence of two fermion multiplets and the gauge structure of the theory.

RESUMEN

La teoría de norma correspondiente al modelo SU(3) para las interacciones vectoriales de leptones y quarks es desarrollada. Luego de rever las principales características del esquema con simetría global SU(3) espontáneamente rota para leptones, concluimos que la versión de norma de la teoría es incompatible con la fenomenología. Cuando se incluye el sector de quarks en el modelo los problemas de dicha teoría de norma son superados. Esto es así debido a que las matrices de masas de los quarks y de los leptones no pueden ser diagonalizados simultáneamente. Este esquema presenta por lo tanto una conexión natural entre los parámetros angulares característicos de las interaccioneselectrodébiles, la existencia de dos multipletes fermiónicos y la estructura de norma de la teoría.

1. INTRODUCTION

One of the most interesting themes of particle physics, and also of fundamental importance, is the lepton and quark spectroscopy. This spectroscopy involves a wide range of problems, such as the number of leptons and quarks, their masses, properties and classification, the connection between quarks and leptons, the unification of their interactions, features and classification of the gauge bosons and the Higgs, the Cabibbo angle, the Weinberg angle, the CP violation, etc. Many of these still remain open problems, in the framework of the standard model.

The charged and neutral current phenomenology, at least for low energies, is consistent with a global SU(2) symmetry plus electromagnetic corrections, which obviously suggests the existence of a corresponding gauge theory with its intermediate bosons. In the past decades a number of models have been proposed for the electroweak interactions[1], but up to now none has given a complete explanation of all the existing experimental evidence, as it implies very strict constrictions. The simplest model consistent with this phenomenology, is the one of Weinberg-Salam which is based on the gauge SU(2) x U(1)[2]. In its present form, it contains three generations of leptons and quarks[3]. Their left handed states are assigned to SU(2) x U(1) doublets:

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L \begin{bmatrix} u \\ d \end{bmatrix}_L ; \quad \begin{bmatrix} \nu_\mu \\ \mu- \end{bmatrix}_L \begin{bmatrix} c \\ s \end{bmatrix}_L ; \quad \begin{bmatrix} \nu_\tau \\ \tau- \end{bmatrix}_L \begin{bmatrix} t \\ b \end{bmatrix}_L ,$$

whereas the right states for e, μ, τ and the quarks are still not
completely understood; the simplest hypothesis is to assign them to
singlets. Although there is no experimental evidence against this
assignation, there is no clear and direct base for it. On the other hand
there is no direct observation of the ν_τ and, which is more serious, there
exists an important gap in this description, that is not having found the
t quark. The main short-coming of the Weinberg-Salam model is its lack
of account for the connection between the generations. In particular, the
fundamental theoretic problems that remain to be explained are the mass
spectrum of the fermions and the connection between the different genera-
tions. The first generation seems not to present difficulties, at least
qualitatively: we can reasonably suppose that the mass differences $(m_e - m_{\nu_e})$
and $(m_u - m_d)$ are of electromagnetic origin and therefore of the same order
of magnitude, whereas the difference between lepton and quark masses is
due to their different behaviour in the presence of strong interactions,
which implies that the latter difference is greater. But this apparent
qualitative consistence is totally broken in the following generations,
where the previous reasoning should still be entirely valid. The
$(m_\mu - m_{\nu_\mu})$ and $(m_s - m_c)$ differences are much greater than in the first
case and, moreover, are of opposite sign $(m_\mu > m_{\nu_\mu}$ but $m_s < m_c)$. The third
generation complicates even more the problem as it gives place to a new
mass scheme apparently independent of the previous ones.

In brief, although there is no incompatibility between this
model and the experiments, a great number of important ingredients remain
unexplained, as are:

a)   The number of fermions
b)   The mass spectrum of the fermions
c)   The connection between leptons and quarks
d)   The observed values for the Cabibbo and Weinberg angles

At least part of the reply to these questions could be connected
with an enlargement of the gauge group for electroweak interactions,
which could be greater than SU(2) x U(1). This is the motivation that has
driven us to develop in recent works a model that has a SU(3) global group,
in which all the known leptons are within a nonet, and the quarks could
be within either in a octet or a nonet. In both cases, the symmetry is

broken by a Higgs nonet or octet. These are the smallest possible representations for the matter and Higgs fields that lead to a mass spectrum compatible with the observed one. The model here considered is the gauged version for vector-like electroweak interactions; there are several ways of including the axial components, but until now we haven't found a physically motivated and consistent one.

