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ISOSPIN QUINTET IN THE A = 4 SYSTEM

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ABSTRACT

The generalized R-matrix methodology of Lane and Robson has been used to calculate the $(0^+,2)$ isospin quintet mass excesses of ^{4}n(g.s.), ^{4}H, ^{4}He, ^{4}Li and ^{4}Be(g.s.). The mass excess values were found to be 50.784, 50.902, 51.025, 51.606 and 52.292 MeV, respectively. Large d(8.3 keV) and e(33.6 keV) terms are required to obtain good agreement with the isobaric multiplet mass equation $M = a + bT_z + cT_z^2 + dT_z^3 + eT_z^4$. The A = 4 coefficients a, b and c are consistent with the corresponding coefficients derived from quintet data in the A = 8 - 32 systems.

RESUMEN

La metodología generalizada de la matriz R debida a Lane y Robson ha sido utilizada para calcular los excesos de masa del quinteto de isoespín $(0^+,2)$ correspondientes a ^{4}n(n.b.), ^{4}H, ^{4}He, ^{4}Li y ^{4}Be(n.b.). Los excesos de masa encontrados fueron: 50.784, 50.902, 51.025, 51.606 y 52.292 MeV, respectivamente. Se requiere que los términos d(8.3 keV) y e(33.6 keV) sean grandes para obtener un buen ajuste con la ecuación de masa del multiplete isobárico $M = a + bT_z + cT_z^2 + dT_z^3 + eT_z^4$. Los coeficientes a, b y c para A = 4 son consistentes con los coeficientes correspondientes obtenidos a partir de datos de quintetos en los sistemas con A = 8 - 32.

1. INTRODUCTION

Studies of the systematics of nuclei have led to an improved understanding of nuclear properties. For example, studies of the systematics of binding energies led to the shell model description of nuclei. A more modest success has been the parameterization of mass excess values of isospin multiplets in terms of formulas with mass dependent coefficients.

Herein, we will consider the T = 2 isospin multiplet in the A = 4 system and attempt to determine if this multiplet is related to T = 2 multiplets in heavier systems. Our A = 4 study will be based on model calculations because of the limited amount of T = 2 data for the mass-four system. However, we will calculate more information than the relative energy shifts of the T = 2 states composing the multiplet. The model calculations will present the first theoretical calculation of the excitation energies and widths of the lowest lying $(0^+,2)$ levels in the ^{4}n, ^{4}H, ^{4}He, ^{4}Li and ^{4}Be systems.

The mass-4 isospin quintet represents an interesting test for the applicability of the quadratic isobaric multiplet mass equation (QIMME)[1]:

$$M(A, T, T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2 \quad , \qquad (1)$$

where the coefficients a, b and c are determined by fitting a given isospin multiplet. Deviations of theoretical calculations of isospin quintets from the QIMME arise from a number of causes. These are: (a) multibody forces, (b) Coulomb effects which expand the radial extent of the wave function[3], (c) Coulomb mixing with $T \neq 2$ levels which give rise to an eT_z^4 term in isospin quintets (T = 2) and a dT_z^3 term in isospin quartets (T = 3/2)[3], and (d) model space truncation[3]. The form of Eq. (1) assumes the aforementioned effects are suppressed. Deviations from the QIMME are represented by additional terms such as $d(A,T)T_z^3$ and $e(A,T)T_z^4$. In a recent review of T = 3/2 quartets[3], the quadratic form of the IMME was found to work well for 21 or 22 complete quartets.

For the T = 2 quintet, fewer systems have been studied. In par-

ticular, data for the A = 8, 12, 16, 20, 24 and 32 systems are available[4-7]. Significant cubic and quartic terms are needed to fit the data for the A = 8 quintet[4,5], but not for the heavier systems[6,7]. It is of interest to investigate the systematics of isospin quintets by determining the applicability of the QIMME in the A = 4 system.

