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THE STOCHASTIC ROAD TO QUANTUM MECHANICS: AN EXPERIENCE †

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ABSTRACT

The work performed by our group in the last 16 years in connection with the foundations of quantum mechanics can be roughly divided in four stages: The preparatory or introductory stage, the period of stochastic quantum mechanics, the period of stochastic electrodynamics and finally, the present stage. In this paper we present a summary of the development and main results of the various stages; however, in view of the minor significance of the first one, and of the excellent recen summary by Brody[1] of the second and third, we shall pay special attention to our current work, which consists basically in the construction of a new version of stochastic electrodynamics.

RESUMEN

El trabajo realizado por nuestro grupo en los últimos 16 años relativo a los fundamentos de la mecánica cuántica, puede dividirse a grandes rasgos en cuatro etapas: la preparatoria o introductoria, la dedi

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cada a la mecánica cuántica estocástica, la dedicada a la electrodinámica estocástica y, finalmente, la etapa actual. En este trabajo se presenta un resumen breve del desarrollo y los resultados principales de cada una de ellas; sin embargo, dados el significado menor de la primera etapa y la publicación reciente de un excelente trabajo de Brody[1] en que se revisan la segunda y tercera etapas, aquí se presta atención especial al trabajo actual del grupo, el que consiste en lo fundamental en un primer intento de construcción de una nueva versión de la electrodinámica estocástica.

1. PRELIMINARY STAGE: BROWNIAN ANALOGY.

Our work on the foundations of quantum mechanics initiated in the late 60's, was motivated by a general state of intellectual dissatisfaction with the orthodox views on quantum mechanics. By that time extensive discussions and numerous controversies on the interpretations of quantum mechanics had arisen[2]; but there existed few, scarcely known and unaccomplished efforts to construct a fundamental theory that would explain the quantum formalism. On analysing the various facts of quantum mechanics it gradually became clear that stochasticity should play an important role in any causal and objective description of the quantum phenomenon. This feeling was reinforced by our rather accidental acquaintance with Fényes' work[3], which represents a serious effort to understand quantum mechanics in terms of a stochastic Markovian behaviour of the electron.

This line of research was stimulated, but at the same time limited, by the analogy with Brownian motion, by far the best-known stochastic process at the time. Indeed, it led to the identification of common features of quantum mechanics and Markov processes, and to the use of a methodology that had been essentially foreign to quantum theory[4]. However, the close analogy between the stochastic electron and the Brownian particle raised several questions which in the framework of this primitive treatment could not be solved satisfactorily, namely: Which is the essential difference between a classical and a quantum Brownian motion? In the frictionless quantum motion, how is it that a stationary state can be reached?

It soon became clear that some fundamental points were wrong or

at least obscure, since any theory aimed at a better understanding of quantum mechanics should permit to distinguish clearly a quantum system from a classical one.

Unfortunately the confusion still prevails -it is present even in Jammer's excellent book-[2] to the extent that special terminology has been suggested to describe the "Brownian motion" of the electron; this widespread confusion adds its share to the aprioristic attitude of many nonspecialist against the possibility of a causal, stochastic explanation of quantum mechanics.

2. STOCHASTIC QUANTUM MECHANICS

As a result of this work it seemed necessary to construct a more elaborate formalism that could serve to describe a more general stochastic process (in the Markovian approximation), and to distinguish between Brownian motion and quantum mechanics as two different physical situations. The intention was to show that quantum mechanics can indeed be understood as the result of a specific stochastic process added upon otherwise classical laws of motion, without yet inquiring into the origin of stochasticity; stochastic quantum mechanics has therefore an explicitly declared phenomenological character.

