*Investigación* *Revista Mexicana de Física* **34** *No. 1(1988) 69–73*

# A Bound for the photon mass from the angular distribution of cosmic background radiation

**José Leonel Torres and Víctor M. Yépez García**

*Departamento de Física, Universidad Michoacana, Morelia, Michoacán*

(recibido el 23 de febrero 1987; aceptado el 11 de septiembre 1987)

**Abstract.** Cosmic background radiation (CBR) provides relevant information on a possible photon mass. From its spectral composition a lower limit $\sim 10^{-37}$ grams was previously obtained for this quantity, a bound several orders of magnitude above those coming from purely electromagnetic, *i.e.*, non-thermal experiments ($\sim 10^{-47}$ grams). From CBR's angular anisotropy another lower bound on the photon mass is now calculated, and it does not appreciably improve on that inferred from its spectral profile.

**PACS: 98.80.Vc; 14.80.A**

## 1. Introduction

Due to experimental errors, measurements cannot determine the exact value of a physical quantity. These errors can be reduced as equipment and methods get refined, but never beyond the fundamental limits imposed by the uncertainty relations of quantum mechanics. The ultimate bound on the photon mass can be calculated from $(\Delta p)(\Delta x) \sim \hbar$, using for $\Delta x$ the size of the observed universe. This value of $\Delta x$ produces the minimum fluctuation in its momentum, which combined with the expression for the total energy, $E = h\nu = (p^2c^2 + m^2c^4)^{1/2}$, gives $m \simeq 10^{-65}$ grams as the minimum measurable mass for the photon and for any physical entity: below this value any mass effect would be masked by the momentum fluctuations from the uncertainty principle.

Bounds on the photon mass are obtained looking for possible deviations from the Maxwell equations, and interpreting such departures in terms of some extension of the theory containing a mass term. The usual covariant extension is that provided by the Proca equations [1], and the accepted bounds on the photon mass are the following:

a) From measurements of the speed of light at various frequencies, using astronomical sources [2], $m \leq 10^{-44}$ grams.
b) From measurements of charge inside a conductor [3], testing the validity of Coulomb's law, $m \leq 2 \times 10^{-47}$ grams.
c) From the spatial behavior of Jupiter's magnetic field over long distances, looking for a possible Yukawa-type term produced by a mass in the field equation [4], $m \leq 8 \times 10^{-49}$ grams.

Still lower bounds have been proposed, but they are based on non-trivial assumptions about the galactic magnetic field [5], or imply such drastic changes in our view of the world [6] that, in the absence of further supporting evidence, they will not be considered here.

Recently a new method to obtain bounds on the photon mass was proposed by one of us [7], looking for deviations from Planck's spectral distribution in a radiation cavity in thermal equilibrium. The Planck formula is strictly valid for infinitely large cavities and a null mass of electromagnetic radiation. The resulting expression for the radiance in finite-size cavities and with massive fields is (*cf.* Appendix):

$$I(\nu,T,V,m) = I_p(\nu,T)\left[1 - \frac{\Lambda}{8\pi V}\left(\frac{c}{\nu}\right)^2 - \frac{1}{2}\left(\frac{mc^2}{h\nu}\right)^2\right], \tag{1}$$

where terms of higher order in $(\Lambda/V)(c/\nu)^2$ and $(mc^2/h\nu)^2$ have been neglected. $I_p(\nu,T) = (2\pi/c^2)[h\nu^3/(e^{h\nu/kT} - 1)]$ is the Planck formula, $V$ the volume, $m$ the photon mass and $\Lambda$ a quantity with the dimension of a length that depends on the type of cavity: $\Lambda = 3L$ for a cube of side $L$, $\Lambda = L_1 + L_2 + L_3$ for a parallelepiped, $\Lambda \simeq 6R$ for a sphere of radius $R$, etc.

Hence to observe a photon mass effect one must use large cavities. It follows from equation (1) that a mass effect will dominate the geometrical term only if $L > h/mc$, *i.e.*, the typical linear dimension in the cavity must be larger than the Compton wavelength of the massive photon. This precludes the use of man-made cavities: in order to improve on the bounds listed above a cavity with sides about the earth-moon distance would be needed! One is then forced to consider cosmic background radiation (CBR) to avoid the geometrical term in equation (1).

Using recent measurements of the CBR spectral distribution, with experimental errors in the 10% range, a bound $m \leq 10^{-37}$ grams was obtained for the photon mass, in the sense that any effect due to a smaller mass than this would be concealed by the experimental errors. This bound is ten orders of magnitude worse than the non-thermal ones mentioned in the introduction, and there is little hope of greatly improving it, as the experimental error term is related to the square of the mass.

