COLLECTIVE MODES IN THE ^{4}He SYSTEM

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(recibido julio 17, 1985; aceptado enero 30, 1986)

ABSTRACT

A new class of states, collective excitations, appear to exist in the ^{4}He system. The states are derived from generalized R-matrix calculations and include levels with $J^\pi = 0^+, 2^+, 4^+, 6^+$, and 8^+. The agreement of the model calculations, when compared with the expected $\alpha J(J + 1)$ rule, improves as the excitation energy increases.

RESUMEN

Un nuevo tipo de estados, excitaciones colectivas, existen en el sistema ^{4}He. Los estados se obtienen por cálculos de matriz R generalizados e incluyen niveles con $J^\pi = 0^+, 2^+, 4^+, 6^+$ y 8^+. El acuerdo de los cálculos del modelo, comparado con la regla esperada $\alpha J(J + 1)$, mejora con el crecimiento de la energía de excitación.

1. INTRODUCTION

Collective phenomena in nuclei is a well established concept[1] and has been observed in many nuclei. Although studies usually focus on heavy nuclei, collective phenomena has been observed or predicted in light nuclei[2,3]. However, collective phenomena has yet to be reported in the lightest nucleus which exhibits a wealth of energy levels, namely the ^{4}He system. This paper will attempt to determine the extent to which 0^+, 2^+, 4^+, 6^+, and 8^+ states obey the general rule[1]: $E(J^+) = E(0^+) + \alpha J(J + 1)$ in the ^{4}He system.

Experimental efforts have confirmed 0^+ and 2^+ levels in ^{4}He[4] and recent data of Grüebler *et al.*[5]. Have suggested a 4^+ level. To date, states of higher spin have yet to be experimentally suggested. Theoretical studies have predicted states of up to $J^\pi = 6^+$ [6,7] and it is possible to extend these studies to calculate states of higher spin. Such an extension will be used to determine how well the ^{4}He system fits the $\alpha J(J+1)$ rule.

The approach noted above assumes that rotational bands will be built upon the 0^+ states in ^{4}He. For example, the first rotational band would have the ground state as its common intrinsic state. Since the ground state of ^{4}He is dominantly spherical[7] in nature, the $\alpha J(J+1)$ model should not work exceptionally well for the ground state rotational band. However, the excited 0^+ states in ^{4}He become non-spherical, and it is expected that rotational bands will be built upon these excited, non-spherical 0^+ states[7].

2. FORMALISM

The formalism wich will be used to calculate the 0^+, 2^+, 4^+, 6^+, and 8^+ states is based on the generalized R-matrix methodology of Lane and Robson[8], and the application of this model to the ^{4}He system has been discussed in Ref. 6. It will be sufficient to state that harmonic oscillator expansion states $|\lambda\rangle$ for the ^{4}He system are defined in terms of internal coordinates[6]. The wave function takes the form

$$\psi = \sum_{\lambda=1}^{N} A_{\lambda} |\lambda> \tag{1}$$

where the number of basis states N depends on both the total angular momentum(J) and parity of the basis state. The specification of the basis state is provided in Ref. 6. The expansion amplitudes A_{λ} are obtained from the solution of the generalized R-matrix equation (with all channel radii at infinity):

$$\sum_{\lambda'} [<\lambda|H - E|\lambda'> A_{\lambda'}] = 0 \quad , \tag{2}$$

where H is the Hamiltonian for the ^{4}He system[6]. The nuclear interaction term in the Hamiltonian is based on a modification to the Sussex interaction[9], and includes only a two-body component[6]. Eq.(2) approaches the shell-model solution because all channel radii are at infinity[6].

The size of the model space used for the $J^{\pi} = 0^{+}$, 2^{+}, 4^{+}, and 6^{+} calculations is limited to $4\hbar\omega$ of total oscillator excitation energy[6,7]. However, it is not possible to calculate states of $J^{\pi} > 6^{+}$ for the $4\hbar\omega$ model space. For the purposes of this paper, a $6\hbar\omega$ model space has been used to calculate the $J^{\pi} = 8^{+}$ spectrum. Calculations for the 8^{+} spectrum are also performed following Refs. 6 and 7, except for the fact that the 8^{+} basis states are permitted additional oscillator excitation energy.