The closest antecedent of our work in this direction is Nelson's well-known Markovian theory of quantum mechanics[5,6], which had clearly established the possibility of a phenomenological treatment in coordinate space similar to the Einstein-Smoluchowski treatment of Brownian motion. Based on the introduction of two different derivatives of a local function with reference to the initial and to the final point, we developed a kinematics that would serve to describe a general Markovian process in coordinate space[7]. In this description, in addition to the drift or systematic time derivative:

$$\mathcal{D}_c = \partial/\partial t + \vec{V}\cdot\nabla \tag{1a}$$

there appears the osmotic or stochastic time derivative:

$$\mathcal{D}_s = \vec{\mu}\cdot\nabla + D\nabla^2 \tag{1b}$$

where

$$\mu = D\nabla \ell n\ \rho \tag{2}$$

is the stochastic velocity, D is the diffusion coefficient and ρ is the density of particles. Four different accelerations may therefore be defined, in terms of which one can write the stochastic generalization of Newton's second law:

$$m(\lambda_1 \mathcal{D}_C + \lambda_2 \mathcal{D}_S)(\vec{V} + \eta\vec{\mu}) = \vec{F} \quad . \tag{3}$$

As a result of imposing three physical conditions, namely: The time-reversal invariance of Eq.(3) when $\vec{F}(t)=\vec{F}(-t)$, the continuity equation for ρ, and the recovery of the classical description in the Newtonian limit, Eq.(3) tranforms into

$$m(\mathcal{D}_C\vec{V} - \lambda\mathcal{D}_S\vec{\mu}) = \vec{F} \tag{4a}$$

$$\mathcal{D}_C\vec{\mu} + \mathcal{D}_S\vec{V} = 0 \tag{4b}$$

with $\lambda = -\eta^2$ still undefined.

The system of Eqs.(4) admits a first integration and linearization in terms of the new variables

$$\Psi_{\pm} = e^{R\pm S/\sqrt{\lambda}} = \rho^{1/2}\, e^{\pm S/\sqrt{\lambda}} \quad , \tag{5}$$

where

$$\vec{u} = 2D\nabla R, \qquad \vec{V} = 2D\nabla S. \tag{6}$$

As a result of this integration one obtains the linear, uncou-

pled equations

$$\pm 2mD\sqrt{-\lambda}\,\frac{\partial \Psi_{\pm}}{\partial t} = -2mD^2\,\lambda\nabla^2\Psi_{\pm} + V\Psi_{\pm} \quad . \tag{7}$$

Since the free parameter λ appears only in the combination $D\sqrt{-\lambda}$ one may take $\lambda^2 = 1$ by an adequate selection of the phenomenological parameter D. With $\lambda = -1$ Eq.(7) is parabolic and the amplitudes Ψ are real; with $\lambda = +1$ it is hyperbolic and the amplitudes Ψ are in general complex. To stress the physical differences we speak in the first case of an Einstein process (a classical, Brownian-type process in the limit of negligible friction) and in the second case of a de Broglie (non-classical, wave-like) process. The Schrödinger equation is obtained from Eq.(7) for $\lambda = 1$, with a suitable selection of the diffusion coefficient:

$$D = \hbar/2m \quad . \tag{8}$$

The above results show that quantum mechanics can indeed be interpreted as a Markov process, but irreducible to Brownian-type stochastic motion. This formalism has allowed a variety of extensions and generalizations, such as the introduction of spin[8], relativistic treatments[9], the extension to mixed states[10,11] and variational formulations[6,12-14]; the corresponding path-integral formulation has been developed[14,15] and applied to the problem of barrier penetration[15]. More general schemes have been developed, either as a theory of the electron[16] or even as a field theory[17]. An important offspring of stochastic quantum mechanics in the so-called stochastic quantization, a stochastic treatment of field theory developed in recent years[17,18].

The theory has also received much criticism. An analysis of the objections raised reveals that they are most frequently the result of a traditional, classical approach to stochastic processes and the consequent lack of distinction between Einstein and de Broglie processes, though in some cases they are related to a specific technical

detail, such as considering the backward derivative $\mathcal{D}_c - \mathcal{D}_s$ as the time inversion of $\mathcal{D}_c + \mathcal{D}_s$. The main shortcoming of stochastic quantum mechanics is its phenomelogical character: Being a formal theory, it is unable to elucidate the physical mechanism responsible for stochasticity and, in the last instance, for quantization. However, it has the merit of indicating the way along which one should look for a deeper and more fundamental theory; this is the subject of the next section.