In sections 2 and 3 of this work we present a brief review about the most general SU(3) invariant renormalizable model for a fermion and a scalar nonet whose fermionic sector satisfies the mass spectrum of leptons. In the next section we study the gauge bosons corresponding to the mentioned model, and show that they are completely incompatible with the electroweak phenomenology. In section 5 we propose an enlargement of the model by the inclusion of a new fermionic multiplet, corresponding to the quark sector. This heals the structure of the bosonic sector, that now can be made compatible with the phenomenology. Finally, in section 6, we make a summary and a discussion of the main results and difficulties of this approach to the lepton and quark spectra.

2. THE LAGRANGIAN FOR THE LEPTON SECTOR

The most general SU(3) invariant renormalizable lagrangian for a fermion and a scalar nonet is[4]

$$L = Tr(\overline{\psi}\phi\psi + \partial_\mu\phi\partial^\mu\phi + g_A \overline{\psi}[\phi,\psi] + g_S \overline{\psi} \{\phi,\psi\}) + g_1 Tr(\overline{\psi}\psi) Tr\phi +$$

$$+ g_2 (Tr\overline{\psi}Tr (\phi\psi) + Tr(\overline{\psi}\phi) Tr\psi) + g_3 Tr\overline{\psi} Tr\phi Tr\psi + V(\phi) \quad , \tag{1}$$

where

$$\psi = \sum_{i=0}^{8} \psi_i\lambda_i \quad , \qquad \phi = \sum_{i=0}^{8} \phi_i\lambda_i \quad , \tag{2}$$

$\lambda_1,\ldots,_8$ are the Gell-Mann matrices, and $\lambda_0 = \sqrt{\frac{2}{3}}$ **1** ; the ψ field is a fermion nonet and ϕ is a scalar one. The indices of SU(3) have been defined so that the Gell-Mann-Nishijina relation for the electric charge, the weak isospin and the weak hipercharge[5]

$$q = I_3 + \frac{1}{2} Y \tag{3}$$

is satisfied.

3. THE MASS SPECTRUM OF LEPTONS

The fermions gather mass through a spontaneous breaking of symmetry[6]. This is obtained because some of the scalar fields are asummed to have a non-zero vacuum expectation value. The electric charge is a good quantum number associated to the physical states; therefore, due to the mixtures that this symmetry breaking generates, only the neutral scalar fields can satisfy such a condition:

$$\langle \phi_i \rangle = 0 \qquad (i = 1,2,4,5) \ ,$$

$$\langle \phi_i \rangle = m_i \qquad (i = 0,3,6,7,8).$$

Within the lagrangian (1) the mass terms that contain g_A and g_S contribute as much to diagonal components as to non-diagonal ones; g_1 generates a contribution common to all the fermions; g_2 appears in the singlet and in the mixture terms of the singlet with the neutral compo- nents of the octet; and finally g_3 contributes only to the singlet.

One of the simplest parametrization that can be tried is to choose $m_6 = m_7 = 0$, which leads to a mass relation of the Gell-Mann-Okubo type with first order electromagnetic corrections taken into account. This relation is evidently not satisfied by the leptons's masses[7], which makes it necessary to explore a more general scheme of symmetry breaking, considering m_6 and m_7 different from zero (this will lead to mixtures between equal charge elements of the nonet). In preceeding works[8] we demostrated that there are only two solutions (related by $b \to -b$) for the mass spectrum of the nonet, consistent with the spectrum of leptons, that are (see Fig. 1)