## 2. Angular distribution of the CBR and the photon mass

The CBR perceived on earth is not isotropic. It shows a large-scale angular dependence [9] due to the Doppler shift produced by the movement of the observer with respect to the CBR rest, or co-moving frame. And it shows small-scale anisotropy [10], attributed to a granular (inhomogeneous) distribution of the last scatterers of the radiation, just before its decoupling from matter.

The intensity measured by an observer moving with speed $v$ through the CBR rest frame is, for low values of $\beta = v/c$ (*cf.* Ref. 9, and De Bernardis *et al.*, Ref. 6), with $\theta$ the angle between $\mathbf{v}$ and the antenna's axis,

$$I_{\text{obs.}}(\nu_{\text{obs.}}) = I(\nu)[1 + \beta(3-\alpha)\cos\theta], \tag{2}$$

where $I(\nu)$ would be the intensity detected by an observer at rest with respect to the CBR; $\nu_{\text{obs.}}$ and $\nu$ are related by the Doppler formula $\nu_{\text{obs.}} = \gamma\nu(1 - \beta\cos\theta)$; $\gamma = (1 - \beta^2)^{-1/2}$ and $\alpha$ is the spectral index, defined by $I(\nu) = I_0\nu^\alpha$, or

$$\alpha = \frac{\nu}{I(\nu)} \frac{dI(\nu)}{d\nu}, \tag{3}$$

Although only valid for small $\beta$, equation (1) illustrates the general result that radiation with a cubic spectral distribution ($\alpha = 3$) would look the same to all observers, and so define the zero-point electromagnetic field for the classical vacuum [11].

Assuming a Planck expression for $I(\nu)$, Gorenstein and Smoot (Ref. 9) report a value $v = 360 \pm 50$ Km/sec for the absolute speed of the earth through the CBR. Similarly, Uson and Wilkinson (Ref. 10) report the bound $\Delta T/T < 4.5 \times 10^{-5}$ on anisotropies of size $\sim 4.5$ arc-minutes in the temperature of the CBR.

From equation (2) the amplitude of $[I_{\text{obs.}}(\nu_{\text{obs.}}) - I(\nu)]/I(\nu)$ is given by

$$\frac{\Delta I}{I} = \beta(3 - \alpha) \tag{4}$$

This quatity has been measured with errors of order $10^{-4}$ (*cf.* De Bernardis *et al.*, Ref. 6, and Fig. 1), much smaller than those with which the spectral distribution is known (of order $10^{-1}$). From equation (2) the spectral index $\alpha$ depends on the derivative of the spectral distribution. These two facts suggested that experimental results on $\Delta I/I$ could provide a better bound on the photon mass than the one reported in reference 7, obtained using the spectral distribution of the CBR.

From equation (1), for CBR

$$I(\nu) = I(\nu, T, m) = I_p(\nu, T)\left[1 - \frac{1}{2}\left(\frac{mc^2}{h\nu}\right)^2\right], \tag{5}$$

and one gets

$$\alpha = 3 - \frac{xe^x}{e^x - 1} + \left(\frac{mc^2}{h\nu}\right)^2, \tag{6}$$

where $x = h\nu/kT$.

Using equation (4) one then should proceed to fit the experimental results (Fig. 1) with three adjustable parameters, namely $\beta$, $T$ and $m$. But a quick estimate of a bound for $m$ will show this to be unnecessary. Equating the experimental error on the left-hand-side of equation (4) with the mass term on its right-hand-side,

