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INFLUENCE OF THREE-BODY FORCES ON T=0 SPECTRUM IN ^{4}HE

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ABSTRACT

A ^{4}He shell-model formalism, including two- and three-body forces is used to calculate the energy and width of the following T = 0 levels: 0^+(g.s.), 0^+(20.1 MeV), 4^+(24.6 MeV), 1^+(25.5 MeV), and 2^+(33.0 MeV). Three-body plus two-body forces lead to significant improvements, in comparison with only two-body forces, in both level width and energy estimates for the T = 0 states noted above.

RESUMEN

Usando un formalismo de modelo de capas en ^{4}He y tomando en consideración fuerzas de dos y tres cuerpos, se calculan la energía y la anchura de los niveles de T = 0 siguientes: 0^+(g.s.), 0^+(20.1 MeV), 4^+(24.6 MeV), 1^+(25.5 MeV) y 2^+(33.0 MeV). Al tomar en cuenta las fuerzas de tres cuer-

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pos se obtienen resultados considerablemente mejores en la estimación, tanto de la energía como de la anchura de dichos niveles con respecto a los que se obtienen considerando únicamente fuerzas de dos cuerpos.

1. INTRODUCTION

Levels in the ^{4}He system below about 50 MeV are of a T = 0 or T = 1 character[1]. The T = 1 levels consist of a 1p - 1h (shell-model) character and are relatively well described by two-body interactions[2-6]. However, the T = 0 spectrum involves more complex structures[7-9] and is distorted relative to the experimental spectrum[10,11]. This distortion is amplified when the model ground state binding energy agrees with experiment[5]. Improvements in the 0^+ T = 0 spectrum, as well as an improved charge form factor and rms radius, were obtained by introducing a shell-model Hamiltonian which includes both two-body plus three-body forces[12]. The applicability of three-body forces in the T = 0 spectrum in ^{4}He has yet to be established. However, the poor agreement between two-body model calculations and data for the $(1^+,0)$ 25.5 MeV and $(2^+,0)$ 33.0 MeV levels[5,10] and the $(4^+,0)$ 24.6 MeV level[9,11] suggest other areas where three-body forces may lead to improvements in the calculated T = 0 spectrum in ^{4}He. If the three-body model of Ref. 12 is valid, it should also lead to an improved representation of the 4^+, 1^+, and 2^+ levels noted above.

The approach of using three-body forces within a shell-model framework will lead to highly model-dependent results. Even though our results will be model dependent, they will provide an indication of the applicability of three-body forces in the ^{4}He T = 0 spectrum.

2. FORMULATION

The three-body force of Ref. 12 was formulated by only considering the 0^+(g.s.) and 0^+(20.1 MeV) levels. In formulating the three-body model only $(0s)^3$ and $(0s)^2(1s)$ configurations (in internal coordinates) were considered because the dominant components of the aforementioned 0^+ states involve $0\hbar\omega$ and $2\hbar\omega$ excitations[7]. In a simplified view, the $J^\pi = 0^+$ three-body forces only contained $0\hbar\omega$ and $2\hbar\omega$ components. In order to calculate the positions of the 1^+(25.5 MeV), 2^+(33.0 MeV), and 4^+(24.6 MeV) levels (which contain significant $4\hbar\omega$ components), three-body forces with $4\hbar\omega$ content must be determined[5,8,9]. Herein, the $4\hbar\omega$ three-body interaction strength will be determined by considering the $(4^+,0)$ 24.6 MeV level[11]. Once the $4\hbar\omega$ three-body interaction strength is determined, it will be used with the $0\hbar\omega$ and $2\hbar\omega$ three-body components[12] to calculate the positions of 1^+ and 2^+ levels. Model eigenenergies for the 1^+ and 2^+ levels will provide a critical test of the adequacy of the proposed three-body approach. Level width comparisons will provide additional criteria to determine the adequacy of the proposed model.

