A relativistic 3-dimensional extended object:
the terron

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Abstract. The theory of a free relativistic 3-dimensional extended object, which will be called a terron, is discussed. Using Dirac's method of constrained Hamiltonian systems a preliminary investigation of the quantization of the free terron is performed.

1. Introduction

It seems that the idea of considering a relativistic extended object as a fundamental physical system was proposed for the first time by Dirac [1]. Dirac realized that the muon has properties so similar to the electron that it may be considered to be merely an electron in excited state. This observation lead him to make the following interesting remark: "If one works with a point charge model of the electron, there is no place in the theory for the muon".

In the context of hadrons, ideas similar to Dirac's where proposed [2]. In particular, the idea introduced by Nambu [3] and others [4] of a 1-dimensional extended object, called a string, has been by far the most successful in describing "elementary" particles. It seems that at the present time superstrings [5] are the best candidates to unify all "elementary" particles.

Nevertheless, the alternative idea of considering a relativistic 3-dimensional extended object (instead of a string) as a fundamental physical system is an interesting possibility [6,7,8]. In fact, it has been pointed out [6,8] that a 3-dimensional extended object, which I will call a terron [6] (also known in the literature as a jelly [9]), has a number of interesting features. One of these features is the fact that a terron is the natural source for a four-index antisymmetric gauge field $(A_{\mu\nu\alpha\beta}; \mu, \nu, \alpha, \beta = 0, 1, \ldots, D - 1)$ [6,8]. One should recall that the gauge field $A_{\mu\nu\alpha\beta}$ appears explicitly and is an important object in a limiting case of string theory, chiral $N = 2$, 10-dimensional supergravity [10].

In addition to their direct physical interest terrons offer a useful system that gives a better understanding of some subtleties of string theory. In particular, a terron may be used to clarify the phenomena of the so called critical dimensions. As is known, one of the most intriguing features of string theory is the fact at the quantum level the Lorentz group algebra does not close unless the dimension D

of the spacetime is 26 (for the bosonic string) or $D = 10$ (for the spinning string or superstring). This property of the quantized string is certainly interesting, since it implies a connection between the dimension D of spacetime and the dimension $d = 1$ of a d-dimensional extended object when Lorentz covariance is required. From this result the question arises whether Lorentz covariance also determine a connection between D and d when d is grater than one. (This question is related to the problem suggested by Scherk [4]: find out which extended system can be quantized for $D = 4$). Of particular interest is the determination of the critical dimension D of spacetime when $d = 3$, corresponding to a terron. One may hope that result obtained for the terron will shed some light on the question of the critical dimension for the string.

The main purpose of this work is to establish the first steps in the quantum theory of the terron. The central idea is to use Dirac's method of constrained Hamiltonian systems to develope the quantum theory of the terron. Specifically, the primary constraint which follow from the definition of the momenta associated with the terron, are derived. In future research, the final goal will be to determine the critical dimension of spacetime for the terron.

Before discussing the theory of the terron I should mention that a preliminary investigation to determine the critical dimension for a 2-dimensional extended object, called a membrane, has been performed by Collins and Tucker [11]. In fact, using Dirac's procedure for constrained Hamiltonian systems, Collins and Tucker developed in some detail the quantum mechanics of membranes. However, due to the fact that quantum membrane theory is extremaly complicated, they do not solve the question of the critical dimension for the membrane. From their results for the quantum membrane, it is reasonable to expect that the quantum theory of a free terron will be also very complicated. Furthermore, since a terron is an extension of the membrane concept, one may think that the quantum theory of the terron will be more complicated than that of the membrane. There is, however, at least one indication, pointed out in reference [6], that suggested the possibility of finding some simplifications in the process of quantizing the terron. Such an indication will be also discussed in this work.

Finally, I should also mention that terron theory has been considered, from a classical point of view in a very general context by a number of authors [12,13,14]. In particular, Teitelboim [13], has explored gauge invariance for extended objects. It is interesting that one of the examples considered by Teitelboim as interesting is precisely the terron. In fact, Teitelboim shows from the relation:

$$D = 2(d + 2),$$

that the case $d = 3$ is interesting, since the spacetime must have dimension $D = 10$, which is the critical dimension in superstring theory. The relation $D = 2(d + 2)$ means that the coupling constant associated with the completely antisymmetric gauge field $A_{\mu\nu\alpha\beta}$ is a pure number (see reference [13], for more details).

2. The free relativistic terron

Terrons are 3-dimensional, relativistic, extended objects. They provide an interesting generalization of pointlike systems, strings, and membranes, corresponding to objects of dimensionality $d = 0, d = 1$, and $d = 2$ respectively. As a free terron moves through flat spacetime it sweeps out a 4-dimensional surface called the world-sheet for the terron, in analogy with the world-line for a pointlike particle.