$$m_e = m_\mu = 0 \quad ,$$

$$m_\tau = m_{\overline{K}^0} = -3(1+\Gamma)a \quad , \tag{5}$$

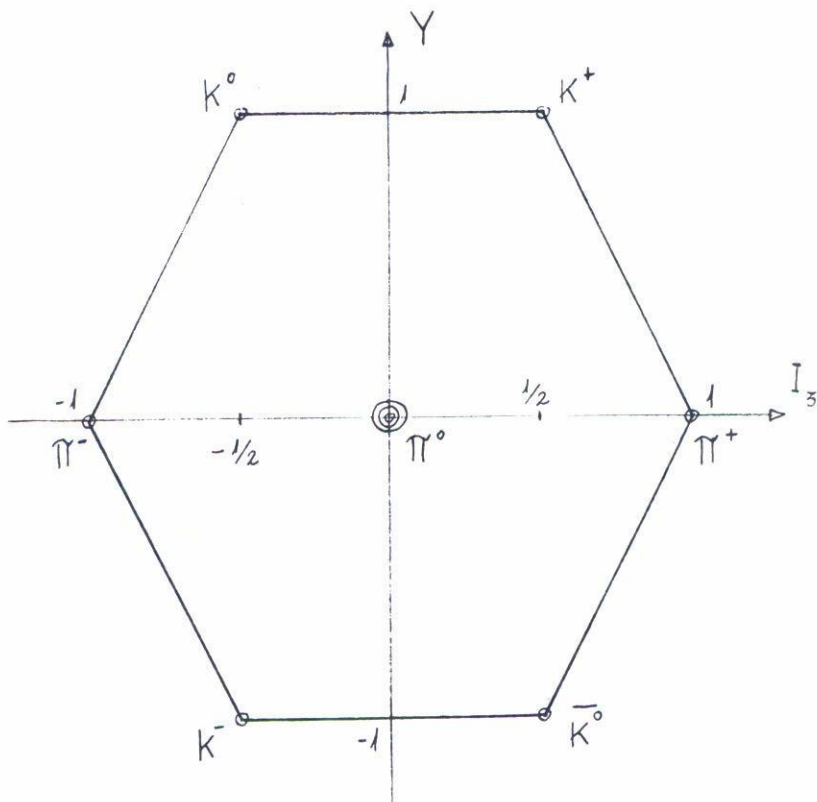$$m_\lambda = m_{k^0} = -3(1-\Gamma)a \quad .$$

Fig. 1. Weight diagram for the leptons.

The parameters of the model:

$$a = \tfrac{1}{2} \, g_S(m_3 + \frac{1}{\sqrt{3}} \, m_8), \qquad\qquad m_{67} = m_6 + im_7 \quad ,$$

$$\Gamma = g_A/g_S \quad , \qquad\qquad\qquad \mu = \sqrt{\tfrac{2}{3}} \, (g_S + 3g_1)m_0 \quad , \qquad (6)$$

$$b = \tfrac{1}{2} \, g_S[\sqrt{3} \, m_8 - m_3)^2 + 4|m_{67}|^2]^{1/2}, \quad \mu_0 = \sqrt{6} \, (2g_2 + 3g_3)m_0$$

satisfy in this solution the relations $\quad a = -\frac{1}{3} \, b = -\frac{1}{2} \, \mu \cong -\frac{2}{5} \, \mu_0.$

The electron and the muon have a null mass at this order; their non-null masses would stem from the contribution of higher order terms[9]. In the center of the nonet there are two neutral leptons without mass (that are identified with the neutrini ν_e and ν_μ), and a third neutral lepton of very great mass. The scale of the parameter a is fixed by the mass of the τ :

$$|a| = \frac{m_\tau}{3(1-|\Gamma|)} \qquad , \qquad (7)$$

and the lower bound for the λ mass given a range for Γ:

$$0.8 < |\Gamma| < 1.2 \qquad . \qquad (8)$$

Finally the lower bound for the mass of the remaining neutral lepton is:

$$|m| > 4|a| \cong 6\, m_\tau \qquad . \qquad (9)$$

In brief, it is possible to make the leptonic mass spectrum compatible with a SU(3) nonet in which the symmetry is broken by a Higgs nonet. The main predictions are:

1) A neutral lepton with a mass close to the τ mass

2) Another charged lepton, λ, with a mass greater than 18 GeV

3) And, furthermore, two new neutral leptons, one with a mass close to the λ mass, and another with a lower bound of

$$\frac{1}{3} \left(\frac{2}{3} m_\lambda + m_\tau \right)$$

4) A massless τ neutrino does not exist. The neutral partner in τ decay would be an admixture of ν_e , ν_μ and higher mass neutral components of the SU(3) multiplet.

