Optimization of the receiver's diameter on a solar vapor generator

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Abstract. A computer-aided method is employed to find the optimum size of the thermal insulation put on the receiver of a solar vapor generator. The receiver is situated in the focal zone of a cylindrical parabolic mirror. The size of the focal zone depends on the imperfections of the mirror, and on the finite size of the sun. The absorbed radiation grows with the receiver's diameter until the optimun size is reached. Beyond this point the thermal losses grow faster than the absorbed radiation. The absorbed power, and the thermal losses due to conduction, convection and radiation, are calculated as functions of the receiver's diameter.

Resumen. Se usa un método de cálculo computarizado para encontrar el tamaño óptimo del aislamiento térmico del captador de un generador solar de vapor. El captador se coloca en la zona focal de un espejo parabólico cilíndrico. El tamaño de la zona focal depende de las imperfecciones del espejo y del tamaño finito del Sol. La radiación absorbida crece con el diámetro del captador hasta llegar a un tamaño óptimo. Después de este valor, las pérdidas térmicas crecen más rápido que la radiación absorbida. Se calcula la radiación absorbida, y las pérdidas por conducción, convección y radiación como función del diámetro del absorbedor.

1. Introduction

The design of a solar vapor generator can be considered as a first step in order to take advantage of solar energy. The vapor produced

in this way may be used to move motors, to produce heat and, as in the case considered here, to produce distilled water. In this case the vapor is obtained at a pressure of 1 atm., a temperature of 100°C, and is used as a heater and as a producer of distilled water in a previously designed high efficiency distiller [1]. For this purpose we have used a parabolic-cylindrical solar concentrator, because of its simplicity and the high concentration ratio attainable.

The diameter and isolation of the absorber are the two most important parameters to be optimized as functions of the heat fluxes in the system in order to maximize the amount of vapor obtained with a fixed solar energy collecting area. The methodology used to attain this optimization is explained below.

2. Theoretical analysis

In a parabolic-cylindrical concentrator, assuming that the absorber has a constant temperature, the useful power (P_u) is given by [2]

$$P_u = P_i - P_{\text{cond}} - P_{\text{conv}} - P_{\text{rad}}, \tag{1}$$

where the first term on the right (P_i) is the power captured by the absorber, and the last three terms represent the power losses due to the difference in temperature between the absorber and the surroundings, *i.e.*, losses by conduction, convection, and radiation.

The power captured by the absorber (P_i) is given by

$$P_i = \rho \gamma \tau \alpha R_d A_c \tag{2}$$

and depends on the reflectivity of the mirror (ρ), its form factor (γ), the transmitivity of the cover (τ), the absorptivity of the collector (α), the intensity of the radiation (R_d), and the effective area of the solar concentrator (A_c).

The maximum efficiency attainable with such a concentrator is therefore

$$(\eta) = \frac{P_u}{R_d A_c} \times 100. \tag{3}$$

According to Eqs. (1) and (3), to improve the efficency of the concentrator it is necessary to minimize the power losses P_{cond}, P_{conv} and P_{rad}.

The total power losses due to the thermal conduction through a wall of conductivity k and of area A are given by

$$P_{\text{cond}} = -kA\frac{dT}{dx}. \tag{4}$$

The radiated power from a surface of area A, emissivity ϵ, and at a temperature T_1, surrounded by an environment at a temperature T_2, is given by the Stefan-Boltzmann equation:

$$P_{\text{rad}} = \epsilon A\sigma(T_1^4 - T_2^4), \tag{5}$$

where T_1 and T_2 are in K and σ is the Stefan-Boltzmann constant.

The losses due to convection of a surface of area A and temperature T_1 immersed in a fluid with a temperature T_2 can be calculated as follows:

$$P_{\text{conv}} = hA(T_1 - T_2), \tag{6}$$

where h is the convection factor. This coefficient depends, among other things, on the natural or forced movement of the fluid around the surface.

The use of pre-defined adimensional quantities such as the Nusselt (Nu), Reynolds (R_e), Grashof (G_r) and Prandtl (P_r) numbers facilitates the calculation of the convection factor, h, in several circumstances. The exact solution for the coupled fluxes of heat and mass involved in the convection process can be attained only for very simple geometries.

