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THE USE OF FRESNEL ZONES FOR DISTINGUISHING BETWEEN FRESNEL AND FRAUNHOFER DIFRACTION

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ABSTRACT

In diffraction problems where the source and the center of the diffraction pattern are on the normal line to the center of a diffraction aperture that has a constant pupil function, a simple calculation of the number of Fresnel zones serves as an easy criterion for the determination of the type of diffraction present. This decision is very important because there are cases of Fraunhofer diffraction at the near field and cases of Fresnel diffraction at the far field. The criterion that is established here is more understandable and easier to calculate than those given in the majority of the optics texts and has the additional advantage of being applicable to arbitrary finite apertures whenever the maximum and minimum dimensions are not very different.

RESUMEN

En aquellos problemas en que la fuente y el centro del patrón de difracción están en la línea normal al centro de una apertura difractora con función de pupila constante, se puede utilizar un cálculo simple del número de zonas de Fresnel como criterio sencillo para determinar el tipo de difracción presente. Esta decisión es muy importante porque haya casos de difracción de Fraunhofer en el campo cercano y casos de difracción de Fresnel en el campo lejano. El criterio aquí propuesto es más entendible y fácil de calcular que los propuestos en la mayoría de los textos de óptica, teniendo la ventaja adicional de poderse aplicar a

aperturas finitas arbitrarias siempre que no sean muy distintas las dimensiones máxima y mínima.

I. INTRODUCCION

We consider an incident spherical wavefront of wavelength λ from a point source P_o that falls on a circular aperture of radius R, and let P be the central point (zero order) of the diffracted pattern to be considered. The line P_oP is perpendicular to the opening at it center (see Fig. 1).

For the situation described in Fig. 1, a commonly used criterion[3-5] is that if r and b are infinite or satisfy r>>R and b>>R, then the considered situation is a case of Fraunhofer diffraction; otherwise, we will have Fresnel diffraction.

Nevertheless, there are experimental and theoretical situations where the preceding criterion or other similar ones fail or are contradictory, as we will show in the following examples.

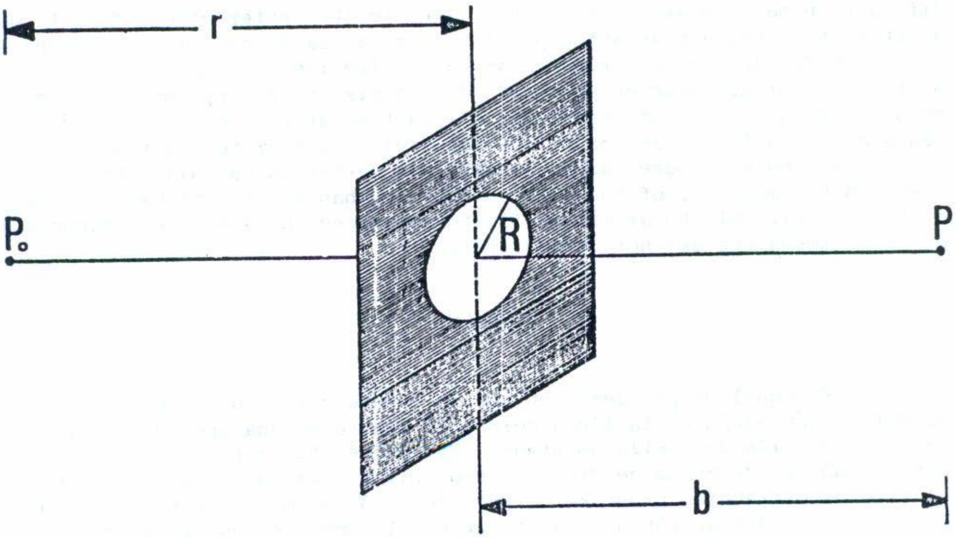

Fig. 1 r - distance from the source to the aperture center.
b - distance from the observation point P to the aperture center.

2. CRITICAL EXPERIMENTAL SITUATIONS

For the following situations we will use He-Ne laser. Therefore, the wavelength will be 6.328×10^{-7} meters.

First case:

Here $r = 1.00$ meters; $b = 10^{99}$ meters, which is 10^{73} times the radius of the universe (approximately 10^{26} meters) and $R = 10^{-3}$ meters.

In this case r is 10^3 times greater than R and b is 10^{102} times greater than R, and we have here a case of Fresnel diffraction.

Second case:

Here $r = 0.50$ meters; $b = 2.00$ meters and $R = 5 \times 10^{-4}$ meters.

In this case r is 10^3 times greater than R and b is 4×10^3 times greater than R, but this is a case of Fraunhofer diffraction.

If we observe both cases, we can see that the usual condition $r \gg R$ and $b \gg R$ is satisfied more clearly in the first case than in the second case; however, the first situation is that of Fresnel diffraction and not Fraunhofer diffraction, as one would deduce on the basis of the conventional criterion.

The analysis of the type of diffraction was realized by means of[2]

$$q = \left(\sqrt{r^2 + R^2} + \sqrt{b^2 + R^2} - r - b \right) / (\lambda/2) \qquad (1)$$

under the following condition:

if $q > 1$, we have Fresnel diffraction; and

if $q \leq 1$, we have Fraunhofer diffraction.