The $4\hbar\omega$ component of the model three-body interaction may be determined by considering the difference between the measured and calculated (two-body) position of the 4^+ level. This difference may be minimized by including a more general Hamiltonian which includes a three-body term in addition to the usually considered two-body term[12]

$$H' = H + U \quad , \tag{1}$$

where H is the two-body Hamiltonian[5] and U is the three-body Hamiltonian. The choice of the three-body term is motivated by a recent study of the splitting of the ground and first excited state (FES) in ^{4}He[12]. The three-body term is defined in terms of a projection operator which selects A = 3 triton $|p_1\rangle$ or ^{3}He $|p_2\rangle$ clusters from the ^{4}He basis state[5,12]:

$$U = \sum_{j=1}^{2} \sum_{i=2}^{2} |p_i\rangle \, \Delta_{ji} \, \langle p_i| \quad . \tag{2}$$

The quantities appearing in Eq. (2) are discussed in more detail in Ref. 12. This form for U was chosen for the FES problem because the dominant ground and FES configurations were based on a limited number of configurations. However, the 4^+ problem is complicated because the $4\hbar\omega$ content of this state admits many more configurations $|p_i\rangle$ than the $0\hbar\omega$ and $2\hbar\omega$ structure of the ground and first excited states. For this reason, we choose to make the following simplifying assumptions for the strengths $\Delta_{ji}(N_{12}, L_{12}, N_B, L_B, i)$:

$$\Delta_{ji}(N_{12}, L_{12}, N_B, L_B, i) = \Delta_{\rho'} \quad , \tag{3}$$

where $\rho' = \rho/2$ and

$$\rho = 2(N_{12} + N_B) + L_{12} + L_B \quad . \tag{4}$$

The three-body force strengths are dependent on the total oscillator content (ρ) of the A = 3 cluster state which is defined by the radial (N_{12} and N_B) and orbital (L_{12} and L_B) quantum numbers[5,12]. For example, all A = 3 clusters with $4\hbar\omega$ of internal excitation, such as (0g)(0s), $(0d)^2$, (2s)(0s), etc., have the same three-body strength within the framework of our model. For consistency with Ref. 12, we use the same values of $\Delta_{\rho'}$ for $0\hbar\omega$ and $2\hbar\omega$ three-body strengths:

$$\Delta_0 = +1.86 \text{ MeV} \quad , \tag{5}$$

$$\Delta_1 = -3.60 \text{ MeV} \quad .$$

Following the methodology of Ref. 12, Δ_2 may be obtained by fitting the model 4^+ eigenenergy to the experimental value of 24.6 MeV. This is achieved with the value $\Delta_2 = -5.02$ MeV.

The Δ_0, Δ_1, and Δ_2 values complete the specification of the model three-body force. When this force is combined with the model two-body interaction[5,13], the general Hamiltonian (Eq. (1)) is completely specified. The general Hamiltonian can be used in an analogous manner to the standard Hamiltonian $H^{(5)}$ in the generalized R-matrix equation[14,15]

$$\sum_{\lambda'} \left[\langle\lambda|H' - E|\lambda'\rangle + \sum_{c} \gamma_{\lambda c}(b_{\lambda' c} - b_c)\gamma_{\lambda' c} \right] A_{\lambda'} = 0 \quad . \tag{6}$$

The quantities appearing in Eq. (6) are defined in detail if Ref. 5 and will not be discussed further herein. The determination of level energies and widths is achieved from the information appearing in Eq.(6).[5]

The Δ_0 and Δ_1 values noted above and the Δ_2 value derived from the 4^+ level lead to a three-body model prediction for the 4^+ level which has a width of 2.5 MeV and occurs at 24.6 MeV excitation energy. Although these results are highly model dependent, they do suggest that a large three-body component is required to describe the $(4^+,0)$ level within a $4\hbar\omega$ model space[5]. Although Δ_2 is large, it is not inconsistent with three-body strengths (Δ_0 and Δ_1) extracted from a consideration of the ^{4}He 0^+ spectrum[12]. However, the importance of three-body effects will not become clear until the 1^+ and 2^+ eigenenergies are calculated.