3. STOCHASTIC ELECTRODYNAMICS

The search for an explanation of quantum stochasticity in terms of a physical cause represents obviously a departure from the orthodox views on quantum mechanics. Stochastic electrodynamics (SED) in such an attempt: Its purpose is to interpret the quantum phenomenon as originating in the interaction with the allpervading zero-point radiation field produced by the far matter in the universe. Intuitively one can view this field as the result of the superposition of the uncorrelated fields emitted be all accelerated charges in the universe. This assumption can in principle help to solve an old dilemma: Recall that at the beginning of the century, the planetary model of the atom was dismissed because of the instability produced by the radiation of the accelerated electron. SED attempts to revindicate this radiation as the ultimate cause of atomic stability and stochasticity at the same time, by proposing that a state of equilibrium can be reached in which the energy absorbed by the electron from the stochastic field is compensated in the average by the radiated energy. In this picture, radiation is necessary to stabilize the (stochastic) atomic orbits. As opposed to earlier models, the atomic electron is now considered an open system that continuously exchanges energy with the random zero-point radiation field.

The idea of this field is certainly not new: It was already envisaged by Planck[20] and some of its cosmological consequences were studied by Nernst[21]. In present-day physics it appears as the vacumm of quantum electrodynamics, usually-but not always inpractice[22]- regarded as a virtual field, though with fluctuations giving rise to observable effects (see, e.g., the discussion in Ref. 23). The assumption of

reality of the vacumm field is not free of difficulties, even very serious ones as the cosmological implications of its enormous energy density; this is an unsolved problem, which strictly speaking affects quantum electrodynamics as well.

The idea of a close connection between quantum stochasticity and the zero-point radiation field is more than three decades old and has been rediscovered by several independent workers. It is mentioned explicitly by Kalitsin[24] already in 1953, but the first attempts to elaborate a theory based on it are due to Braffort and coworkers[25] and Marshall[26]. A short review of SED is presented in Ref. 27 and a more detailed one can be found in Ref. 28

The zero-point field is normally assumed to be a solution of Maxwell's equations without sources, that can be expressed in terms of Fourier components with stochastic amplitudes averaging to zero and having independent Gaussian distributions. By the requirement of Lorentz invariance, its spectral density must be of the form $\lambda\omega^3$. With $\lambda = 4\hbar/3\pi c^2$, the energy of the field mode ω is $\frac{1}{2}\hbar\omega$, as it should at zero temperature.

Before entering into the dynamics, let us illustrate the fundamental role that the hypothesis of the zero-point field may play as a foundational cornerstone of quantum theory.

Consider a blackbody in equilibrium at temperature T. Its mode of frequency ω will have the average energy

$$\langle\varepsilon(\omega)\rangle = \langle\varepsilon_T(\omega)\rangle$$

and assuming the field components to be Gaussianly distributed with zero mean, the variance of the energy is

$$\sigma_\varepsilon^2(\omega) = \sigma_T^2 = \langle\varepsilon_T\rangle^2 \quad .$$

Using the well-known Einstein formula for the fluctuations of the energy of a system in thermal equilibrium with a heat bath at tem-

perature T:

$$\sigma_T^2 = -\frac{\partial \langle \varepsilon_T \rangle}{\partial \beta} \quad , \quad \beta = 1/KT \tag{9}$$

we get the differential equation

$$\frac{\partial \langle \varepsilon_T \rangle}{\partial \beta} + \langle \varepsilon_T \rangle^2 = 0$$

with solution

$$\langle \varepsilon_T \rangle = \frac{1}{\beta_0 + \beta}$$

where β_0 is an integration constant; since for $T \to \infty$ $(\beta \to 0)$ $\langle \varepsilon_T \rangle$ should become infinite, we must take $\beta_0 = 0$ and thus we get the classical formula

$$\langle \varepsilon_T \rangle = \frac{1}{\beta} = KT \tag{10}$$

which corresponds to the Rayleigh-Jeans law. Now let us add to these elementary considerations a single new ingredient, namely, the zero-point field. For this we assume the most natural hypotheses, namely:

i) The mean energy is a sum of the thermal and zero-point contributions:

$$\langle \varepsilon(\omega) \rangle = \langle \varepsilon_T \rangle + \langle \varepsilon_0 \rangle$$

ii) The zero-point field is Gaussian:

$$\sigma_0^2 = \langle \varepsilon_0 \rangle^2$$

iii) The zero-point field and the thermal part are statistically independent, and thus the variance of the energy is

$$\sigma_\varepsilon^2 = \langle \varepsilon \rangle^2 = \sigma_T^2 + \sigma_0^2$$

whence

$$\sigma_T^2 = \langle\varepsilon\rangle^2 - \sigma_0^2 = \langle\varepsilon_T\rangle^2 + 2\langle\varepsilon_0\rangle\langle\varepsilon_T\rangle \quad .$$