We obtained $q = 1.58$ in the first case and for the second case $q = 0.98$.

It is very important to observe that q is the exact number N of Fresnel zones calculated for the given situation in the Fig. 1. But this calculation of q by means of the Eq. (1) and the mentioned condition are used in Ref. 2 for circular apertures only and the author does not state explicity that q is the number N of Fresnel zones.

3. PARABOLIC APPROXIMATION FOR N[3,5]

It is well known that the number of Fresnel zones for the situation shown in Fig. 1 can be computed from the formula

$$N = R^2(r+b)/(r\cdot b\cdot \lambda) \quad ; \qquad (2)$$

this is the parabolic approximation for N, valid when $R \ll r$ and $R \ll b$.

As long as $R \ll r$ and $R \ll b$, the calculation of N using Eq. (2) is easier than with Eq. (1), because in the latter we must retain many significant figures; since the wavelength is around 10^{-7} meters. Moreover, it is a theoretical and experimental fact that is Fraunhofer diffraction the zero order coincides with the point of maximum intensity of the diffraction pattern; whenever we have a pupil function which is constant over all points of the aperture.

In terms of the number of Fresnel zones, this situation is obtained only when $N \leqslant 1$, since if we had calculated for instance 2.50 zones at the center of the pattern, then we could find at least one other point of the pattern where the intensity is greater; thus according to above implicit definition, this would not be a case of Fraunhofer diffraction.

Therefore the criterion we will use is that

$N > 1$ implies Fresnel diffraction; and

$N \leqslant 1$ implies Fraunhofer diffraction.

4. APPLICATION OF THE CRITERION TO OTHER FORMS OF APERTURES

Physically, the criterion we are using to distinguish between the two types of diffraction is that (supposing a constant pupil function) the diffraction is Fraunhofer when the maximum intensity occurs at the central point (zero order), and Fresnel if this is not the case.

Consequently, we can apply this criterion to non-circular apertures.

Thus, for rectangular apertures we consider R as the circle that circumscribes this rectangular orifice (see Fig. 2(a)) and similary, we can apply this criterion to arbitrary finite apertures, whenever the maximum and minimum distances to the center of the aperture are not very different. For these cases R can be taken equal to the maximum distance from the center to the border of the aperture, in others words, R will be the radius of the circle that circumscribes the aperture from its center (see Fig. 2(b)).

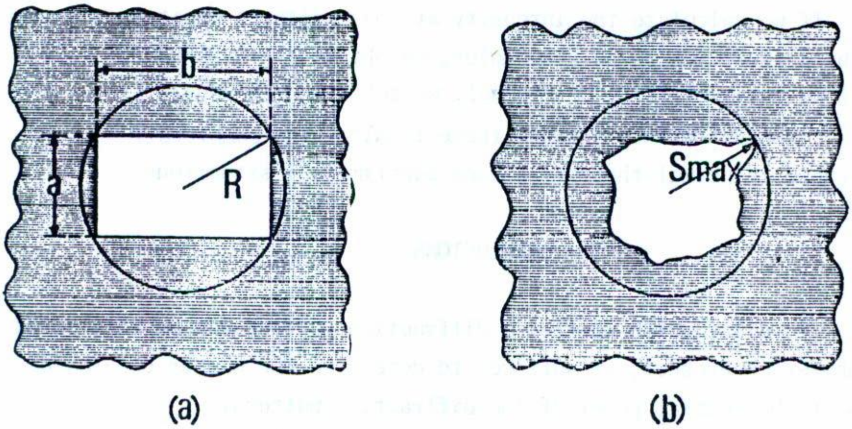

Fig. 2 (a) Rectangular aperture, $R = \sqrt{a^2 + b^2}/2$;
(b) Arbitrary finite aperture, $R = S_{max}$.

When N = 1 it is very important to note that we can calculate the intensity distribution considering the diffraction to be either Fraunhofer or Fresnel, since the difference between these two calculations is negligible. For this reason we will consider the case of N = 1 as Fraunhofer diffraction, because the intensity calculations for Fraunhofer diffraction are easier.

Example:

> With normal incidence a plane wave-front falls on a square aperture 2 millimeters on a side. The screen is placed 4 meter from the aperture, their planes parallel. Calculate the intensity of the point 0.10 millimeters to left of the center of the pattern (λ=500 nm).

In this situation N = 1 for the center of the pattern.

Let I(0) be the intensity of the center of the pattern (zero order). If we calculate the intensity at the indicated position considering the diffraction to be Fraunhofer, we obtain I = 0.967531209 I(0); and if we consider it to be Fresnel, we get I = 0.967900722 I(0).

The difference between these results is completely negligible in normal theoretical and experimental situations.

5. CONCLUSIONS

To distinguish whether a diffraction pattern can be considered Fraunhofer or Fresnel, it suffices to determine the number of Fresnel zones at the central point of the diffraction pattern.

If $N \leq 1$ the pattern can be considered of the Fraunhofer type; if $N>1$, for intensity calculations the pattern must be considered of the Fresnel type. In the great majority of cases, Eq. (2) is sufficient for calculating N.

We can apply the criterion to circular, rectangular and arbitrary finite apertures, in the case the maximum and minimum dimensions are not very different. It should be remembered that the criterion stated here is valid only if the pupil function is constant.

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