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RELATIVISTIC EFFECTS IN NUCLEAR DISTORTED-WAVE BORN-APPROXIMATION CALCULATIONS

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ABSTRACT

An approximation is formulated to account for relativistic effects in core + cluster wave functions. The approximation can be implemented by altering the input to distorted-wave Born-approximation codes such as DWUCK and MERCURY. Approximation results for the ^{24}Mg(p,d)

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^{23}Mg(2.36 MeV,$\frac{1}{2}^+$) cross section and associated reaction form factors agree to within 20 percent when compared to the predictions of the Dirac Equation. The approximation also permits the incorporation of center-of-mass and relativistic corrections in a consistent manner.

RESUMEN

Se formula una aproximación para tomar en cuenta efectos relativistas en funciones de onda de núcleo más cúmulo. La aproximación se puede implementar alterando la entrada a códigos de aproximación de Born de onda distorsionada tales como DWUCK y MERCURY. Los resultados de esta aproximación para la sección eficaz de la reacción ^{24}Mg(p,d) ^{23}Mg(2.36 MeV, $\frac{1}{2}^+$) y para los factores de forma asociados con la reacción concuerdan dentro del 20% con las predicciones de la ecuación de Dirac. La aproximación también permite la incorporación de correcciones de centro de masa y relativistas de una manera consistente.

1. INTRODUCCION

Calculations of distorted-wave Born-approximation (DWBA) form factors (wave functions) for nuclear core plus valence nucleons are usually calculated using a Schrödinger equation formulation in standard codes such as DWUCK[1] or MERCURY[2]. However, recent work suggests that modifications to the standard DWBA approach which include center-of-mass corrections[3-6] and relativistic effects[7,8] are important considerations in the calculation of reaction properties such as transfer and inelastic scattering cross sections.

The importance of center-of-mass effects in the zero-range form factor was noted by Pinkston[3] and Pinkston and Iano[4]. Feng *et al.*[5] also observed improved DWBA results with the proper treatment of the nuclear structure. Usually, the implementation of the center-of-mass correction approaches requires significant modification to DWBA codes. However, it is possible to obtain a center-of-mass correction approach which is implemented by only altering the input to existing DWBA codes. This type of approach was followed in Ref. 6 and an approximate center-of-mass representation was shown to reproduce the rigorous center-of-mass form factor correction model results.

The inclusion of a Dirac type relativistic correction would require even more extensive modifications to the standard DWBA codes noted above. Therefore, an approximate approach of including relativistic

wave function effects generated by the Dirac equation would be highly desirable. Such a model would permit estimates of Dirac relativistic effects without significant error. It would be even more desirable if both Dirac and center-of-mass effects could be obtained within the same DWBA model by only altering the input to the wave function potential parameters[6].

The present paper will attempt to accomplish this task. Specifically, our approximation will be compared with a more rigorous DWBA model which used Dirac wave functions[8]. In addition, model results including Dirac wave functions will be compared with models for center-of-mass and center-of-mass plus Dirac corrections in the wave functions. The corrections will be incorporated into both core plus cluster form factors and into the entrance and exit channel wave functions.

2. FORMALISM

The form factor $\langle B,b|V|A,a\rangle$ contains all the nuclear structure information required to calculate the cross section for the reaction

$$B + b \longrightarrow A + a \quad , \tag{1}$$

where B + b are the entrance channel particles and A + a are the particles in the exit channel. For transfer reactions, the identifications

$$B = A + x$$

and (2)

$$a = b + x \quad ,$$

are made in order to facilitate the description of the B(b,a)A reaction. The quantity x appearing in Eq.(2) is the transferred cluster or particle. As noted in Ref. 6, the form factor involves integration over all target and projectile coordinates,

$$\langle B,b|V|A,a\rangle = \Psi^*_B(\vec{r}_{Ax})\ V(\vec{r}_{bx})\ \Psi\ (\vec{r}_{bx}) \quad , \tag{3}$$