Inserting this result into Einstein's formula (9) we get the new differential equation

$$\frac{\partial\langle\varepsilon T\rangle}{\partial\beta} + \langle\varepsilon_T\rangle^2 + 2\langle\varepsilon_0\rangle\,\langle\varepsilon_T\rangle = 0 \quad . \tag{11}$$

The solution of Eq.(11) that goes to (10) in the classical limit is

$$\langle\varepsilon_T\rangle = \frac{2\langle\varepsilon_0\rangle}{e^{2\beta\langle\varepsilon_0\rangle}-1} \tag{12}$$

This is Planck's law for the blackbody for $\langle\varepsilon_0\rangle \neq 0$; it suffices to take $\langle\varepsilon_0\rangle = 0$ as corresponds to the absence of the zero-point field, to recover classical physics. This derivation, due to Boyer[29] and Theimer[30], allows also to learn a little more about $\langle\varepsilon_0\rangle$ from first principles. In fact, Wien's law for the spectral density of radiation in equilibrium with matter at temperatura T

$$\rho(\omega) = \omega^3 f(\tfrac{\omega}{T})$$

gives when compared with Eq.(12), $\langle\varepsilon_0\rangle \alpha\omega$ and $\rho_0(\omega)\ \alpha\omega^3$, in agreement with the arguments mentioned above, In writing $\varepsilon_0 = \frac{1}{2}\hbar\omega$ Planck's constant acquires a physical meaning: It measures the fluctuations of the random field and hence also the fluctuations impressed by this field upon the particle (as expressed, e.g., in the Heisenberg inequalities). Other derivations of Planck's law from SED are presend in Ref. 31.

To develop a theory of the motion of electrons in interaction with the random field we can in principle start for the Hamiltonian of

the entire (field plus particle) system:

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + V(\vec{x}) + \frac{1}{2}\sum_{n\lambda}\left(p_{n\lambda}^2 + \omega_n^2 q_{n\lambda}^2\right) \tag{13}$$

whith

$$\vec{A} = \sqrt{\frac{2\pi\hbar c}{L^3}}\sum_{n\lambda}\frac{1}{\sqrt{\omega n}}\hat{e}_{n\lambda}a_{n\lambda}e^{i(\vec{k}n\cdot\vec{x}-\omega_n t)} + c\cdot c\cdot$$

$$\vec{k}_n\cdot\hat{\varepsilon}_{n\lambda} = 0, \qquad \omega_n = c|\vec{k}_n|; \qquad a_{n\lambda} + \frac{1}{\sqrt{2\hbar\omega_n}}\left(p_{n\lambda} + i\omega_n q_{n\lambda}\right) \tag{14}$$

$$\langle p_{n\lambda}^2\rangle = \omega_n^2 \quad \langle q_{n\lambda}^2\rangle = \frac{1}{2}\hbar\omega_n$$

and derive the Hamilton equations for both the particle and the field variables. An approximate procedure used to eliminate the latter leads to a stochastic extension of the Abraham Lorentz equation for the particle:

$$m\ddot{\vec{x}} = \vec{F}(\vec{x}) + \frac{2e^2}{3c^3}\dddot{\vec{x}} + e\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right) \quad . \tag{15}$$

This highly nontrivial stochastic differential equation with coloured noise is somewhat simplified by neglecting the magnetic force and taking the long-wavelength limit, in which $\vec{E} = \vec{E}(t)$; Eq.(15) reduces then to

$$m\ddot{\vec{x}} = \vec{F}(\vec{x}) + m\tau\dddot{\vec{x}} + e\vec{E}(t) \tag{16}$$

with $\tau = 2e^2/3mc^3$. This is the Braffort-Marshall equation, usually taken as the starting point of SED. It has been applied to simple linear problems, such as the free particle[32], the particle subject to a constant force[32] and the harmonic oscillator[25,26,33,34] (see Ref.28 for a more complete list of references). Let us briefly mention some of the results for the harmonic oscillator: Owing to the Gaussian distribution of the random field amplitudes, the particle variables x and

p are Gaussian as well and the stationary phase-space distribution is

$$P(x,p) = \frac{1}{\pi\hbar} \exp\left[-\frac{2}{\hbar\omega}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2\right)\right] .$$