In the case of natural convection (free convection), we have

$$Nu = C(G_r P_r)^n, \tag{7}$$

where C and n are experimentally determined constants. For uncovered horizontal tubes, we have according to [3]:

$$C = 0.525 \quad \text{and} \quad n = 0.25 \quad \text{if} \quad 10^4 < (G_r P_r) < 10^9. \tag{7a}$$

On the other hand, in the case of forced convection:

$$Nu = C(R_e)^n, \tag{8}$$

where

$$C = 0.24 \quad \text{and} \quad n = 0.6 \quad \text{if} \quad 10^3 < R_e < 5 \times 10^4 \tag{8a}$$

for uncovered horizontal tubes in a transversal flow [4].

The convection coefficient can be obtained from the Nusselt's number by means of the equation

$$h = \frac{Nuk}{D}, \tag{9}$$

where D is the diameter of the tube, and k is the thermal conductivity evaluated at the mean temperature:

$$T = \tfrac{1}{2}(T_1 + T_2),$$

where T_1 and T_2 are the temperatures of the surface and the surroundings, respectively.

In the Grashof's and Reynold's numbers, the tube's diameter is taken into account as characteristic parameter. Therefore, any scale change in the size of the system will be affected by the exponent n. The coefficient C could then depend on the geometry of the surface and its surroundings. When we modify the geometry without changing the absorber's diameter, we can use Eqs. (7) and (8) adjusting the corresponding C value to the measured Nusselt number, assuming the exponent n to be the same.

3. Results

Uncovered tubes

The evaluation of the total power collected by an uncovered tube can be made using the equations presented above. Such calculations
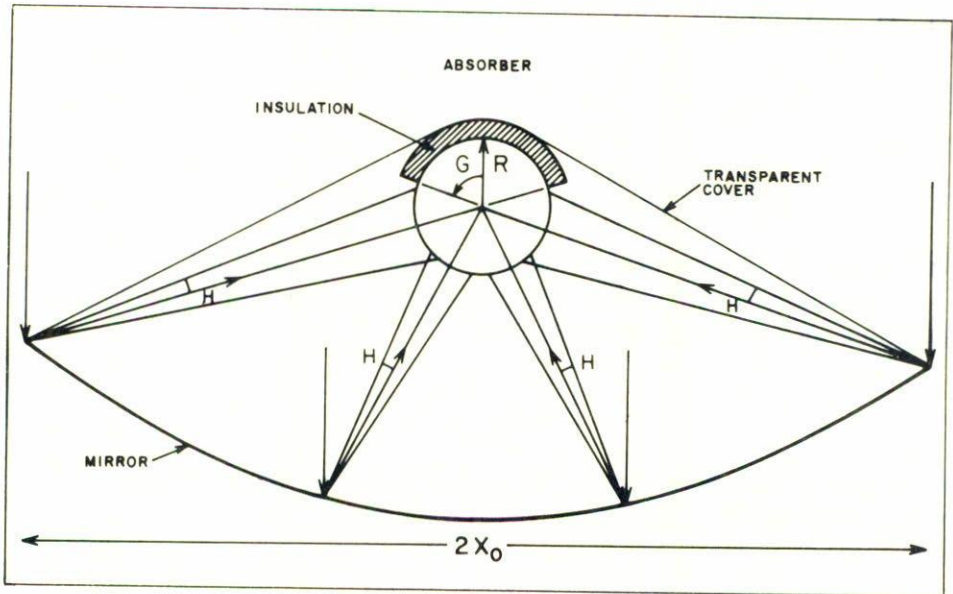
FIGURE 1. Diagram of the mirror and absorber tube. The radiation captured depends on the radius R and the angle G covered by the thermal insulation layer.

are simplified by the fact that the temperature in a vapor generator at atmospheric pressure is constant (and equal to $100°C$), and so are the thermal losses. The values of the constants used in this case are given in the appendix.