Specific applications to the A = 4 system are warranted because the isospin quintet encompasses the ground states of the tetraneutron (^{4}n) and tetraproton (^{4}Be) and A = 4 isospin studies lead to information concerning the possible deviation from the QIMME. Furthermore, extending quintet studies to the A = 4 system is desirable because it is the lightest system in which T = 2 states can exist, and therefore provides an end point for a systematic study of isospin quintets. Finally, it is hoped that a study of the A = 4 system will foster further study of the T = 2 levels in light nuclei.

This paper will investigate the $(J^\pi,T) = (0^+,2)$ quintet of levels in the A = 4 system which includes the ground state of the tetraneutron and tetraproton. The tetraneutron ($T_z = 2$) ground state and the lowest lying T = 2 level in ^{4}He($T_z = 0$) were previously discussed[8]. The energies of the remaining multiplet members ^{4}H($T_z = 1$), ^{4}Li($T_z = -1$) and ^{4}Be($T_z = -2$) follow from the formalism of Ref. 8.

2. METHODOLOGY

The calculations presented herein are based on the generalized R-matrix methodology of Lane and Robson[9]. Model basis states are expanded on a basis of properly symmetrized translationally invariant harmonic oscillator eigenstates including atates of up to $4\hbar\omega$ of oscillator excitation[8,10]. The model interaction is based on a modification of the Sussex matrix elements[10,11]. Once the basis states $|\lambda\rangle$ and model interaction are chosen, the mass-four eigenspectrum is determined by the solution of the equation

$$\sum_{\lambda'} \left[\langle \lambda | H - E | \lambda' \rangle \right] A_{\lambda'} = 0 \quad , \tag{2}$$

where H is the mass-four Hamiltonian which is expressed as

$$H = -\sum_{K} (\hbar^2/2\mu_K)\nabla_k^2 + \sum_{i<j} V_{ij} + \sum_{i<j} V_{ij}^{Coulomb} \quad . \tag{3}$$

In Eq. (3), K runs over the internal coordinates and i and j run over nucleon coordinates (see Fig. 1, Ref. 10). The μ_K are reduced masses in the K-th coordinate[10], V_{ij} is the nuclear interaction between the i and j nucleon, and $V_{ij}^{Coulomb}$ is the Coulomb interaction. Using standard techniques, the desired many-body matrix elements of the Hamiltonian can be expressed in terms of standard one —or two— body matrix elements evaluated over all space[8,10,12]. Additional details of the model, choice of basis states and method of solution are found in Refs. 8, 10 and 12.

3. RESULTS AND DISCUSSION

Level energies for the $(0^+,2)$ states were calculated following the methodology of Ref. 8. The energies E were converted into mass excess values M by means of the relationship.

$$M = ZM_p + NM_n - E \quad , \tag{4}$$

where Z is the number of protons in the nucleus, N is the number of neutrons, M_p is the mass excess of the proton and M_n is the mass excess of the neutron. The results of the model calculations are summarized in Table I.

Before discussing specific mass excess values, we will discuss the level widths of the members of the $(0^+,2)$ quintet in the A = 4 system. Model widths are obtained from a modification[9,13,14] of Eq. (2):

$$\sum_{\lambda'} \left[\langle \lambda | H - E | \lambda' \rangle + \sum_{c} \gamma_{\lambda c}(b_{\lambda' c} - b_c)\gamma_{\lambda' c} \right] A_{\lambda'} = 0 \quad , \tag{5}$$

where $\gamma_{\lambda c}$ and $b_{\lambda c}$ are the reduced widths[15] and logarithmic derivatives associated with the expansion states $|\lambda\rangle$. The quantities b_c are related to radial wave functions $U_c(r_c)$ in the physical two-body channels:

TABLE I

Nucleus	T_z	Mass Excess (MeV)
^4n	2	50.784
^4H	1	50.902
^4He	0	51.025
^4Li	-1	51.606
^4Be	-2	52.292

Table I. $A = 4$, $J^\pi = 0^+$, $T = 2$. Quintet mass excess values.

$$b_c = \left(\frac{r_c}{U_c} \frac{dU_c}{dr_c} \right)_{r_c = a_c} . \tag{6}$$

The channels (c) are limited to binary break-up clusters A and B. Multibody break-up channels involving three or more clusters are not included in Eq. (5) because of orthogonality difficulties. The inclusion of binary channels requires A and B to be bound systems[9,10,13,14].