At temperatures $T>0$ the average energy of the field modes is $\frac{1}{2}\hbar\omega(1+\theta)/(1-\theta)$ with $\theta = \exp(-\hbar\omega/kT)$ and the phase-space distribution takes the form

$$P(x,p) = \frac{1}{\pi\hbar}\,\frac{1-\theta}{1+\theta}\exp\left[-\frac{2}{\hbar\omega}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2\right)\frac{1-\theta}{1+\theta}\right] \qquad (17)$$

which is just the Wigner distribution for a mixture in equilibrium with a heat bath at temperature T, as described by quantum statistical mechanics (see, e.g., Ref. 35). The Lamb shift and the radiative decay rate are obtained when the radiation reaction is introduced as a perturbation.

In spite of the remarkable coincidence with quantum mechanics, there are conceptural differences that deserve a close attention. For instance, for SED the ground state is the state in which the rates of absorption and radiation of energy are equal. The Heisenberg inequality reflects the fluctuations impressed by the field upon x and p in the stationary state. Owing to the relatively long correlation time of the single field components, the particle fluctuations are highly correlated, which explains the narrow emission and absorption lines. Here we have a theory that can explain in principle the successes of the formal stochastic theories of quantum mechanics, but we also face some problems. For instance, in Eq.(17) the excited states appear simply as mathematical components of the equilibrium distribution, devoid of a direct physical meaning. Also a fundamental epistemological question, namely, why Schödinger's equation provides a correct description of the system, remains unanswered. It is clear that some basic elements are still lacking in the theory.

In addition to those mentioned above, other problems have been successfully approached, such as the study of the free radiation field[26,34,36] the van der Walls forces[37] and diamagnetism[26,38,39]. SED has also served to suggest an explanation of the origin and meaning

of the electron spin[39].

However, the few attempts to solve nonlinear problems have been unsuccessful. The usual procedure is to construct a Fokker-Planck equation for the (multiply periodic) classical system perturbed by the random field and the radiation reaction[40,41] (See also Refs. 28 and 41 for more complete references). For the hydrogen atom, for instance, this procedure leads to a ground state of zero energy, which implies spontaneous ionization. Moreover, the system turns out to be non-ergodic and the Fokker-Planck equation admits several coexisting stationary solutions[39,41,42] (See also Refs. 1 and 28).

For the quartic oscillator, the results are not as aberrant: The ground-state energy is correctly predicted to first order in the perturbation, but the second order result is incorrect[41]. Another unsatisfactory result, common to all nonlinear problems, is the lack of detailed energy balance at every single frequency of radiation; it would thus seem that the mechanical system pumps energy from some field modes into others, contradicting Kirchhoff's law on the universality of the equilibrium spectral density. This problem was first discussed by Boyer[43] and has been rediscussed afterwards[44].

4. THE PRESENT STAGE: SED ANEW

A general feeling of frustration has involved SED in recent years, due to its inability to produce new positive results. But a careful revision of the situation tends to suggest that the difficulties encountered are methodological rather than a matter of principle. Even though we cannot afirm at this stage that SED is an essentially correct theory, there are two points that strengthen our confidence. First, the simplicity and physical clarity of its postulates and second, the positive results it has furnished, which can hardly be considered a mere coincidence with quantum mechanics. It seems therefore opportune to explore anew the possibilities of SED and try to obtain a better understanding of its implications. This has been the purpose of our most recent work[45].

