Revista Mexicana de Física 30 no. 2 (1984) 217-224

INFLUENCE OF THREE-BODY FORCES ON THE ELECTRIC MONOPOLE MATRIX ELEMENT AND THE EO EWSR IN THE ^{4}He SYSTEM

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(recibido sept. 22, 1983; aceptado oct. 7, 1983)

ABSTRACT

The EO matrix element, connecting the ground and first excited states in ^{4}He, is evaluated using both two-body and two-body plus three-body forces. Realistic agreement with the measured EO matrix element is obtained by utilizing two-body plus three-body forces in the model Hamiltonian. A consideration of a variety of ^{4}He data, including the ground state binding energy, ground state rms radius, ground state charge form factor, first excited state energy, EO matrix element, and percent depletion of the energy weighted sum rule for the EO transition, suggest that three-body forces are required to properly describe both ground and first excited state properties in the ^{4}He system.

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RESUMEN

El elemento matricial E0, que conecta el estado base y el primer nivel excitado de ^{4}He, es evaluado usando fuerzas entre dos cuerpos y fuerzas entre dos cuerpos más tres cuerpos. Se obtiene un acuerdo realista con el elemento matricial E0 medido al utilizar el Hamiltoniano modelo de fuerzas entre dos cuerpos más tres cuerpos. La consideración de una variedad de datos del ^{4}He que incluyen: la energía de enlace del estado base, el radio rms del estado base, el factor de forma de la carga en el estado base, la energía del primer nivel excitado, el elemento matricial E0, y el defecto porcentual de la EWSR, sugiere que son necesarias las fuerzas entre res cuerpos para describir apropiadamente las propiedades del estado base y del primer estado excitado del ^{4}He.

1. INTRODUCTION

The introduction of three-body forces into the ^{4}He nuclear Hamiltonian has recently been shown to provide significant improvements in the description of the ground state (GS) and first excited state (FES)[1]. Three-body forces improved the calculated GS rms radius, GS charge form factor, and position of the ^{4}He FES. Although individual GS and FES properties were improved, no test of the overlap of these wave functions was performed.

An ideal test of the overlap of GS and FES wave functions is obtained from the electric monopole (E0) matrix element and indirectly from the depletion of the energy-weighted-sum-rule (EWSR) strength for the E0 transition. The E0 matrix element and EWSR strength have been experimentally measured[2,3], but no theoretical efforts have been published to utilize these data to assess the adequacy of theoretical models and associated wave functions.

The present paper will use the two-body plus three-body shell model approach of Ref. 1 in order to evaluate the E0 matrix element and the percent depletion of the EWSR in ^{4}He. The calculations will serve as a stringent test of the GS and FES wave functions and will provide additional evidence for the relevance of three-body forces in the ^{4}He system.

2. FORMULATION

The approach utilized in this paper involves a modification to the usual two-body shell-model Hamiltonian[4] (H). Herein, a more general Hamiltonian (H') is utilized:

$$H' = H + cU \quad , \tag{1}$$

where U is the three-body Hamiltonian and c is a constant which assumes the value $c = 0$ if only two-body forces are included in H' and $c = 1$ if two-body plus three-body forces are to be included in H'. The two-body Hamiltonian H is expressed as[4]

$$H = -\sum_{K} (\hbar^2/2\mu_k)\nabla_k^2 + \sum_{i<j} V_{ij} + \sum_{i<j} V_{ij}^{Coulomb} \quad , \tag{2}$$

where K runs over the internal coordinates of the ^{4}He system (see Fig. 1, Ref. 4). The first term in Eq. (2) represents the kinetic energy, the second term is the two-body nuclear interaction term which is based on the Sussex or modified Sussex interactions[4,5], and the third term is the Coulomb interaction.