$$\beta\left(\frac{mc^2}{h\nu}\right)^2 \simeq 10^{-4}, \tag{7}$$

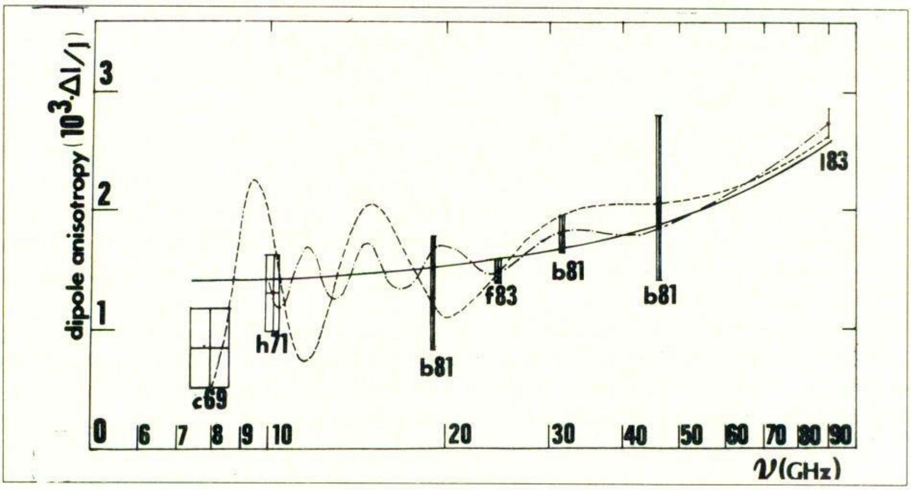


FIGURE 1. Experimental results for CBR anisotropy. Number under each error bar indicates year of experiment. The three curves shown correspond to fits to the data assuming diverse forms for $I(\nu)$ (*cf.* eq. (2)), the spectrum of the radiation as seen by an observer at rest with respect to the CBR, and they need not concern us here. (Taken from De Bernardis *et al.*, Ref. 6).

from which a bound $m \leq 10^{-38}$ gram emerges, because larger values of $m$ would be incompatible with the magnitude of the reported errors in $\Delta I/I$. This bound slightly improves on the one from the spectral distribution ($m \leq 10^{-37}$ gram, *cf.* Ref. 7), but it remains so far above non-thermal ones (*cf.* introduction), that a finer analysis of the experimental data in figure 1 is not warranted. Although errors in $\Delta I/I$ are much smaller than those in the intensity $I(\nu)$, it turns out that the $\beta$ factor in equation (7) essentially destroys this gain in precision.

## 3. Conclusion

Experimental results on the angular distribution of CBR yield bounds on the photon mass comparable with those from spectral profile measurements. The same negative outlook on substantial improvements applies in both cases.

## 4. Appendix

Here we outline the method how equation (1) was obtained. It combines deviations from the Planck spectrum due to a finite cavity size and a possible photon mass.

The first correction ($\Lambda c^2/8\pi V\nu^2$) was calculated [8] adding the first $10^6$ normal modes corresponding to cavities with several shapes of interest: cubes, cones, spheres, etc. An interpolating function among the discret summands was employed in each case to express the result as a simple combination ($\Lambda$) of the geometrical parameters.

The second correction $(1/2)(mc^2/h\nu)^2)$ was calculated in reference 7 assuming the photon has a tiny mass $m$, and using the Proca equations to describe the behavior of massive electromagnetic waves in a cavity. Its negative sign is related to the fact that part of the energy goes into the rest mass of light, so it is not available for radiation.

## Acknowledgement

Useful conversations with A. Nieto and R. Zuazua are gratefully acknowledged.

## References

1. For a review of work on the photon mass up to 1971 see A.S. Goldhaber, and M.M. Nieto, *Rev. Mod. Phys.* **43** (1971) 277. For more recent work on the subject see Refs. 6 and 7 below.
2. G. Feinberg, *Science* **166** (1969) 879.
3. E.R. Williams, J.E. Faller, and H. Hill, *Phys. Rev. Lett* **26** (1971) 72.
4. L. Davis, Jr., A.S. Goldhaber and M.M. Nieto, *Phys. Rev. Lett.* **35** (1975) 1402.
5. *Cf.* Goldhaber and Nieto, Ref. 1 above and, for related work, L.B. Okun and Ya.B. Zeldovich, *Phys. Lett.* **78B** (1978) 597.
6. H. Georgi, P. Ginsparg and S.L. Glashow, *Nature* **306** (1983) 765; P. De Bernardis, S. Masi, F. Melchiorri and A. Moleti, *Ap. J. (Letters)* **284** (1984) L21.
7. J. Torres-Hernández, *Phys. Rev.* **A32** (1985) 623.
8. H.P. Baltes and F.K. Kneubühl, *Helv. Phys. Acta* **45** (1972) 481.
9. M.V. Gorenstein, and G.F. Smoot, *Ap. J.* **244** (1981) 361.
10. J.M. Uson and D.T. Wilkinson, *Ap. J. (Letters)* **277** (1984) L1.
11. T.H. boyer, *Phys. Rev.* **182** (1969) 1374; P.W. Milonni, *Phys. Rep.* **25** (1976) 1.

**Resumen.** La radiación cósmica de 3°K proporciona información relevante sobre una posible masa del fotón. De su composición espectral se obtuvo anteriormente un límite inferior para esta cantidad ($\sim 10^{-37}$ gramos), mucho mayor que límites provenientes de análisis relacionados puramente con las ecuaciones de Maxwell ($\sim 10^{-47}$ gramos). Analizando ahora la anisotropía angular de esta radiación se obtiene un nuevo límite para la masa del fotón, que no mejora apreciablemente el inferido de su espectro.