Let us introduce the four arbitrary internal coordinates to parametrize the world-sheet of the terron:

$$\xi^a = (\tau, \sigma, \lambda, \rho), \qquad a = 0, 1, 2, 3. \tag{1}$$

Here, τ is a timelike evolution parameter while σ, λ, and ρ are spacelike parameters used to label points within the terron. Thus, the motion of a terron through spacetime can be described using the coordinates

$$\chi^\mu = \chi^\mu(\xi^a), \qquad \mu = 0, 1, \ldots, D - 1, \tag{2}$$

where D is the dimension of spacetime. Consider the induced metric on the world-sheet for the terron

$$h_{ab} = \frac{\partial \chi^\mu}{\partial \xi^a} \frac{\partial \chi^\nu}{\partial \xi^b} \eta_{\mu\nu}, \tag{3}$$

where $\eta_{\mu\nu} = diag(-1, 1, 1, \ldots, 1)$ represents the flat Minkowski spacetime metric. Let h represent the determinant of h_{ab}:

$$h = \det h_{ab}. \tag{4}$$

One postulates the following action for the terron

$$S = -p \int d^4\xi \sqrt{-h}, \tag{5}$$

where p is a constant of the motion measuring the inertia of the object analogous to the rest mass of a point particle or the tension of a string, and has dimensions of a $[length]^{-4} (\hbar = c = 1)$.

Consider now the equations of motion of the free terron. In order that the equations of motion be suitable for quantization they should be derivable, of course, from the action (5) using the principle of least action. Provided certain boundary conditions are satisfied, one finds the following equations of motion from the action (5):

$$\Box \chi^\mu = 0, \tag{6}$$

where the operator $\Box$ represents the D'Alembertian in a 4-dimensional curved space with metric (3):

$$\Box \equiv \frac{\partial}{\partial \xi^a}\left[\sqrt{-h}\,h^{ab}\frac{\partial}{\partial \xi^b}\right]. \tag{7}$$

An important feature of the equations of motion (6) is that they are non-linear differential equations for χ^μ. In fact, from the definition of h_{ab} in (3) one learns that the equations of motion are non-linear functionals of χ^μ. One may understand this observation better if one introduces the notation

$$\dot{\chi}^\mu \equiv \frac{\partial \chi^\mu}{\partial \tau}, \quad \chi'^\mu \equiv \frac{\partial \chi^\mu}{\partial \sigma}, \quad \bar{\chi}^\mu \equiv \frac{\partial \chi^\mu}{\partial \lambda}, \quad \tilde{\chi}^\mu \equiv \frac{\partial \chi^\mu}{\partial \rho}$$

and rewrites the equations of motion (6) in the following form:

$$\frac{\partial}{\partial \tau}\begin{vmatrix} \dot{\chi}^\mu & \chi'^\mu & \bar{\chi}^\mu & \tilde{\chi}^\mu \\ \dot{\chi}\cdot\chi' & (\chi')^2 & \chi'\cdot\bar{\chi} & \chi'\cdot\tilde{\chi} \\ \dot{\chi}\cdot\bar{\chi} & \chi'\cdot\bar{\chi} & (\bar{\chi})^2 & \bar{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\tilde{\chi} & \chi'\cdot\tilde{\chi} & \bar{\chi}\cdot\tilde{\chi} & (\tilde{\chi})^2 \end{vmatrix}\frac{1}{\sqrt{-h}} + \frac{\partial}{\partial \sigma}\begin{vmatrix} (\dot{\chi})^2 & \dot{\chi}\cdot\chi' & \dot{\chi}\cdot\bar{\chi} & \dot{\chi}\cdot\tilde{\chi} \\ \dot{\chi}^\mu & \chi'^\mu & \bar{\chi}^\mu & \tilde{\chi}^\mu \\ \dot{\chi}\cdot\bar{\chi} & \chi'\cdot\bar{\chi} & (\bar{\chi})^2 & \bar{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\tilde{\chi} & \chi'\cdot\tilde{\chi} & \bar{\chi}\cdot\tilde{\chi} & (\tilde{\chi})^2 \end{vmatrix}\frac{1}{\sqrt{-h}}+$$