4. THE INTERACTIONS BETWEEN LEPTONS

In the preceeding section, we recalled that there are solutions with all the known leptons in a nonet of SU(3) consistent with its mass spectrum. This constitutes an interesting result, but it is far from being a sufficiently solid base for the viability of the model. In fact, a more

detailed test for this scheme must consider the interaction between the leptons and the structure of the gauge boson sector of the model, that must satisfy strong constrictions of both experimental and theoretical character.

In this contex, we will study the gauge bosons corresponding to the lagrangian (1) that appear when we consider a local, gauged, SU(3) symmetry group instead of a global one. In particular, we will center our attention on its mass spectrum, which is a crucial point to be considered. From the fermion and scalar boson lagrangian, studied in the former section, we obtain the corresponding gauge theory replacing the derivatives with respect to the space-time coordinates by the corresponding covariant derivatives[6]:

$$\partial_\mu \xi_k \to D_\mu \xi_k = \partial_\mu \xi_k - g(F_\ell)_{km} A^\ell \xi^m \quad , \tag{10}$$

where ξ^k fields are members of an octet, and the A^ℓ form the gauge boson octet. The components of the SU(3) generators in the regular representation are the structure constants of the group:

$$(F_\ell)_{km} = f_{k\ell m} \quad . \tag{11}$$

The singlet doesn't contribute to the leptons' dynamics because it is decoupled with the matter fields of the octet.

The mass terms for the gauge bosons come from

$$\partial_\mu \phi^k \partial^\mu \phi_k \to D_\mu \phi^k D^\mu \phi_k = \partial_\mu \phi^k \partial^\mu \phi_k + g^2 f_{k\ell m} f_{kst} \phi^m t \phi^t A^\ell A^s +$$
$$+ 2g \partial_\mu \phi^k f_{k\ell m} A^\ell \phi^m \quad . \tag{12}$$

Through the mechanism of spontaneous symmetry breaking, one generates in the lagrangian the following mass terms:

$$L_{bm} = g^2 f_{k\ell m} f_{kst} m_m m_t A^\ell A^s \quad . \tag{13}$$

The resulting mass matrix has the structure

$$M = \begin{pmatrix} M^+ & & \\ & M^- & \\ & & M^0 \end{pmatrix} \quad , \tag{14}$$

where

$$M^+ = \frac{\delta^2}{4} \begin{pmatrix} (3a+y)^2 + |m_{67}|^2 & -6a\ m_{67}^* \\ -6a\ m_{67} & (3a-y)^2 + |m_{67}|^2 \end{pmatrix} \quad , \tag{15}$$

$$M^- = M^+ \ (m_{67} \longleftrightarrow m^*_{67}) \quad , \tag{16}$$

$$M^0 = \frac{\delta^2}{4} \begin{pmatrix} |m_{67}| & -\sqrt{3}|m_{67}|^2 & -\sqrt{2}\ y\ m_{67}^* & -\sqrt{2}\ y\ m_{67} \\ -\sqrt{3}|m_{67}|^2 & 3|m_{67}|^2 & \sqrt{6}\ y\ m_{67}^* & \sqrt{6}\ y\ m_{67} \\ -\sqrt{2}\ y\ m_{67} & \sqrt{6}\ y\ m_{67} & 4y^2 + 2|m_{67}|^2 & -2\ m_{67}{}^2 \\ -\sqrt{2}\ y\ m_{67}^* & \sqrt{6}\ y\ m_{67}^* & -2\ m_{67}^*{}^2 & 4y^2 + 2|m_{67}|^2 \end{pmatrix} \quad , \tag{17}$$

with

$$\delta = \ ^-g/g_S \quad ; \qquad y = \tfrac{1}{2}\ g_S(m_3 - \sqrt{3}\ m_8) \quad . \tag{18}$$

The order adopted for rows and columns is

$$M^\pm : \qquad B_\pi^\pm = \frac{1}{\sqrt{2}}\ (A_1 \pm iA_2) \quad ; \qquad B_k^\pm = \frac{1}{\sqrt{2}}\ (A_4 \pm iA_5)$$

$$M^0 : \qquad B_3 = A_3 \ ; \quad B_8 = A_8 \ ; \quad B_{k^0} = \frac{1}{\sqrt{2}}\ (A_6 + iA_7) \ ; \quad B_{\overline{k}^0} = \frac{1}{\sqrt{2}}\ (A_6 - iA_7).$$

$$\tag{19}$$

The gauge fields B form the octet showed in Fig. 2, with the weak strangeness and the electric charge there indicated.

