EFFICIENCY OF THE TWO-STAGE RC LOW-PASS BAND FILTER IN COMPARISON WITH THE MATCHED FILTER

Francisco F. Medina E.

Departamento de Física

Universidad de Antioquia, A.A. 1226, Medellín, Colombia

(recibido julio 24, 1984; aceptado noviembre 8, 1984)

ABSTRACT

Matched Filters for detection the most usual signals (*i.e.*, triangular, rectangular and trapezoidal pulses) are complex and expensive. This work presents a general method to calculate suboptimal filters to be constructed using simple RC circuits, which exhibit high efficiency and low cost in comparison with those Matched Filters. We use a triangular pulse in basic band as input of the filters but there is no loss of generality because the procedures of calculation are completely general and are related with detection in a carry-band.

RESUMEN

Los filtros acoplados para la detección de las señales más usuales (*i.e.*, pulsos triangulares, rectangulares y trapezoidales) son complicados y costosos. Este trabajo presenta un método general para el cálculo de filtros subóptimos construidos usando circuitos RC, los cua-

les exhiben una alta eficiencia y son poco costosos en comparación con los filtros acoplados. Utilizamos un pulso triangular en banda básica como señal para los filtros, lo que no significa pérdida de generalidad ya que los procedimientos de cálculo son completamente generales y están relacionados con la detección en una banda de transporte.

1. DETERMINATION OF THE SNR_O

The signal-to-noise ratio for the Matched Filter is defined as

$$SNR_0 = \frac{|S_o(t)|^2_{MAX}}{\langle n_o^2(0)\rangle} \quad , \tag{1}$$

with $|S_o(t)|^2_{MAX}$ the maximum power of the output and $\langle n_o^2(0)\rangle$ the noise average power at the output of the device (Fig. 1).

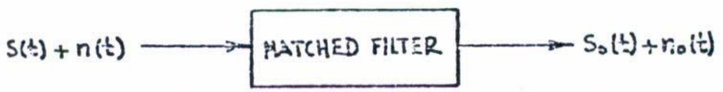

Fig. 1 Block diagram of the Matched Filter.

The impulse response function (IRF) of the Matched Filter for the signal summerged into white noise at the input is

$$h(t) = S^*(-t) \tag{2}$$

and

$h(-t) = S(t)$, because it is a real and even symmetric function.

Then,

$$|S_o(t)|^2_{MAX} = \left| \int_{-\infty}^{\infty} S(t)h(\tau-t)\, dt \right|^2_{MAX} = \left| \int_{-\infty}^{\infty} S(t)\, S^*(t-\tau)\, dt \right|^2_{MAX} \tag{3a}$$

and

$$|S_o(t)|^2_{MAX} = \Gamma^2_{ss}(0) \quad . \tag{3b}$$

Now, the average power of noise is

$$\langle n_o^2(0)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} |N(\omega)\,H(\omega)|^2 d\omega \quad . \tag{4}$$

with $|N(\omega)|^2$ the noise power spectrum and $H(\omega)$ the transference function of the Matched Filter. In white noise we have $|N(\omega)|^2 = 1$ and the Eq. (4) is

$$\langle n_o^2(0)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega = \Gamma_{kk}(0) \tag{5}$$

using the Wiener-Khintchine theorem. But, replacing (2) into (5) we have

$$\langle n_o^2(0)\rangle = \Gamma_{ss}(0) \quad ; \tag{6}$$

and using (3) and (6) in (1), we obtain

$$SNR_o = \Gamma_{ss}(0) \quad . \tag{7}$$

For a triangular pulse, defined as

$$S(t) = \begin{cases} 1 - \frac{|t|}{T} & (-T \leq t \leq T) \\ 0 & (\text{otherwise}) \end{cases} \quad , \tag{8}$$

summerged into white noise, we have

$$SNR_o = \frac{2}{3}\ T \quad . \qquad (9)$$

2. DETERMINATION OF THE SNR

Now we will calculate the signal-to-noise ratio for the two-stage RC low-pass band (SNR) as suboptimal filter (Fig. 2). In this case, the IRF of the device and its related transfer function are

$$h(t) = \frac{t}{\tau^2}\, e^{-t/\tau} \xleftrightarrow{\text{FOURIER'S TRANSFORM}} H(\omega) = \frac{1}{(1+i\omega\tau)^2}, \qquad (10a,b)$$

with $\tau = RC$ and $\omega = 2\pi\nu$. The signal-to-noise ratio is defined as

$$SNR = \frac{|S_o(t)|^2_{MAX}}{\langle n_o^2(0)\rangle} \quad . \qquad (11)$$

Replacing (10a,b) into (3a) and (4) and this result into (11) we obtain

$$RSN = \frac{4\tau}{A^2}\ \{ [Ae^{-A} - (1-e^{-A})(2+x)]e^{-x} + 2(1-e^{-x}) - x(1+e^{-x}) + A \}^2 , \qquad (12)$$

with $A = T/\tau$ and $x = (t-T)/\tau$. For each A there is an x which maximizes the SNR. All solutions with $x>A$ implies that the pulse does not exist.