The three-body term was derived in Ref. 1 and has the form

$$U = \sum_{j=1}^{2} \sum_{i=1}^{2} |p_i\rangle \Delta_{ji} \langle p_i| \quad , \tag{3}$$

where $|p_i\rangle$ represents the three-body cluster wave function and the Δ_{ji} are defined in terms of the excitation energy in the three-body cluster and represents the strength of the three-body interaction. The choice of U is simplified by noting that the dominant configurations in ^{4}He (in internal coordinates) of the GS and FES are $(0S)^3$ and $(0S^2(1S)$[6]. For simplicity, Eq. (3) leads to a three-body force which only involves relative S states.

In Eq. (3), the sum over i labels the $A = 3$ wave function and the sum over j labels the excitation in the $A = 3$ cluster[7]. Specifically, $j = 1$ implies $(0S)^2$ and $j = 2$ labels a (0S)(1S) configuration in the internal coordinates of the $A = 3$ cluster. The triton wavefunction ($|p_1\rangle$) is obtained by transforming the ^{4}He wavefunction into a triton plus proton configuration-i.e., transforming from the $|\lambda\rangle$ to $|\xi\rangle$ basis of Ref. 4. The ^{3}He wavefunction ($|p_2\rangle$) is obtained in a similar fashion. The specification of the A = 4 three-body Hamiltonian is further defined with the values

$$\Delta_{11} = \Delta_{12} = +1.86 \text{ MeV}$$

and (4)

$$\Delta_{21} = \Delta_{22} = -3.60 \text{ MeV}.$$

The terms in Eq. (4) are obtained when the three-body force of Eq. (1) is used in conjunction with the modified Sussex interaction[(4,5)]. Additional details of the model Hamiltonian and supporting wavefunctions are defined in more detail in Refs. 1, 4, 5, and 7.

3. MODEL INTERACTIONS

A variety of interactions, summarized in Table I, will be utilized in order to assess the impact of three-body forces on 0^+ (FES, 20.1 MeV) $\rightarrow 0^+$(GS) E0 transitions in ^{4}He. The Sussex interaction[(5)] is the two-body interaction which is used as the basis of the model two-body force. The Sussex interaction leads to a reasonable ^{4}He spectrum (see Fig. 3, Ref. 4), but underbinds the ^{4}He ground state and does not lead to a realistic charge form factor[(4)]. The gap between the GS and FES is also overestimated by the Sussex interaction.

A second two-body interaction, the modified Sussex interaction[(4,5)], is derived from the Sussex interaction:

$$V^{\text{modified Sussex}} = C\, V^{\text{Sussex}} \quad , \tag{5}$$

where C has the value 1.168 for the b = 1.60 fm Sussex matrix elements[(5)]. Modifications to the Sussex interaction gave the correct GS binding energy, but introduced distortions into the ^{4}He spectrum[(4)]. In addition, the modified Sussex interaction did not improve the charge form factor[(1,4)].

The final interaction considered in this study utilizes the modified Sussex interaction plus the three body force of Ref. 1. The two-body plus three-body force leads to the correct ground state binding energy, FES eigenvalue, and a charge form factor which is qualitatively similar to the experimental charge form factor[1,4]. If the two-plus three-body (TPTB) interaction is to be considered as a viable interaction for describing the ^{4}He GS and FES, then it must also provide a realistic description of the E0 transition between these states.

TABLE I

Interaction	c	E_B(MeV)[a]	$E_{0'}$(MeV)	ρ(E0)	Depletion of EWSR (%)
Modified Sussex	0	28.3	30.9	0.44	43
Sussex	0	17.6	24.2	0.44	34
Modified Sussex plus three-body (TPTB)	1	28.3	20.2	0.53	41
Experiment	—	28.3[b]	20.1[b]	0.55[c]	46[c]

[a] Ground state binding energy.
[b] Ref. 9.
[c] Refs. 2 and 3.

Table I. Two body and two-body plus three-body force predictions for ground state and FES energies, E0 matrix element, and percent depletion of the EWSR in the ^{4}He system.