If the spectrum of levels generated by Eqs. (1) and (2) admits a collective(rotational) structure, it should be described by the rule[1]

$$E(J^{+}) = E(0^{+}) + \alpha J(J + 1) \quad . \tag{3}$$

The rotational constant α is expected to have a value given by

$$\alpha(A) = 30.985 \text{ MeV}/ (A^{5/3}) \quad , \tag{4}$$

where the constant is chosen such that it metches the experimental α value in ^{240}Pu[10]. The result of Eq. (4) for the A=4 system suggest an α value of 3.074 MeV. It should be noted that an extrapolation from the A=240 system to the A=4 system will at best lead to qualitative results.

3. COMPARISON OF MODEL RESULTS WITH THE EXPERIMENTAL ^4He SPECTRUM

The model results summarized in this section represent the results of model calculations for specific basis sizes. The dimensionality of the basis states considered in this study are summarized in Table I.

TABLE I

J^π	Maximum Oscillator Excitation Energy ($\hbar\omega$)	Size of basic
0^+	4	45
2^+	4	108
4^+	4	46
6^+	4	3
8^+	6	6

Table I. Dimensionality and maximum excitation energy for 0^+, 2^+, 4^+, 6^+, and 8^+ states.

The model has been previously compared with available experimental data in Refs. 6 and 11 - 16. A summary of the comparison of model and experimental[4] levels was provided in Figs. 3 and 4 and Table 4 of. Ref. 6. The model generally predicts the order and position of the experimental levels. As noted in Ref. 6, the T=1 spectrum is predicted quite well in level order and in energy. The T=0 spectrum tends to lie higher in energy than the experimental T=0 spectrum, but the 0^+ ground state is predicted to lie at the correct energy. The major discrepancy in the T=0 calculation is that the first excited level lies about 10 MeV above the corresponding experimental level. Model calculations(11) also show favorable agreement with measured proton and neutron polarization differences in the reactions $^3H(\vec{p},p)^3H$ and $^3He(\vec{n},n)^3He$ which support a new level near 37 MeV excitation energy.

A detailed comparison of model scattering results for the $^3H(p,p)$, $^3He(n,n)$, $^3H(p,n)$, $^2H(d,d)$, $^2H(d,p)$, and $^2H(d,n)$ reactions show good agreement with data(12). Comparisons of model and experimental analyzing powers and polarizations for the charge symmetric reactions $^2H(\vec{d},p)^3H$ and $^2H(\vec{d},n)^3He$ were provided in Ref. 13. The calculated(13) differences in polarization and analyzing power values are approximately an order of magnitude smaller than measured differences for $E_d \leq 10.0$ MeV. Model results(14) for the ^4He giant dipole resonance are also in reasonable agreement with data.

Model calculations also accurately represent a new proposed level at 40 MeV excitation energy in the ^4He system(15). Calculations of structure effects in the $^4He(\gamma,p)^3H$ E2 cross section correspond favorably with available data(16). The results of the aforementioned calculations suggest the model level spectra are generally providing a realistic representation of both level and scattering data in the ^4He system. The comparisons noted above are for excitation energies below 40 MeV. An assessment of the model spectra for excitation energies greater than 40 Mev will require a comparison with data which does not yet exist. Therefore, confirmation of predictions of model levels for $E_X > 40$ MeV must await further experimental measurements.

4. RESULTS AND DISCUSSION

The results of our calculations are summarized in Tables II, III and IV. Each of these tables presents 0^+, 2^+, 4^+, 6^+, and 8^+ levels predicted by the model of Eqs. (1) and (2), and a least squares fit of these levels to the $\alpha J(J + 1)$ rule. In addition, the difference Δ between the energy predicted by the $\alpha J(J + 1)$ model and R-matrix model is presented.

Table II summarizes the model results for the lowest lying members of the 0^+, 2^+, 4^+, 6^+, and 8^+ spectra. In a similar fashion, Tables III and IV summarize the second and third members of each spectrum, respectively.