where V is the effective interaction between b and x, and the Ψ's are the bound state wave functions which describe the motion of the transferred body x relative to A and b. For convenience, the spectroscopic strengths are absorbed into the bound state wavefunctions. As a specific example and to compare our results directly with the Dirac approach, x is assumed to be a single nucleon. Therefore, the zero-range DWBA approximation can be utilized and the radial dependence of the interaction V can be replaced by a delta function[6,9]:

$$V(\vec{r}_{bx})\ \Psi_a(\vec{r}_{bx}) = D_o\ \delta(\vec{r}_{bx}) \quad , \tag{4}$$

where

$$D_o = \int d\vec{r}_{bx}\ V(\vec{r}_{bx})\ \Psi_a(\vec{r}_{bx}) \quad . \tag{5}$$

By introducing the delta function, the T-matrix can be written as

$$T_{fi} = D_o \int d\vec{r}\ \chi_f^{(-)*}\ (A\vec{r}/B)\ \Psi_B^*(\vec{r})\ \chi_i^{(+)}(\vec{r}) \quad , \tag{6}$$

where $\Psi_B(\vec{r})$ is the form factor and the remaining quantities appearing in Eq.(6) are defined in Ref. 6. The form factor $\Psi_B(\vec{r})$ is defined within the DWBA formalism by solving the Schrödinger equation:

$$(T + U - E)\Psi = 0 \quad , \tag{7}$$

where T is the kinetic energy, U is the nucleon-nucleus potential which includes the Coulomb interaction, and E is the binding energy of the nucleon-nucleus system. A similar approach is used to obtain the entrance ($\chi_i^{(+)}$) and exit ($\chi_f^{(-)}$) channel wave functions[9].

The Dirac equation wave function Ψ is obtained from the Dirac equation ($\hbar = c = 1$):

$$\{\vec{\alpha} \cdot \vec{p} + \beta[m + V_s(r)] + V_v(r) + V_c(r) - E\}\,\Psi = 0 \quad , \tag{8}$$

where real Dirac scalar (V_s), Dirac vector (V_v), and Coulomb potentials (V_c) are added to the free Dirac Hamiltonian ($\vec{\alpha} \cdot \vec{p} + \beta m$) with total energy E. The mass m is taken to be the reduced total energy to account for recoil in a minimum relativistic manner[10]. Standard manipulations[8] transform Eq.(8) into a second-order differential equation for the upper component Ψ, which is just the Dirac form factor. The transformation contains the Darwin term[11] in $\vec{r} \cdot \vec{\nabla}$ which is transformed away[8] be setting $\Psi(\vec{r}) = B^{\frac{1}{2}}(\vec{r})\,\phi(\vec{r})$, where $\phi(r)$ is obtained from the equation

$$\left\{-\frac{\nabla^2}{2E} + U_{eff}(r) + V_{s.o.}(r)\,\vec{\sigma} \cdot \vec{L} - \frac{E^2 - m^2}{2E}\right\}\phi(\vec{r}) = 0 \tag{9}$$

and

$$B(r) = 1 + [V_s(r) - V_v(r)]\,/(E + m) \quad , \tag{10}$$

$$U_{eff}(r) = V_{cen}(r) - \frac{1}{2B^2 r}\,\frac{d}{dr}\left(r^2 \frac{dB}{dr}\right)$$

$$+ \frac{3}{4B^2}\left(\frac{dB}{dr}\right)^2 + iW(r) \tag{11}$$

The term $B^{\frac{1}{2}}$ is defined as the damping factor which suppresses the wave function in the nuclear interior. The factor $B^{\frac{1}{2}}$ is typically about 0.75 near the origin and rises to unity in the nuclear surface[8]. The other terms appearing in Eqs.(9)-(11) are defined by Rost *et al.*[8].