The form factor γ mentioned in Eq. (2) is the fraction of the mirror's reflected energy impinging on the absorber. Because of the finite dimension of the sun and the imperfection of the mirror, the radiation can not be concentrated in a single point, but in a diffuse focal zone. If we suppose that the imperfections follow an angular Gaussian distribution, with H as the angular standard deviation, the reflected beams will follow the same distribution (see Fig. 1). Those beams reflected from the outer parts of the mirror travel a longer distance compared with those reflected from the center, and therefore the dispersion for these rays is greater. It has

been demonstrated [5] that when the focal distance is equal to one quarter of the linear aperture ($f = X_0/2$), the focal zone tends to a minimum, and therefore this is the optimum shape for a solar collector when an uncovered tube is used. In Ref. 5 it is also shown that the dispersion of the radiation reflected from a parabolic mirror, with a Gaussian distribution of defects on the mirror's surface is similar to a Gaussian curve, but with a more expanded tail. This result justifies the common approximation for the resulting radiation as a Gaussian curve with a dispersion equal to that of the middle point in the mirror. This approximation is valid if the profundity of the mirror is small ($f > X_0/2$). However, if the mirror is very deep ($f < X_0/2$) this approximation cannot be used. In the following, such an approximation was not used. The form factor was calculated for each of the sizes of the absorber, considering the dispersion of the rays as Gaussian and being reflected from the middle point of each of the N zones in which the mirror has been divided ($N = 30$). The width of each zone is $2X_0/N$.

Figure 2a and 2b show the dependence of each of the terms in Eq. (1) as a function of the absorber radius normalized to the standard deviation S of the beam reflected from a point in the middle of the mirror ($X = X_0/2$). P_0 is the radiation power intercepted by the mirror, P_i is the fraction of that power captured by the absorber, and P_u is the useful power. In Fig. 2a, the focal distance is equal to $1/4$ of the linear aperture ($f = X_0/2$) for natural and forced convection. The existence of an optimum radius can be observed from this figure. In Fig. 2b we have ($f = 0.7X_0$) and free and forced convection. For forced convection, the useful power rapidly decreases and the dominant term in Eq. (1) is the convection term. The wind velocity used in the calculations was always 2.5 m/s.

Figure 3 shows the improvement of the useful power as the mirror's geometry is improved, therefore reducing the dispersion angle H down to the value of 0.26' (the apparent semidiameter of the Sun). Figure 3 shows only the values calculated for a forced convection case, the worst case of all.
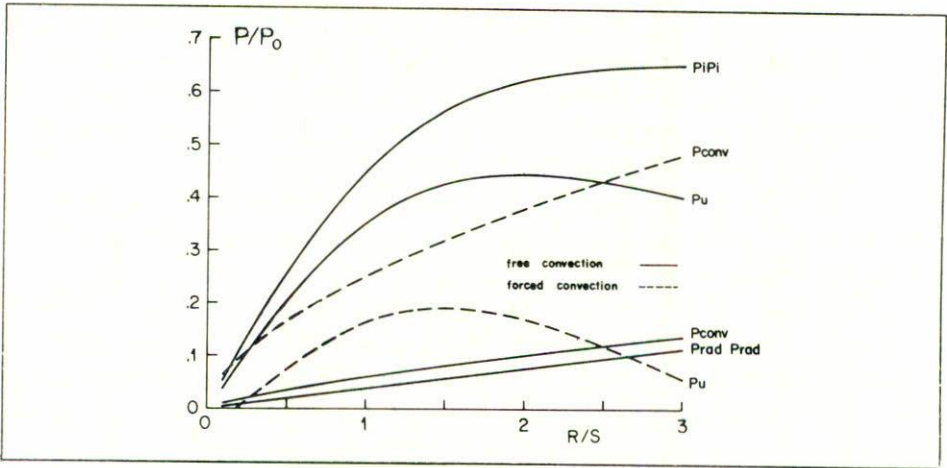
FIGURE 2. a) Calculated power incident in the absorber (P_i), power losses by convection (P_{conv}) and radiation (P_{rad}), and useful power (P_u) as a function of the absorber radius normalized to the standard deviation S of the beam reflected from a point in the middle of the mirror $(X = X_0/2)$. For a naked absorber, without any transparent cover. The focal distance is $f = 0.5X_0$ and $H = 1.25°$.
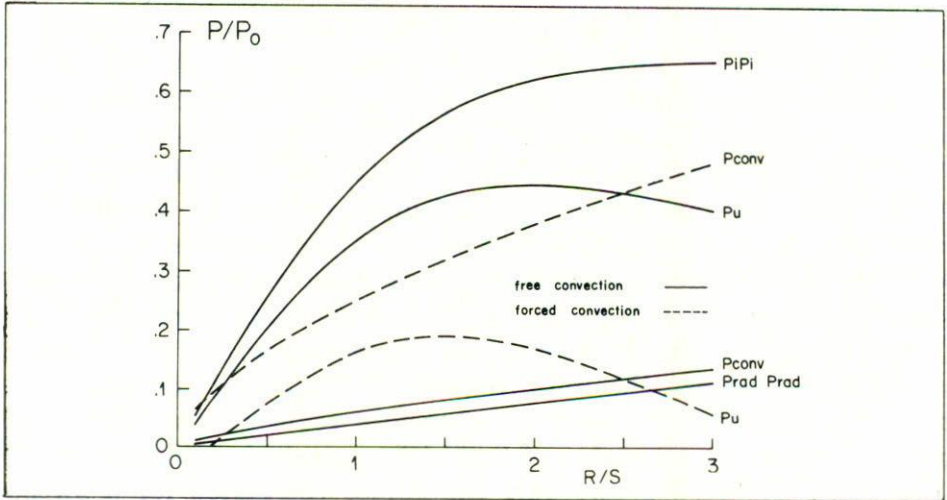