Let us start by examining a fundamental point, namely, the origin of quantization. For this purpose, we first consider a system subject to an external binding force $F(x,\dot{x},t)$:

$$m\ddot{x} = F \tag{18}$$

such that it can reach a stationary periodic state of motion (we shall consider one-dimensional motion, for simplicity). By writing

$$x = A \sin \theta, \quad \dot{x} = \Omega A \cos \theta$$

with

$$A = A(t), \quad \theta = \Omega t + \phi(t)$$

one obtains

$$\langle (F + m\Omega^2 x)x \rangle_\Omega = 0 \quad , \quad \langle (F + m\Omega^2 x)\dot{x} \rangle_\Omega = 0 \tag{19}$$

where $\langle \ \rangle_\Omega$ denotes averaging over a period $T = 2\pi/\Omega$, under the assumption that A and ϕ are essentially constant during that period. This approximate description in terms of a harmonic oscillation is valid only in the stationary state of motion. We can therefore speak of a local linearization, that changes from state to state: For every value of the amplitude (or the energy), there will be a different value of Ω for Eqs. (19) to hold, which reflects the asynchrony of the system.

Let us now look at Eq.(16) from point of view. We can rewrite it by adding $m\Omega^2 x$ on both sides:

$$m\ddot{x} + m\Omega^2 x = [F + m\Omega^2 x] + m\tau\dddot{x} + eE \quad . \tag{20}$$

If we select Ω so as to comply with Eqs.(19), then the effect of the terms within square brackets is simply a periodic alteration

which averages to zero in the stationary state and Eq.(20) can be approximated by the linear equation

$$m\ddot{x} = - m\Omega^2 x + m\tau\dddot{x} + eE \quad . \tag{21}$$

Here we are taking into account that the radiation reaction and the random electric force are small compared with the external force and hence do not alter the stationary periodic motion significantly. These terms play actually a stabilizing role: As seen from Eq.(20) the field mode of frequency Ω sustains the oscillation and the radiation reaction force prevents an infinite resonant response to this mode.

The solution of the linearized Eq.(21) can be used to derive some interesting results, such as the Heisenberg relation

$$\sigma_x^2 \, \sigma_p^2 = \frac{\hbar^2}{4} \tag{22}$$

for the dispersions of x and p, and

$$\langle K \rangle = \frac{1}{4} \hbar\Omega \tag{23}$$

for the average kinetic energy per degree of freedom. (The system is assumed to have the right ergodic properties for the time averages to coincide with the ensemble averages). We recall that these results hold only in the stationary state. This means that Eq.(23) must hold in addition to the relation between $\langle K \rangle$ and Ω obtained from the linearization condition: The value of Ω (and that of $\langle K \rangle$) is thus uniquely fixed and the motion be comes quantized. This is the proposed mechanism of quantization. In other words, of all classically allowed states, only one is stationary under the influence of the random electromagnetic field.

Consider, e.g., the circular orbits of the hydrogen atom. Eq.(20) goes into (21) for $F(x) = - \frac{e^2}{a^3} x$ and Eqs.(19) give

$$\Omega = (e^2/ma^3)^{1/2} \quad .$$

The kinetic energy associated to this orbital motion is therefore, according to classical mechanics,

$$\langle K \rangle = \frac{e^2}{2a} = \frac{1}{2} (me^4\Omega^2)^{1/3} \quad . \tag{24}$$

On the other hand, Eq.(23) gives

$$\langle K \rangle = \frac{1}{2} \hbar\Omega \tag{25}$$

for two degrees of freedom. Combining Eqs.(24) and (25) we obtain

$$\Omega = \frac{me^4}{\hbar^3} \quad , \qquad \varepsilon = - \langle K \rangle = - \frac{me^4}{2\hbar^2} \tag{26}$$

which are the quantum mechanical results for the ground state.

By virtue of the nonlinearity of the force, it is natural to assume that the system can respond preferentially also to one of the harmonics, say $n\Omega$, of the field; in this case one has for two-dimensional motion:

$$\langle K \rangle = \frac{1}{2} \hbar\Omega n \tag{27}$$

instead of Eq.(25), and this leads to the correct values for the energy levels of the excited states:

$$\varepsilon_n = - \frac{me^4}{2\hbar^2 n^2} \quad . \tag{28}$$

These results can be generalized to elliptic ortibs and similar methods can be applied to other simple nonlinear problems, with satisfactory results. They can also be rephrased in a more familiar form, by noticing that the average kinetic energy is related to the

action variable:

$$\langle K \rangle = \frac{m\Omega}{4\pi} \oint \dot{x}^2 \, dt = \frac{\Omega}{4\pi} \oint p dx = \frac{\Omega J}{4\pi}$$

whence from Eq.(27):

$$J = n\hbar \quad . \tag{29}$$

This approximate calculation suggests a physical explanation of the phenomenological Wilson-Sommerfeld rules.