The mass matrix (14) is diagonalized by two consecutive rotations in the R^8 space, that can be

$$\begin{cases} \text{a rotation with axis 7 and angle } \ \operatorname{tg}\theta_7 = \dfrac{m_6}{y} \\ \text{a rotation with axis 6 and angle } \ \operatorname{tg}\theta_6 = \dfrac{m_7}{\sqrt{y^2 + m_6^2}} \end{cases} \tag{20}$$
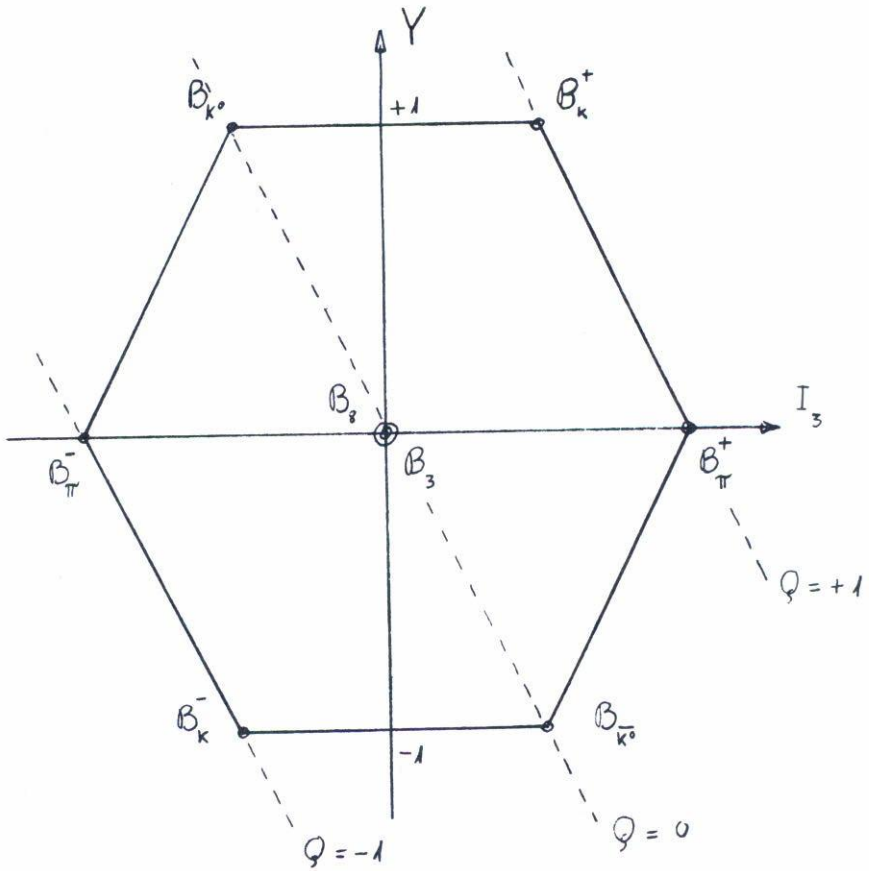
Fig. 2 Weigth diagram for gauge bosons.

or, similarly, rotating first with axis 6 and next axis 7 (6⟵⟶7). The fields corresponding to the mass matrix diagonalized by the transformation (20) are

$$W_1^{\pm} = \alpha_- \, B_\pi^{\pm} - \alpha_+ \, B_k^{\pm} \quad ,$$

$$W_2^{\pm} = \alpha_- \, B_\pi^{\pm} + \alpha_+ \, B_k^{\pm} \quad ,$$

$$W_1^0 = \frac{1}{4} \left[(3+\eta_6) \, B_3 + \sqrt{3} \, (1-\eta_0) B_8 + \sqrt{2} \, \beta_+ \, B_{k^0} + \sqrt{2} \, \beta_- \, B_{\bar{k}^0} \right] \quad ,$$