Specific formulas for the positions and widths of R-matrix resonances are available in the literature[15,16]. In particular, the resonance energy E_R^μ corresponding to the level E_μ may be defined as the solution to the equation

$$E_R^\mu = \mathrm{Re}\left[E_\mu - \xi_\mu(E_R^\mu) \right] . \tag{7}$$

The total width of the resonance is then obtained from the equation

$$\Gamma_R = -2\ \mathrm{Im}\left[E_\mu - \xi_\mu(E_R^\mu) \right] , \tag{8}$$

where E_μ is obtained by transforming Eq. (5)[17] and ξ_μ is defined in terms of known R-matrix energies and reduced widths E_μ, $\gamma_{\mu c}$ and standard Coulomb radial functions[15,16].

In order to calculate widths at least one decay channel is required in the c sum of Eq. (5). For ^{4}He, the n + ^{3}He, p + ^{3}H and d + ^{2}H channels are included in the c sum, but the n + p + ^{2}H and n + n + p + p(^{2}n + ^{2}p) channels are omitted[10]. For ^{4}H, only the n + ^{3}H channel is included. In the ^{4}H system, the ^{2}n + ^{2}H channel is omitted because the ^{2}n cluster is unbound. Similarly, only the p + ^{3}He channel is included in the ^{4}Li system. The ^{4}n and ^{4}Be systems have no bound break-up clusters —i.e., ^{2}n, ^{3}n, ^{2}p and ^{3}p are unbound[8]. Therefore, the formalism of Eq. (5) can only provide a crude approximation to the level widths in ^{4}n and ^{4}Be. Since the ^{2}n and ^{2}p systems are more bound than ^{3}n and ^{3}p, the $|T_z| = 2$ systems will only include ^{2}n + ^{2}n or ^{2}p + ^{2}p channels. The aforementioned channel sum limitations also impact the resonance positions E_R^μ of the quintet members. Therefore, shell model energies which are based on Eq. (2) will be used in the isospin multiplet analysis. The use of shell model level energies eliminates artificial level positions changes due to the incomplete treatment of break-up channels by the model. However, model truncation errors are still present because of the $4\hbar\omega$ basis space limitation. The truncation errors can not be further evaluated because $4\hbar\omega$ is a practical computational limit[10]. However, as noted in Ref. 3, these errors will be small and are observed in quartets through the introduction of a dT_z^3 term with a d coefficient which is only a fraction of a keV.

The use of Eq. (5), Eq. (8) and the channels noted above leads to width information for the $(0^+,2)$ quintet. The widths for ^{4}n, ^{4}H, ^{4}He, ^{4}Li and ^{4}Be are 600 keV, 13 keV, 9 keV, 12 keV and 480 keV, respectively. The $(0^+,2)$ levels are narrow enough to be considered states and their widths are of the same order of magnitude as those for T = 2 states in the $A \geq 8$ systems. The A = 4, T = 2 states are also more narrow than most of the known resonances in the ^{4}H, ^{4}He and ^{4}Li systems[10,18]. In particular, T = 1 resonances in ^{4}He have widths of about 10 MeV and even with widths this large the T = 1 resonances are considered to be states[10,18].

In order to facilitate the comparison with $A \geqslant 8$, $T = 2$ multiplets, the mass excess values, summarized in Table I, were fit to the QIMME and to the equations which contain cubic $(a + bT_z + cT_z^2 + dT_z^3)$, quartic $(a + bT_z + cT_z^2 + eT_z^4)$ and cubic plus quartic $(a + bT_z + cT_z^2 + dT_z^3 + eT_z^4)$ terms. Table II summarizes the results of a least squares (LSQ) fit to the calculated mass excess values in terms of a parameter Δ defined as

$$\Delta = \left[\sum_{i=1}^{5} (M^i_{LSQ} - M^i)^2 \right]^{1/2} , \qquad (9)$$

where M^i is the calculated (model) mass excess value of Table I for the i-th system in the quintet, M^i_{LSQ} is the mass excess derived from the coefficients of the least squares fit and the sum is over all five members of the $T = 2$ quintet. The addition of the term dT_z^3 to the QIMME provides only a 5 percent reduction in Δ. However, the addition of the eT_z^4 terms yields about a 70 percent improvement (decrease in Δ). The five parameter fit suggests d and e values which are essentially the same as the individual four parameter fits. In any event, the need for sizeable d and e coefficients is suggested.