3. RESULTS AND DISCUSSION

In order to evaluate the Dirac-DWBA model approximation, the core plus nucleon wave function was generated from the potential

$$U = V_o f(r,r_o,a_o) + V_1 f(r,r_1,a_1) + V_2 f(r,r_2,a_2)$$

$$+ V_{Coulomb} + \frac{\lambda V_o}{45.2}\,\frac{1}{r}\,\frac{df(r,r_{so},a_{so})}{dr}\,\vec{L}\cdot\vec{S} \quad . \tag{12}$$

Following Ref. 8, similar terms are also added to the entrance and exit channel potentials, but herein we concentrate our discussion on the core plus nucleon form factor. The approximation of Eq.(12) only requires the addition of a few bound state potential cards to the input deck for DWBA codes such as DWUCK[1] or MERCURY[2]. In Eq.(12), V_o is the central potential, λ is the Thomas spin-orbit term, a_i is the diffuseness parameter, r_i is the radius parameter, and $V_{Coulomb}$ is the Coulomb interaction between the core and the transferred cluster. The term $f(r,r_i,a_i)$ has the Woods-Saxon shape:

$$f(r,r_i,a_i) = \left[\, 1 + \exp\left(\frac{r - r_i A^{1/3}}{a_i}\right) \right]^{-1} , \tag{13}$$

where A is the core mass. The term involving V_1 is the center-of-mass correction term[6] and will be taken to be zero initially. The V_2 term represents the Dirac approximation, and will be used to approximate the Dirac form factor.

A number of free parameters (namely V_2, r_2, and a_2) are used to simulate the Dirac solution. Since the Dirac solution is known to depress the interior wave function[8], the parameters r_2 and a_2 are chosen following the selection of Ref. 6:

$$r_2 = 0.44\ r_o$$

and (14)

$$a_2 = 0.31\ a_o$$

In order to justify this selection, we attempt to obtain the results of Rost *et al.*[8] for the $^{24}Mg(p,d)^{23}Mg(2.36\ MeV, \frac{1}{2}^+)$ cross section for $T_p = 94$ MeV. The model noted above, using the parameters of Eq. (14), the value $V_2 = +V_o$ and the reaction channel parameters of Ref. 8 reproduce the cross section results of Rost *et al.* within about 20%. Although this in itself an important result, an even stronger conclusion concerning the validity of our approximation can be drawn if the form factor were depressed in accordance with the predictions of the Dirac solution.

A comparison of model Dirac approximations and the Dirac solution for the $^{23}Mg + n$ form factor is summarized in Table I. The results again confirm that the model (DWBA with Dirac simulation) approximation provides a representative description of the Dirac solution. The model solution is within 6% of the Dirac solution at $r = 0$, and within about 1% of the Dirac form factor value at the nuclear surface. The cross section and form factor results provide confidence that the model is providing a reasonable description of the Dirac wave fuction corrections.

In addition to relativistic(Dirac) corrections, Table I summarizes the impacts of center of mass corrections. The center-of-mass corrections alter the form factor considerably more than the Dirac corrections. However, a physical for factor must necessarily include both Dirac and center-of-mass corrections. The present approach provides a means of obtaining physical form factors and this approach can be used to provide quick and representative cross section estimates for evaluating the effects of relativistic and center-of-mass corrections.

TABLE I

Method	V_1 [a]	V_2 [b]	Value of form factor [c] r = 0	r=surface	Reference
Dirac(exact)	——	——	1.00	0.75	8
Dirac(approx.)	0	$+V_0$	0.94	0.74	This work
Center-of-Mass (approx.)	$+5V_0$	0	0.86	0.36	This work
Dirac+Center-of-Mass (approx.)	$+5V_0$	$+V_0$	0.86	0.34	This work

a) Center-of-mass potential strength, Eq.(12).

b) Dirac potential strength, Eq.(12).

c) Normalized to the Ref. 8 Dirac form factor value at r=0.

Table I. Comparison of Approximation Methods for the ^{23}Mg+n form factor.

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