$$W_2^0 = \frac{1}{4} \left[\sqrt{3}(1-\eta_0) B_3 + (1+3\eta_0) B_8 - \sqrt{6}\beta_- \, B_{k^0} - \sqrt{6} \, \beta_+ \, B_{\bar{k}^0} \right] \quad , \quad (21)$$

$$W_3^0 = \frac{i}{4} \left[\sqrt{2} \, \gamma_+ \, B_3 - \sqrt{6} \, \gamma_- \, B_8 + 2(\eta_s - \kappa_+) B_{k^0} + 2(\eta_s + \kappa_-) B_{\bar{k}^0} \right],$$

$$W_4^0 = \frac{i}{4} \left[\sqrt{2} \, \gamma_- \, B_3 - \sqrt{6} \, \gamma_+ \, B_8 - 2(\eta_s - \kappa_-) B_{k^0} - 2(\eta_s + \kappa_+) B_{\bar{k}^0} \right],$$

with

$$\alpha_{\pm} = \cos \frac{\theta_6}{2} \cos \frac{\theta_7}{2} \pm i \, \text{sen} \frac{\theta_6}{2} \, \text{sen} \frac{\theta_7}{2} \quad ,$$

$$\beta_{\pm} = \cos \theta_6 \, \text{sen} \, \theta_7 \pm i \, \text{sen} \, \theta_6 \quad ,$$

$$\gamma_{\pm} = \text{sen} \, \theta_6 \cos \theta_7 \pm i \, \text{sen} \, \theta_7 \quad , \quad (22)$$

$$\eta_6 = \cos \theta_6 \cos \theta_7 \quad ,$$

$$\eta_s = \text{sen} \, \theta_6 \, \text{sen} \, \theta_7 \quad ,$$

$$\kappa_{\pm} = i(\cos \theta_6 \pm \cos \theta_7) \quad .$$

The corresponding structure of the mass spectrum is very simple:

$$\mu_{w_1^+} = \mu_{w_1^-} = \frac{\delta}{2} \, (3a + b) \quad ,$$

$$\mu_{w_2^+} = \mu_{w_2^-} = \frac{\delta}{2} \, (3a - b) \quad ,$$

$$\mu_{w_1^0} = \mu_{w_2^0} = 0 \quad , \quad (23)$$

$$\mu_{w_3^0} = \mu_{w_4^0} = \delta b \quad .$$

This model contains two neutral gauge fields with zero mass. One of them is a Goldstone boson, as is well known from the symmetry breaking scheme considered (the only generator under which the vacuum is degenerated is

$q = F_3 + \frac{1}{\sqrt{3}} F_8$). The other massless boson is only a geometric accident and adquires a non-null mass trough higher order contributions. Nevertheless at the lowest order here considered we have two neural long range interactions, which is absolutely incompatible with the experimental evidence. It is pretty well established that there exists only one neutral boson (the photon), and any other neutral boson must have a mass at least of the order of magnitude of the lighter charged bosons. This last result apparently nullifies all the validity that could have the model, and it would be so unless we introduce additional fields that allow us to modify the previous results.

In particular, the boson mass spectrum corresponding to the spectrum (5) for the leptons is

$$\mu_{w_1^+} = \mu_{w_1^-} = 0 \quad ,$$

$$\mu_{w_2^+} = \mu_{w_2^-} = M \quad ,$$

$$\mu_{w_1^0} = \mu_{w_2^0} = 0 \quad ,$$

$$\mu_{w_3^0} = \mu_{w_4^0} = M \quad ,$$

where

$$M = 3_\delta a \qquad , \qquad\qquad\qquad (25)$$

which is clearly incompatible with electroweak phenomenology.

5. ENLARGEMENT OF THE MODEL: THE QUARK SECTOR AND ITS CONSEQUENCES ON GAUGE BOSONS

The model developed in the preceeding sections can easily be extended to include the quarks. The reasons for doing this are:

   i)   the paralelism between the mass and charge spectra of the leptons
        and of the quarks,
   ii) the fact that both families of fermions have electroweak
        interactions, and
   iii) the necessity of introducing new fields so as to obtain interactions
        consistent with the experiment.

The former points have already been exploited in the past and are the basis for the concept of generations.