FIGURE 2. b) Same as Fig. 2a for $f = 0.7X_0$. The maximum useful power (P_u) values are smaller than that for $f = 0.5X_0$.
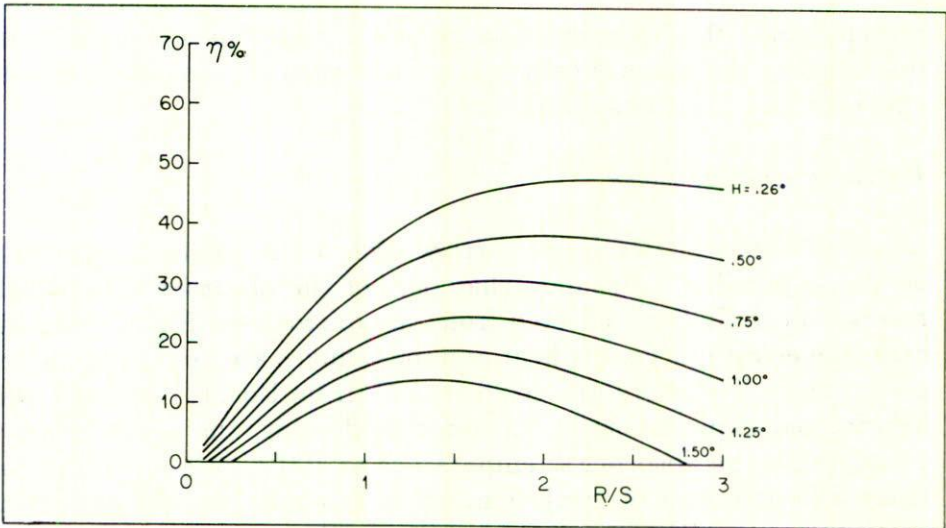
FIGURE 3. The efficiency (η) for a naked absorber, in a mirror with $f = 0.5X_0$, forced convection $(V = 2.5$ m/s$)$. The angular standard deviation (H) of the reflected beam, is the parameter associated to each curve.

An experimental value of $H = 2.50°$ was obtained [5] with an aluminum mirror constructed in our laboratory. Figure 3 shows how sensitive the useful power is to this parameter. The improvement of the shape of the mirror is difficult, and the best value that we could obtain was $H = 1.70°$ [6] .

Totally isolated tubes

Another way to reduce the convection losses is to surround the absorber tube with a transparent tube, and to reduce the pressure between the tubes down to such a pressure that the convection and conduction terms can be neglected. In such a case, the incident power P_i will be reduced due to the reflection in the external tube. Because of that, the diameter of the external tube must be large enough to reduce the reflection losses on its surface. The approximate useful power can be estimated in Fig. 2 by substracting from the incident

power P_i only the losses by radiation. Such a system is very efficient, but has the problems introduced by the vacuum seal between the absorber and the transparent tube.