4. RESULTS AND DISCUSSION

Formulas for the E0 matrix element and percent depletion of the EWSR are defined by Lange, Kumar, and Hamilton[3]. The E0 matrix element ρ(E0) is defined in terms of the wave function of the initial state (FES) $J_1(0^+$, 20.1 MeV), the final state wave function $J_2(0^+$, GS), and the electric monopole operator M(E0)[8]:

$$\rho(E0) = \langle J_2 | M(E0) | J_1 \rangle \quad , \tag{6}$$

where

$$M(E0) = R_0^{-2} \sum_{K=1}^{4} e_K \left[\frac{1}{2} - t\ (K) \right] r_K^2 \quad . \tag{7}$$

In Eq. (7), R_0 is the nuclear radius, e_K is the charge of the K-th nucleon, $t_z(K)$ is the z-component of the isospin operator, and r_K is the coordinate which joins the K-th particle in the ^{4}He cluster to the remaining A = 3 cluster. Actual charges, not effective charges, are used in calculating the electromagnetic matrix elements-e.g., e_K = 1.0e for protons and e_K = 0.0e for neutrons.

The matrix element $\rho(E0)$ is most easily evaluated using the $|\xi>$ states defined in Ref. 4. The wave functions for the ground and first excited states are defined in Refs. 4 and 6.

The electric monopole matrix element, $\rho(E0)$, for $0^+(FES) \rightarrow 0^+(GS)$ transitions and the energy of the first excited state ($E_{0'}$) provide the information needed to determine the percent depletion of the EWSR strength for E0 transitions (D_{EWSR})(3):

$$D_{EWSR} = 2.88\ A^{5/3} E_{0'} \rho^2(E0)/Z^2 \quad , \tag{8}$$

where A is the mass number and Z is the atomic number.

Table I summarizes the results of model calculations for $\rho(E0)$ and D_{EWSR} for both two-body (modified Sussex and Sussex) and two-plus three-body (TPTB) interactions. As noted in Eq. (6), $\rho(E0)$ is sensitive to both the FES and GS wave functions. The experimental result of 0.55 is most closely reproduced by the TPTB potential, which yields a value of 0.53 for the E0 matrix element. Both Sussex and modified Sussex interactions lead to $\rho(E0)$ values of 0.44.

The equality of Sussex(5) and modified Sussex (MS)(4) $\rho(E0)$ results is in contrast to the differences in their predictions of the ground state and FES eigenergies summarized in Table I. The MS interaction leads to the correct binding energy in ^{4}He, but yields a FES which lies 10.8 MeV above the experimental position of Fiarman and Meyerhof(9). On the other hand, the Sussex interaction underestimates the ^{4}He binding energy by 10.7 MeV and leads to a FES which lies 4.2 MeV above the experimental position(10). The TPTB interaction leads to the correct GS and FES eigenvalues. The differences between the ground and FES properties and the $\rho(E0)$ values indicate that the TPTB potential provides the best representation of the experimental $\rho(E0)$ data[2,3].

Other data, such as D_{EWSR} measurements, are dependent on the product of the FES eigenvalue $E_{0'}$ and the square of the E0 matrix element. Therefore, D_{EWSR} represents a product of two individual quantities which may have large errors, but their product may lead to a good representation of the data. For example, the D_{EWSR} experimental value of 46% is best reproduced by the modified Sussex interaction (43%), even though the MS interaction overestimates $E_{0'}$ by 10.8 MeV (54%) and underestimates $\rho(E0)$ by 20%. The TPTB interaction yields a 41% D_{EWSR} value and the Sussex interaction suggest a value of 34%.

5. CONCLUSIONS

The D_{EWSR} results provide additional evidence that a variety of data must be considered in selecting model interactions. A consideration of only the D_{EWSR} data would suggest that the modified Sussex interaction best represents the ^4He system. However, a consideration of a variety of data (ground state binding energy and rms radius, FES energy, charge form factor, E0 matrix element, and percent depletion of the energy-weighted-sum-rule) suggest that the ^4He 0^+ level spectrum is best described by the two-plus three-body interaction, and that three-body forces are required to properly describe both ground and first excited state properties in the ^4He system.

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