$$\frac{\partial}{\partial \lambda}\begin{vmatrix} (\dot{\chi})^2 & \dot{\chi}\cdot\chi' & \dot{\chi}\cdot\bar{\chi} & \dot{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\chi' & (\chi')^2 & \chi'\cdot\bar{\chi} & \chi'\cdot\tilde{\chi} \\ \dot{\chi}^\mu & \chi'^\mu & \bar{\chi}^\mu & \tilde{\chi}^\mu \\ \dot{\chi}\cdot\tilde{\chi} & \chi'\cdot\tilde{\chi} & \bar{\chi}\cdot\tilde{\chi} & (\tilde{\chi})^2 \end{vmatrix}\frac{1}{\sqrt{-h}} + \frac{\partial}{\partial \rho}\begin{vmatrix} (\dot{\chi})^2 & \dot{\chi}\cdot\chi' & \dot{\chi}\cdot\bar{\chi} & \dot{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\chi' & (\chi')^2 & \chi'\cdot\bar{\chi} & \chi'\cdot\tilde{\chi} \\ \dot{\chi}\cdot\bar{\chi} & \chi'\cdot\bar{\chi} & (\bar{\chi})^2 & \bar{\chi}\cdot\tilde{\chi} \\ \dot{\chi}^\mu & \chi'^\mu & \bar{\chi}^\mu & \tilde{\chi}^\mu \end{vmatrix}\frac{1}{\sqrt{-h}}$$

$$= 0 \tag{8}$$

Here, $|A_{ab}|$ is the determinant of a matrix A_{ab}, and the Lorentz indices in the products $a \cdot b$ for any vectors a^μ and b^μ are suppressed. Note that the metric h_{ab} given by (3) can be written using the above notation as

$$h_{ab} = \begin{pmatrix} (\dot{\chi})^2 & \dot{\chi}\cdot\chi' & \dot{\chi}\cdot\bar{\chi} & \dot{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\chi' & (\chi')^2 & \chi'\cdot\bar{\chi} & \chi'\cdot\tilde{\chi} \\ \dot{\chi}\cdot\bar{\chi} & \chi'\cdot\bar{\chi} & (\bar{\chi})^2 & \bar{\chi}\cdot\tilde{\chi} \\ \dot{\chi}\cdot\tilde{\chi} & \chi'\cdot\tilde{\chi} & \bar{\chi}\cdot\tilde{\chi} & (\tilde{\chi})^2 \end{pmatrix} \tag{9}$$

Now, in order to use Dirac's method [15] to quantize a classical constrained system, one should consider the Hamiltonian formalism rather the Lagrangian formalism. The starting point in the Hamiltonian formalism is the definition of the canonical momenta in terms of a Lagrangian associated with the system. According to (5), the Lagrangian for the terron is

$$L = -p\sqrt{-h}. \tag{10}$$

Thus, the canonical momenta are

$$P_\mu^a = \frac{\partial L}{\partial \left(\frac{\partial \chi^\mu}{\partial \xi^a} \right)}, \tag{11}$$

which can also be written as

$$P_\tau^\mu \equiv P^{\mu 0} = \frac{\partial L}{\partial \dot{\chi}_\mu}, \qquad P_\sigma^\mu \equiv P^{\mu 1} = \frac{\partial L}{\partial \chi'_\mu},$$

$$P_\lambda^\mu \equiv P^{\mu 2} = \frac{\partial L}{\partial \bar{\chi}_\mu}, \qquad P_\rho^\mu \equiv P^{\mu 3} = \frac{\partial L}{\partial \tilde{\chi}_\mu}. \tag{12}$$

In terms of the canonical momenta given by (12), the equations of motion (6) or (8) become

$$\frac{\partial}{\partial \tau} P_\tau^\mu + \frac{\partial}{\partial \sigma} P_\sigma^\mu + \frac{\partial}{\partial \lambda} P_\lambda^\mu + \frac{\partial}{\partial \rho} P_\rho^\mu = 0. \tag{13}$$

This manner of writing the equations of motion is useful to show that the total linear momentum

$$\mathbf{P}^\mu = \int d^3\xi \, P_\tau^\mu(\xi^a), \tag{14}$$

and the total angular momentum

$$M^{\mu\nu} = \int d^3\xi \, M_\tau^{\mu\nu} = \int d^3\xi \left(\chi^\mu P_\tau^\nu - \chi^\nu P_\tau^\mu \right) \tag{15}$$

are conserved quantities:

$$\frac{\partial}{\partial\tau}\mathbf{P}^\mu = 0, \qquad \frac{\partial}{\partial\tau}M^{\mu\nu} = 0. \tag{16}$$