The agreement of the R-matrix calculations with the $\alpha J(J + 1)$ rule improves as the excitation energy increases. For example, the sum of the Δ values for the 0^+, 2^+, 4^+, 6^+, and 8^+ levels decreases as the excitation energy increases--*i.e.* Δ = -37.5, -13.1, and -7.7 MeV for the first, second, and third collective bands, respectively. The largest discrepancy between R-matrix and $\alpha J(J + 1)$ model predictions occurs for the 4^+ levels, which have been recently shown to be poorly described by models which only include two-body forces[17]. In the $4^+\Delta$ values are excluded, Δ value sums of -24.8, -0.6, and +1.7 MeV are derived for the first, second, and third collective 0^+ bands, respectively. With increasing excitation energy, the results suggest that a collective structure is present in the ^{4}He system. However, the α_i values of each band are considerably smaller (α_1 = 1.528 MeV, α_2 = 1.153 MeV, and α_3 = 1.036 MeV) than the predicted value of 3.074 MeV[10].

The B(E2) values for the three 0^+ bands noted above are summarized in Table V. As a matter of comparison, the single particle B(E2) value for a $2^+ \rightarrow 0^+$ transition in ^{4}He is given by[1]

$$B_{sp}(E2) = 0.06\ A^{4/3}\ e^2\ fm^4 = 0.4\ e^2\ fm^4 \tag{5}$$

which is essentially in agreement with the values summarized in Table V. Therefore, the values in Table V do confirm the rotational picture suggested herein.

5. CONCLUSIONS

The results of this study suggest that a new class of states, collective excitations, appear to exist in the ^{4}He system. The major disagreement between our model and $\alpha J(J + 1)$ predictions lie in the $J^\pi = 4^+$

levels. The agreement with the $\alpha J(J + 1)$ model improves with increasing excitation energy. This improvement is expected because the ^4He excited states become increasingly non-spherical and appear to become more collective in nature as the excitation energy increases.

TABLE II

J^{π}	Relative Energy (MeV)[a]		Δ(MeV)
	R-matrix	$\alpha_1 J(J + 1)$[b]	
0_1^+	0.0	0.0	0.0
2_1^+	36.9	9.2	-27.7
4_1^+	43.3	30.6	-12.7
6_1^+	71.4	64.2	-7.2
8_1^+	99.9	110.0	+10.1
			Σ = -37.5

[a] Relative to 0_1^+ (ground state).

[b] α_1 = 1.528 MeV.

Table II. Spectrum for the first 0_1^+ (g.s) collective band in ^4He.

TABLE III

J^{π}	Relative Energy (MeV)[a]		Δ(MeV)
	R-matrix	$\alpha_2 J(J+1)$[b]	
0_2^+(29.2 MeV)	0.0	0.0	0.0
2_2^+	10.5	6.9	-3.6
4_2^+	35.6	23.1	-12.5
6_2^+	50.2	48.4	-1.8
8_2^+	78.2	83.0	+4.8
			Σ = -13.1

[a] Relative to 0_2^+ 29.2 MeV level.

[b] α_2 = 1.153 MeV

Table III. Spectrum for the second 0_2^+(29.2 MeV) collective band in ^{4}He.

TABLE IV

J^π	Relative Energy (MeV)[a] R-matrix	$\alpha_3 J(J+1)$[b]	Δ(MeV)
0_3^+(37.6 Mev)	0.0	0.0	0.0
2_3^+	4.2	6.2	+2.0
4_3^+	30.1	20.7	-9.4
6_3^+	42.8	43.5	+0.7
8_3^+	75.6	74.6	-1.0
			Σ = -7.7

[a] Relative to 0_3^+ 37.6 MeV level.

[b] α_3 = 1.036 MeV.

Table IV. Spectrum for the third 0_3^+(37.6 MeV) collective band in ^{4}He.

TABLE V

J_i^π	J_f^π	$B(E2)_{J_i \longrightarrow J_f}$ e^2 fm^4		
		0_1^+ band	0_2^+ band	0_3^+ band
2^+	0^+	0.3	0.2	0.3
4^+	2^+	3.3	5.9	3.0
6^+	4^+	3.1	1.0	16.3

Table V. Calculated B(E2) values in ^4He

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