Partially isolated tubes

In the case of parabolic mirrors with a rim angle of less than 90°, it is possible to insulate that part of the absorber tube which receives no radiation reflected from the mirror (see Fig. 1). In this case, the equations for the heat and mass fluxes are not easy to solve analytically. Therefore, the coefficient C (from Eqs. (7) and (8)) was determined experimentally. In order to do this, the power needed to maintain the absorber's temperature at 100°C was measured for three different sizes of insulation, under free and forced convection, and with and without a transparent plastic cover going from one edge of the mirror to the absorber, and then to the other edge of the mirror. This cover reduces greatly the convective losses, as can be seen from the measured C values:

$C = 0.60$ free convection without transparent cover;

$C = 0.27$ forced convection without transparent cover;

$C = 0.29$ free convection with transparent cover;

$C = 0.052$ forced convection with transparent cover.

A complete description of the system can be found in Ref. [6].

Figure 4a shows how the efficiency changes for free and forced convection, in a partially insulated absorber as a function of the absorber's radius. The normalizing standard deviation of the beam is $S = 6$ mm. The parabolic mirror ($f = 0.7X_0$ and $H = 1.25°$) is without any transparent cover, so when the wind blows the efficiency drops, even when we have 200° of the perimeter insulated. Fig. 4b shows the efficiency calculated with the C parameter measured with the transparent cover ($\tau = 0.9$) over the system. G is half the angle covered by the insulating layer, the optimun value of G is 60°. The
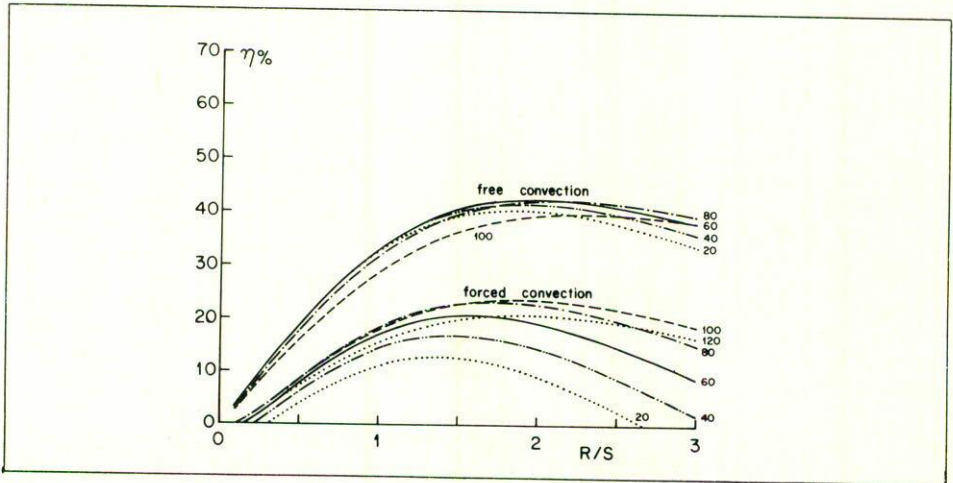
FIGURE 4. a) The efficiency (η) for free and forced convection, for a partially insulated absorber, in a parabolic mirror ($f = 0.7X_0$, $H = 1.25°$) as a function of the absorber's radius. The parameter G is half the angle covered by the insulating layer.
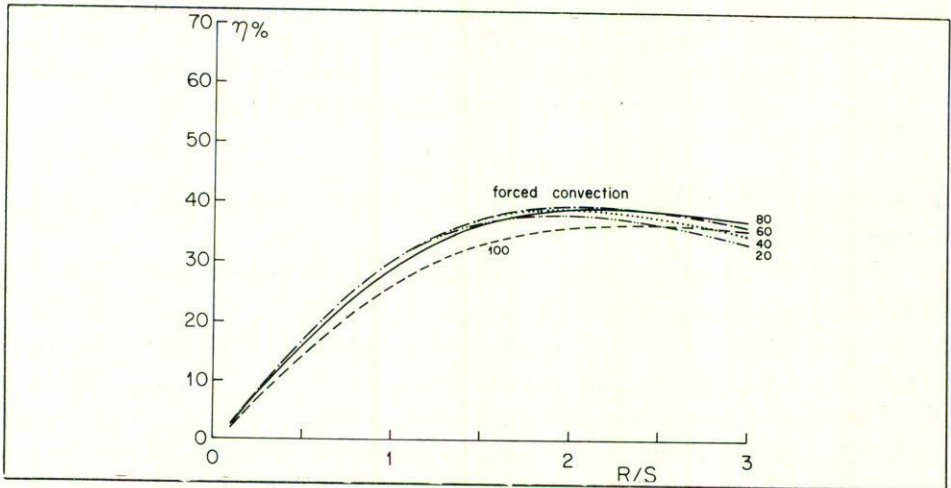