3. RESULTS AND DISCUSSION

Using the Δ_0, Δ_1, and Δ_2 values, calculations for other T = 0 levels will be performed. The limited set of three-body matrix elements (0, 2, and $4\hbar\omega$) restricts our T = 0 calculation to positive parity states. We further restrict consideration to the levels of Fiarman and Meyerhof[10]. With these caveats in mind, two-body plus three-body force calculations will be performed for the $(1^+,0)$ 25.5 MeV and $(2^+,0)$ 33.0 MeV levels.

Table I summarizes the results of two-body (TB) and two-plus three-body (TPTB) forces for the 0^+ (g.s.), 0^+(20.1 MeV), 4^+(24.6 MeV), 1^+(25.5 MeV), and 2^+ (33.0 MeV) levels. Model calculations using the TPTB force are improved considerably in comparison to TB results[5] for all T = 0 states considered herein. The 1^+ and 2^+ calculations show significant improvement. The 1^+ level is shifted from 36.6 MeV using a TB force to 28.1 MeV using the TPTB interaction which is near the experimental position of 25.5 MeV. In a similar fashion, the calculated position of the 2^+ level is shifted from 38.9 MeV to 31.0 MeV with the TPTB force.

The TPTB forces also lead to improved widths for the T = 0 states considered herein. Significant improvement is obtained with TPTB forces for the 0^+(20.1 MeV) level, but the model result (TB width of 11.9 MeV and TPTB width of 2.4 MeV) is still considerably larger than the experimental width of 0.27 MeV[16]. Marked improvements are also obtained for the other T = 0 levels. The 2^+ width for both TB(3.8 MeV) and TPTB(5.4 MeV) interactions both fall within the experimental range of 2.8 - 5.6 MeV[16].

The 1^+ experimental width of 2.9 - 5.6 MeV[16] is also better reproduced by the TPTB force - *i.e.*, a width of 0.4 MeV is derived from the TB force and a width of 2.1 MeV is obtained from the TPTB interaction.

TABLE I

J^π	Excitation Energy (MeV)[a]		
	TB	TPTB	Experiment
0^+	0.0(28.3)	0.0(28.3)	0.0(28.3)[b]
0^+	30.9	20.1	20.1[b]
4^+	42.4	24.6	24.6[c]
1^+	36.6	28.1	25.5[b]
2^+	38.9	31.0	33.0[b]

a) Binding energy in parenthesis.
b) Ref. 10.
c) Ref. 11.

Table I. T = 0 level energies with two-body and three-body forces.

TABLE II

$J^\pi(E_x)$ (MeV)	Level Width (MeV)		
	TB	TPTB	Experiment
0^+(g.s.)	0.0	0.0	0.0
0^+(20.1)	11.9	2.4	0.27[a]
4^+(24.6)	4.0	2.5	several[b]
1^+(25.5)	0.4	2.1	2.9 - 5.6[a]
2^+(33.0)	3.8	5.4	2.8 - 5.6[a]

a) Ref. 16.
b) Ref. 11.

Table II. T = 0 level widths with two-body and three-body forces.

4. CONCLUSIONS

The results of this study are supportive of theoretical contentions that the description of the ^{4}He system with only two-body forces is not sufficient and that multibody forces are needed for a proper description[12,17]. Three-body forces when combined with standard two-body forces lead to improvements in the positions and widths of positive parity T = 0 levels of Fiarman and Meyerhof and the $(4^+,0)$ level of Grüebler *et al.* The model utilized herein suggests that the impact of three-body forces are large and that three-body forces and needed to properly describe the T = 0 spectrum in ^{4}He.

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