The extension of the scheme that we propose for quarks is direct: we simply assume that the quarks form a SU(3) octet or nonet, and therefore their lagrangian has the same form as the leptonic lagrangian, except that

$$\psi^q = \sum_{i=1}^{8} \psi_i^q \lambda_i \qquad \text{and} \qquad \phi^q = \sum_{i=1}^{8} \phi_i^q \lambda_i \qquad (26)$$

Two predictions naturally arise from this scheme: there is no-top quark of electric charge 2/5 and there must exist two new quarks of charge -4/3. This is due to the fact that, according to what is known, there must exist at least three quarks of charge -1/3, the d, s and b quarks, and the only place they can occupy in the multiplet is the $(k^0, \bar{k}^0, \pi, \eta)$ sector. This means that the octet must be displaced in such a way that π^+ and k^+ have a 2/3 charge and π^- and k^- a = -4/3 charge (Fig. 3).
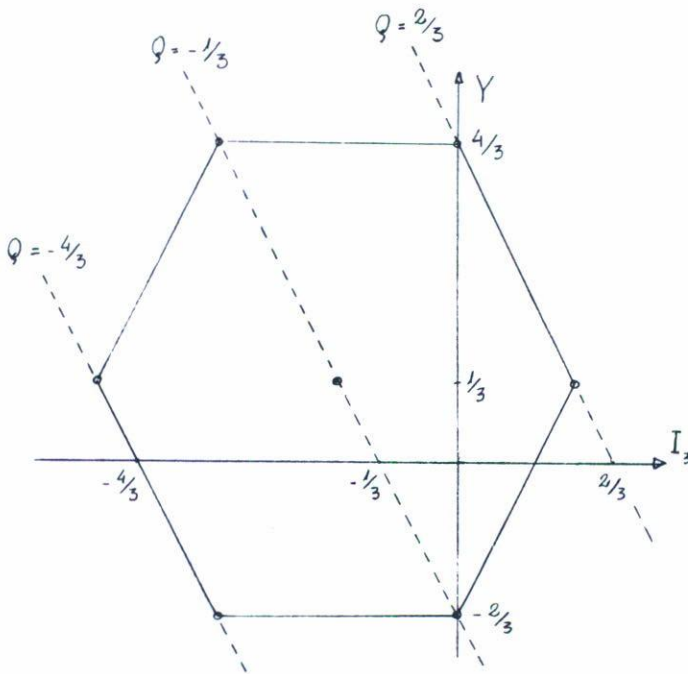

Fig. 3. Weight diagram for quarks.

We obtained in a previous work[8] the mass spectrum for a SU(3) multiplet. Using the same notation as in Refs. 8 we have:

$$m_{k\pm}^q = \mu_q + \frac{1}{2}(1 \pm 3\Gamma_q) a_q - \frac{1}{2}(1 \mp \Gamma_q) b_q ,$$

$$m_{\pi\pm}^q = \mu_q + \frac{1}{2}(1 \pm 3\Gamma_q) a_q + \frac{1}{2}(1 \mp \Gamma_q) b_q ,$$

$$m_{\substack{k^0 \\ \overline{k}^0}}^q = \mu_q - a_q \pm \Gamma_q b_q ,$$ (27)

$$m_{\substack{\pi^0 \\ \eta}}^q = \mu_q \pm \left[a_q^2 + \frac{1}{3} b_q^2 \right]^{\frac{1}{2}}$$

With regard to the gauge bosons, their mass is now due as much to the contribution of the Higgs of the leptons as well as to that of the Higgs of the quarks. Thus we have that their spectrum, in terms of the parameters of the quark sector and M , defined in Eq. (25), is now

$$\mu_{B_{1,2}^+}^2 = \mu_{B_{1,2}^-}^2 = \frac{1}{2}M^2 + (9\,a_q^2 + b_q^2) \pm \left[(\tfrac{1}{2}M^2 + 6a_q b_q)^2 - 6a_q(y_q + b_q)M^2 \right]^{\frac{1}{2}},$$

$$\mu_{B_1^0}^2 = 0 \quad ,$$

$$\mu_{B_2^0}^2 = M^2 + 4 b_q^2 \quad ,$$ (28)

$$\mu_{B_{3,4}^0}^2 = \frac{1}{2}(M^2 + 4b_q^2) \pm \frac{1}{2}\left[M^4 + 16b_q^4 + 8(b_q^2 - 2|m_{67}^q|^2)M^2 \right]^{\frac{1}{2}} \quad .$$

This spectrum contains a neutral boson of zero mass (B_1^0), two charged bosons and a neutral one $(B_2^\pm, B_4^0)$ with masses of a given order of magnitu de, and finally the remaining $(B_1^\pm, B_2^0)$ with masses that can be made, if necessary, much greater that the previous ones. Thus the generated spectrum is compatible with the existing phenomenology.