FIGURE 4. b) The efficiency (η), as in Fig. 4a, but with a transparent cover ($\tau = 0.9$), for forced convection. Wind velocity is 2.5 m/s. The free convection values are about 1% better than those shown for forced convection.
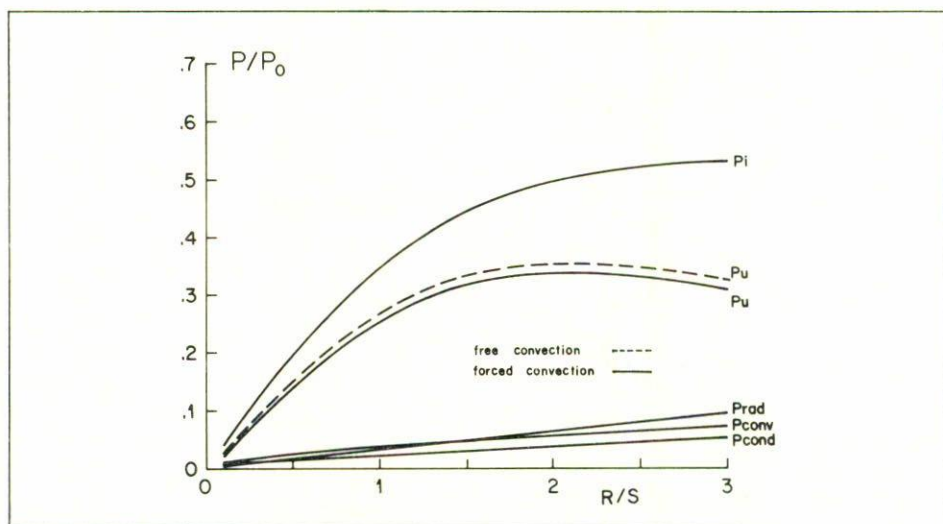
FIGURE 5. Calculated power incident in the absorber (P_i), power losses by convection (P_{conv}), conduction (P_{cond}) and radiation (P_{rad}) and useful power (P_u) as a function of the absorber radius, for forced convection Only the useful power (P_u) for free convection. The experimental set-up has an R/S ratio $= 1.7$. The measured efficiency is between the free convection and forced convection calculated values.

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Figure 5 shows the results calculated for a mirror constructed in Mérida, Yucatán. In this case ($f = 0.24 \pm 0.005$ m, $H = 1.7° \pm 0.1°$, $A = 2.40 \times 0.35 = 1.68$ m^2, direct radiation $680 \pm 1°W/\text{m}^2$) the total power received $P_0 = 1142$ watts. From the figure, the P_u/P_0 ratio for free convection is equal to 0.34 and for forced convection is equal to 0.325, so the useful power should fluctuate between 388 w for no wind and 371 w for a 2.5 m/seg wind speed average value for that city. In those days when the equipment was tested, 250 ± 10 ml of distilled water were obtained every half an hour, which is equivalent to a useful power of 380 ± 15 w, in agreement with the calculated values of 371–388 w.

It is important to point out that the power losses due to the transparent cover and the reflectivity of the mirror alone reduce the useful power to 63% of P_0. Therefore it is imperative to control the thermal losses in order to achieve a reasonable efficiency.

Appendix

Constant's values:
    Absorptance $(\alpha) = 0.98$
    Emittance $(\epsilon) = 0.89$
    Mirror's reflectivity $(\rho) = 0.7$
    Direct radiation $(Rd) = 680$ W/m^2
    Air density $(\rho_1) = 1.048$ kg/m^3
    Air thermal conductivity $(k) = 0.028$ W/mK
    Air viscosity $(\mu) = 2.06 \times 10^{-5}$ Ns/m^2
    Air Prandtl number $(P_r) = 0.72$
    Stefan Boltzmann constant $(\sigma) = 5.67 \times 10^{-8}$ W/m^{2}K^4
    Gravity aceleration $(g) = 9.81$ m/s^2
    Insulation thickness $(x) = 0.01$ m
    Insulation thermal conductivity $(k) = 0.035$ W/mK

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