Notice also that by assigning a wavelength to the orbital motion of velocity v sustained by the field harmonic $n\Omega$:

$$\lambda = \frac{v}{\nu} = \frac{2\,v}{n\,\Omega}$$

and using once more Eq.(27):

$$\langle K \rangle = \frac{1}{2} \hbar n \Omega = \frac{1}{2} m v^2$$

one obtains

$$\lambda = \frac{\hbar}{mv} \tag{30}$$

which indicates that de Broglie's wavelength can be interpreted as a dynamical (average) property acquired by the particle through its interaction with the vacuum field.

Let us now develop a formalism based on the heuristic approach outlined above. It is clear from Eq.(21) that the properties of the field variables will reflect themselves in the stationary solution. The statistical properties are reflected in the calculation of averages and dispersions, such as Eqs.(22) and (23). Similarly, the canonical properties of the field determine a symplectic structure for the particle

variables. To see this, we introduce the Poisson bracket with respect to the random field amplitudes, which we shall call Poissonian:

$$< \; ; \; > \; \equiv \; \sum_{m\lambda} \left(\frac{\partial}{\partial a_{m\lambda}} \frac{\partial}{\partial a^{*}_{m\lambda}} - \frac{\partial}{\partial a^{*}_{m\lambda}} \frac{\partial}{\partial a_{m\lambda}} \right)$$

and calculate the Poissonian of x and p corresponding to the sationary solution of Eq.(21); the result is

$$\begin{aligned} <x_i ; p_j> \; &= i\hbar \delta_{ij} \\ <x_i ; x_j> \; &= <p_i ; p_j> \; = 0 \end{aligned} \tag{31}$$

to lowest order in τ. Note that these results are independent of Ω and hence they apply for any binding force and any (stationary) state of motion. Using Eqs.(31) one obtains for any phase-space variables f,g:

$$<f;g> \; = i\hbar \; [f,g]_{xp} \tag{32}$$

where $[\; , \;]_{xp}$ is the usual Poisson bracket with respect to x and p. Two particular instances of Eqs.(32) are

$$<x_i ; f> \; = i\hbar \frac{\partial f}{\partial p_i} \quad , \quad <p_i ; f> \; = - \, i\hbar \frac{\partial f}{\partial x_i} \quad . \tag{33}$$

Let us now attempt to describe the dynamics. Since the effects of both the radiation reaction and the random electric force are small when the system is close to a stationary state, the dynamics will be essentially determined by the external force, which means that the equation of evolution for any function f(x,p,t) is basically the classical equation

with $H = \frac{p^2}{2m} + V$. However, since x and p are random variables that must satisfy Eq.(31), the phase-space functions f and H are subject to the constriction expressed in (32):

$$i\hbar\,[f,H]_{xp} = \langle f;H\rangle \quad .$$

By introducing this into the above equation we obtain

$$i\hbar\dot{f} = i\hbar\,\frac{\partial f}{\partial t} + \langle f;H\rangle \tag{34}$$

which we propose as the equation of evolution for f. It is interesting to observe that even though our description has been strictly causal, in writing Eq(34) any reference to the cause of stochasticity has been lost; this suggests explaining the apparently non-causal statistical behaviour of quantum mechanics as an artifact of the description.

Eq.(34) applied to the probability density ρ gives

$$i\hbar\,\frac{\partial\rho}{\partial t} = \langle H;\rho\rangle \tag{35}$$

since ρ is a conserved quantity and therefore $\dot{\rho} = 0$. To describe the evolution in terms of stationary states we introduce a Hilbert space with vectors ϕ_n such that

$$\xi = \sum_n C_n e^{-i\varepsilon_n t/\hbar}\phi_n \quad , \quad \xi\xi^* = \rho \ . \tag{36}$$