TABLE II

a	b	c	d	e	Δ*
51.0941	-0.3720	0.1139	———	———	0.1014
51.0941	-0.3437	0.1139	-0.0083	———	0.0963
51.0250	-0.3720	0.2626	———	-0.0336	0.0316
51.0250	-0.3437	0.2626	-0.0083	-0.0336	0.0000

* The deviation Δ could also be defined by dividing the right hand side of Eq. (9) by $(5 - n)$ where n is the number of parameters in the fit $(\Delta' = \Delta/(5 - n))$. With this definition, the deviations noted above become 0.0507, 0.0963, 0.0316 and 0.0000, respectively.

Table II. Predicted coefficients in units of MeV for the IMME in the $A = 4$ system.

Table III summarizes the IMME coefficients and suggests that the five parameter fit to the A = 4 system leads to a, b and c coefficients which are consistent with heavier systems. The a and b coefficients increase with decreasing mass and the A = 4 c coefficient is between the 0.20 - 0.28 MeV range suggested in the A = 8 - 32 data. The three parameter fit leads to a c coefficient of about 0.11 which is outside the 0.20 - 0.28 range. This also suggests that d and e terms are needed to properly fit the A = 4 calculations. The eT_z^4 term is more important than the dT_z^3 term in minimizing Δ and a value e = 43 keV is suggested. This value is considerably larger than e values from A = 8 - 32 systems which range between 2 and 7 keV. The A = 4 d coefficient of 8 keV lies outside the 3 - 6 keV range of the other quintets.

TABLE III

A	a(MeV)	b(MeV)	c(MeV)	d(MeV)	e(MeV)	Reference
4	51.0250	-0.3437	0.2626	-0.0083	-0.0336	This work
8	32.4360	-0.8907	0.2173	0.0044	0.0024	5
12	27.5950	-1.7628	0.2434	0.0044	———	6
16	17.9840	-2.5995	0.2220	0.0025	———	6
20	9.6908	-3.463	0.278	0.005	-0.007	7
24	1.5058	-4.1818	0.2235	0.0060	-0.0021	6
32	-13.9651	-5.4648	0.1996	-0.0033	0.0019	6

Table III. Summary of IMME coefficients for T = 2 quintets.

The second order form of the IMME is required because of the extent to which the isospin symmetry is broken. Coupling of individual quintet levels to neighboring $T \neq 2$ levels is influenced by the characteris-

tics of the neighboring states. The coupling is sensitive to the number of neighboring levels as well as to their location. For example, the isospin mixing ratio at the peak of the ^{4}He giant dipole resonance (GDR) can not be explained by the currently accepted ^{4}He level spectrum[19,20]. However, the addition of a new $(1^-,0)$ level to the spectrum brings the calculated isospin mixing ratio into agreement with experiment[20].

4. CONCLUSIONS

Model calculations have been performed for the mass excess values of $(0^+,2)$ states in the A = 4 isospin quintet. A three parameter fit of the model mass excess values to the QIMME $(a + bT_z + cT_z^2)$ leads to a c coefficient which lies outside the range of values derived from heavier nuclei. The five parameter fit $(a + bT_z + cT_z^2 + dT_z^3 + eT_z^4)$ to the IMME leads to a, b and c coefficients which are consistent with those derived from heavier systems. The d coefficient is somewhat larger in magnitude than values derived from $8 \leqslant A \leqslant 32$ nuclei. The A =4e coefficient has a value of 34 keV which is considerably larger than the 2 - 7 keV range derived from A = 8 - 32 systems. Issues concerning the need for cubic and quartic terms can only be resolved with experimental data. Data involving properties of the $(0^+,2)$ quintet in the A = 4 system would provide insight into the systematics of isospin quintets as well as information concerning the importance of multibody forces in light nuclei.

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