An interesting characteristic of this model is that the different multiplets that appear in it are rotated among themselves. If a rotation with the generator $\overline{q} = F_3 - \sqrt{3}\,F_8$ is done, it is possible to eliminate the dependence of the spectra on m_7 , in such a way that $m_{67} \to m_6$, without any other consequence in the mass expressions. In doing this it is very simple to give the rotations between the multiplets. Taking as reference

the diagonal representation of the lepton multiplet, the diagonal representation of the quark mass matrix is rotated with respect to it by an angle defined by

$$\text{ctg } \theta_q = \frac{y_q}{m_6^q} \quad , \tag{29}$$

with respect to axe 7 and y_q defined as in Eq. (18). Analogously the diagonalized charged sector of the boson octet forms with the leptons an angle defined by

$$\text{ctg } \theta_{cb} = \text{ctg } \theta_q - \frac{1}{3} \frac{b_\ell^2}{a_q \, m_6^q} \quad , \tag{30}$$

whilst the neutral sector is decoupled in two subspaces ($(6,7)$, $(3,8)$) by a rotation also with axe 7, but by an angle

$$\text{ctg } 2\theta_{nb} = \text{ctg } 2\theta_q + \frac{1}{2} \frac{b_\ell^2}{y_q \, m_6^q} \quad . \tag{31}$$

The complete diagonalization of the mass matrix of neutral fields is finally obtained by an additional transformation that is out of the group. The angles that appear here correspond to the angles that are introduced phenomenologically in the standard model. The angle between leptons and quarks gives a contribution to the Cabibbo angle (it can also have a contribution that stems directly from the heavy charged bosons), whereas the fact that the charged and neutral boson sectors are rotated among themselves originates the Weinberg angle.

6. FINAL REMARKS

In this work, we have analized some consequences of gauging the global SU(3) group symmetry that was proposed in earlier works. Two conclusions may be drawn from this analysis. First, if only one Higgs multiplet contributes to the gauge boson mass matrix, then two long range forces arise in lowest order perturbation theory. Because the Goldstone theorem implies that only one massless gauge boson should arise, the second massless gauge boson should acquire mass in higher order perturbation theory. This, in turn, means that the effective coupling constant for the generated neutral current is of the order of $\alpha^2/M^2_{w^\pm}$. No such

neutral current exists and, in consequence, this theory is not acceptable to describe electroweak interactions.

Second, when an additional Higgs multiplet is allowed to contribute to the gauge boson masses, the spurious zero mass gauge boson acquires mass. This is so provided that the orbits of the vacuum expectation values of the two Higgs families are rotated with respect to one another. Such kind of Higgs families are called for when one tries to account for both the quark and lepton mass spectra, and therefore this scheme realizes naturally the existence of angles like the Cabibbo and Weinberg angles.

The connection among the phenomenological angles and the rotation angles was pointed out superficially at the end of section 5. A deeper and more careful study of that connection is obviously needed in order to proceed.

However, we are reluctant to begin such and effort because we think that in the near future some of the predictions of the model will be tested. In particular, the existence of -4/3 electric charge quarks and nonexistence of +2/3 quarks is expected for as a signature of the correctness of this framework. Furthermore, the most direct and naive way to account for the V-A nature of the charged weak interactions, which would be to postulate that the left and right parts of the fermions belong to different group representations is both aesthetically ugly and practically bothersome. This is so because the only way in which triangle anomalies do not destroy the renormalizability of the theory is by assigning the right parts of the fermions to singlets, as is done in the standard model, or to octets[10]. The singlet choice introduces to many parameters in the theory; the octet selection conflicts with the fermion mass spectra.

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