It follows from Eqs.(35) and (36) that the ϕ_n are the solutions of the eigenvalue equation

$$\varepsilon_n\phi_n = \langle H;\phi_n\rangle \tag{37}$$

and the eigenvalues are given by

$$\varepsilon_n = \int \phi_n^* \langle H;\phi_n \rangle \, d\mu \tag{38}$$

where μ is the accesible phase space, assuming the ϕ_n to be properly normalized: $\int \phi_n^* \phi_n = 1$. Now, suppose we want the quantity

$$\int \psi^* \langle H;\psi \rangle d\mu$$

to attain an extremal value under small independent variations of ψ(or ψ), subject to the normalization condition

$$\int \psi^* \psi \, d\mu = 1.$$

The corresponding variational problem reads (when only the ψ^* are varied)

$$\int \delta\psi^* (\langle H;\psi \rangle - \lambda\psi) d\mu = 0$$

and its solutions are, according to Eqs.(37) and (38), $\psi = \phi_n$, $\lambda = \varepsilon_n$. This means that the energy parameter acquires the extremal values ε_n when ψ corresponds to the stationary solutions ϕ_n. This formalism has allowed us to transform the problem of calculating the quantized values of Ω corresponding to the stationary states of motion, into an eigenvalue problem.

As a further step in this attempt to extricate the connection between SED and quantum mechanics, we observe that the present description lends itself to an operator formalism, in which the operators are associated to the corresponding dynamical variables by means of the relation

$$\hat{A}\phi = \langle A;\phi \rangle \quad . \tag{39}$$

From the properties of the Poisson brackets one can derive useful properties for the operators. For instance, the identity

$$\langle A;BC\rangle = \langle A;B\rangle C + B\langle A;C\rangle$$

leads to

$$\hat{A}B\phi = \langle A;B\rangle\phi + B\hat{A}\phi$$

or interchanging A and B:

$$A\hat{B}\phi = \langle A;B\rangle\phi + \hat{B}A\phi \quad .$$

These equations can be combined and rewritten as

$$[A,\hat{B}] = [\hat{A},B] = \langle A;B\rangle \quad . \tag{40}$$

The first (second) expression is the commutator of $\hat{A}$ and $\hat{B}$ in the A(B) representation; Eq.(40) tells us therefore that the value of the commutator is independent of the representation. Thus for instance, the commutator of $\hat{x}$ and $\hat{p}$ is

$$[\hat{x},\hat{p}] = i\hbar$$

in both the x and the p representation. From Eqs.(33),

$$\hat{p}_i = -i\hbar\partial/\partial x_i$$

in the x representation and

$$\hat{x}_i = i\hbar\partial/\partial p_i$$

in the p representacion.

On may further write the Jacobi identity:

$$\langle A;\langle B;\phi\rangle\rangle \; - \langle B;\langle A;\phi\rangle\rangle \; = \langle\langle A;B\rangle;\phi\rangle \quad ;$$

In terms of operators:

$$(\hat{A}\hat{B} \; - \; \hat{B}\hat{A})\phi \; = \langle \hat{A};\hat{B}\rangle\phi$$

whence

$$[\,\hat{A},\hat{B}\,] \; = \langle \hat{A};\hat{B}\rangle \quad . \tag{41}$$

For instance, for the components of the orbital angular momentum this relation gives

$$[\,\hat{\ell}_i,\hat{\ell}_j\,] \; = i\hbar \; \varepsilon_{ijk}\hat{\ell}_k \quad .$$

Finally, the eigenvalue equation (37) takes the form

$$\varepsilon_n\phi_n \; = \hat{H} \; \phi_n$$

and the general solution (36) satisfies the equation

$$i\hbar \; \frac{\partial \xi}{\partial t} \; = \hat{H}\xi$$

with $\hat{H} \; = \; \frac{\hat{p}^2}{2m} \; + \; V(x)$ in the x representation. We thus recover the usual quantum formulation in Hilbert space.

The results presented in this section suggest that a proper consideration of the specific dynamical effects of the random electromagnetic field may solve the problems faced by SED in connection with energy balance and the existence of stationary states. Our arguments have been mainly heuristic and it is obviously necessary to develop a more rigorous tratment in which every assumption is clearly justified.

If this can be done the new theory thus developed should not only explain the well-known quantum phenomenology, but also allow us to extend our knowledge beyond present-day quantum mechanics.

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