3. DETERMINATION OF THE EFFICIENCY

The efficiency of any suboptimal filter is

$$E = \frac{SNR}{SNR_0} \cdot 100\% \quad . \qquad (13)$$

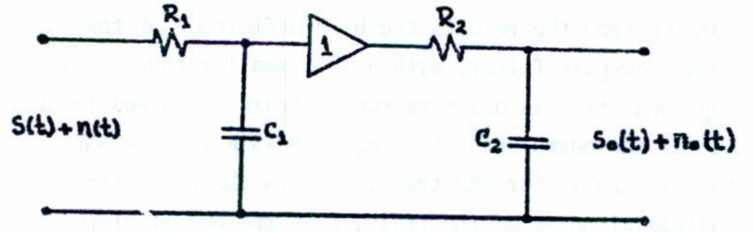

Fig. 2 The two-stage RC low-pass band sub-optimal filter Note the operational amplifier with unitary gain used as accoplator between the two steps of the circuit.

Then, for this case the efficiency is given by

$$E = \frac{6}{A^3}\{[Ae^{-A} - (2+x)(1-e^{-A})]\ e^{-x} + 2(1-e^{-x}) - x(1+e^{-x}) + A\}^2 \quad , \qquad (14)$$

which graph is shown in Fig. 3. After the maximization of the efficiency we obtain the following results:

$$E_{MAX} \cong 88\% \cong -0.56\ \text{db} \ ; \ A = 3.2055$$

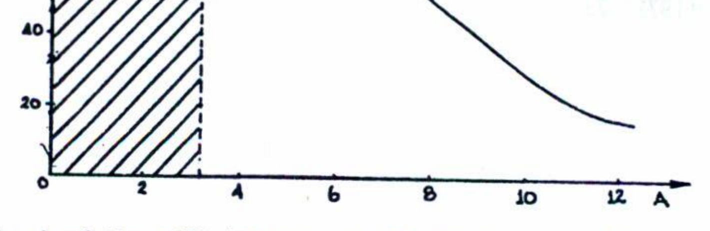

Fig. 3 Graph of the efficiency curve for the two-stage RC low-pass band filter in comparison with the Matched Filter.

4. CONCLUSIONS

i) We can observe from the results the high efficiency of the suboptimal two-stage RC low-pass filter, with a loss smaller than 1 db.

ii) In the implementation is usual to send a train of pulses to obtain a better gain at the output because $SNR_T = (N \cdot BT) SNR_P$, where SNR_T is the signal-to-noise ratio for the train, SNR_P is the signal-to-noise ratio for an individual pulse of the train, N is the number of pulses, B is the bandwidth and T is the duration of any pulse. The product BT is the number of modes transmited.

iii) The forbidden zone for the efficiency function is strictly defined as $x > \alpha A$ with α a real number. In this case, we use $\alpha = 1$.

ACKNOWLEDGEMENTS

The author acknowledges the important comments of Dr. Peter Barlai for this article and the special way the colaboration of Prof. Román Casta ñeda in a realization of the computer programs.

BIBLIOGRAPHY

1. M. Skolnik, *Introduction to Radar Systems*, McGraw-Hill, New York (1962).
2. J. Millman and H. Taub, *Pulse and Digital Circuits*, McGraw-Hill, New York (1965).
3. G.L. Turin, "An introduction to Digital Matched Filters", *Proc. IEEE*, 64 (1976)1092.
4. G.L. Turin, "Minimax strategies for Matched Filter detection", *IEEE Trans. Com.* 23 (1975)1370.
5. T. Yamabuchi, T. Takagi and K. Mano. "Matched filters for rectangular, raised cosine and sine wave signals", *IEEE, Trans.Com.* 19 (1971)369.
6. G.W. Carlock. "The two-stage RC low-pass Matched Filter", *IEEE Trans. Com.* 20 (1972) 73.