The terron is a constrained system since the action (5) is invariant under reparametrization

$$\xi^a \to \xi'^a(\xi^b). \tag{17}$$

So, in Dirac's method for a constrained system, constraints arising from the definition of the momenta (11) are called primary constraints. Its turns out that the computation of these constraints is very lengthly, so we will not describe it. We will simply write the results. From the definition of the momenta P_μ^a given by (11) one can show that the following indentities exist locally:

$$P_\tau^\mu P_{\tau\mu} + p^2 \begin{vmatrix} (\chi')^2 & \chi'\cdot\bar\chi & \chi'\cdot\tilde\chi \\ \chi'\cdot\bar\chi & (\bar\chi)^2 & \bar\chi\cdot\tilde\chi \\ \chi'\cdot\tilde\chi & \tilde\chi\cdot\bar\chi & (\tilde\chi)^2 \end{vmatrix} \equiv 0, \tag{18}$$

$$P_\tau^\mu \chi'_\mu \equiv 0, \qquad P_\tau^\mu \bar\chi_\mu \equiv 0, \qquad P_\tau^\mu \tilde\chi_\mu \equiv 0,$$

$$P_\sigma^\mu P_{\sigma\mu} + p^2 \begin{vmatrix} (\dot\chi)^2 & \dot\chi\cdot\bar\chi & \dot\chi\cdot\tilde\chi \\ \dot\chi\cdot\bar\chi & (\bar\chi)^2 & \bar\chi\cdot\tilde\chi \\ \dot\chi\cdot\tilde\chi & \bar\chi\cdot\tilde\chi & (\tilde\chi)^2 \end{vmatrix} \equiv 0, \tag{19}$$

$$P_\sigma^\mu \dot\chi_\mu \equiv 0, \qquad P_\sigma^\mu \bar\chi_\mu \equiv 0, \qquad P_\sigma^\mu \tilde\chi_\mu \equiv 0,$$

$$P_\lambda^\mu P_{\lambda\mu} + p^2 \begin{vmatrix} (\dot\chi)^2 & \dot\chi\cdot\chi' & \dot\chi\cdot\tilde\chi \\ \dot\chi\cdot\chi' & (\chi')^2 & \chi'\cdot\tilde\chi \\ \dot\chi\cdot\tilde\chi & \tilde\chi\cdot\chi' & (\tilde\chi)^2 \end{vmatrix} \equiv 0, \tag{20}$$

$$P_\lambda^\mu \dot\chi_\mu \equiv 0, \qquad P_\lambda^\mu \chi'_\mu \equiv 0, \qquad P_\lambda^\mu \tilde\chi_\mu \equiv 0,$$

$$P^\mu_\rho P_{\rho\mu} + p^2 \begin{vmatrix} (\dot\chi)^2 & \dot\chi \cdot \chi' & \dot\chi \cdot \bar\chi \\ \dot\chi \cdot \chi' & (\chi')^2 & \chi' \cdot \bar\chi \\ \dot\chi \cdot \bar\chi & \bar\chi \cdot \chi' & (\bar\chi)^2 \end{vmatrix} \equiv 0, \tag{21}$$

$$P^\mu_\rho \dot\chi_\mu \equiv 0, \qquad P^\mu_\rho \chi'_\mu \equiv 0, \qquad P^\mu_\rho \bar\chi_\mu \equiv 0.$$

Note that all of these identities involve the canonical momenta $P^\mu_\tau, P^\mu_\sigma, P^\mu_\lambda$, and P^μ_ρ, as well as the velocities $\dot\chi^\mu, \chi'^\mu, \bar\chi^\mu$, and $\tilde\chi^\mu$. Strictly speaking, $\chi'^\mu, \bar\chi^\mu, \tilde\chi^\mu$ are not velocities and $P^\mu_\sigma, P^\mu_\lambda$, and P^μ_ρ are not momenta, since τ is the evolution parameter and σ, λ, ρ are spacelike parameters. So, one should expect that among the identities (18)–(21), the first four given in (18) will play an important role in both classical level and quantum level. Let us label the identities (18) as follows:

$$\begin{aligned} \phi_1 &\equiv P^\mu P_\mu + p^2 det(h_{ij}) \equiv 0, \qquad i,j = 1,2,3 \\ \phi_2 &\equiv P^\mu \chi'_\mu \equiv 0, \\ \phi_3 &\equiv P^\mu \bar\chi_\mu \equiv 0, \\ \phi_4 &\equiv P^\mu \tilde\chi_\mu \equiv 0, \end{aligned} \tag{22}$$

where, in order to simplify notation, $P^\mu \equiv P^\mu_\tau$. According to the Dirac formalism, the identities (22) are primary constraints, since they follow from the definition (11) of the momenta.

It is not difficult to see that the canonical Hamiltonian

$$H_0 \equiv \int d^3\xi \{ \dot\chi^\mu P_\mu - L \} \tag{23}$$

vanishes identically. Thus, one may write the Hamiltonian for the relativistic terron as

$$H = \int d^3\xi \{ \lambda_1 \phi_1 + \lambda_2 \phi_2 + \lambda_3 \phi_3 + \lambda_4 \phi_4 \}, \tag{24}$$

where $\lambda_1, \lambda_2, \lambda_3$, and λ_4 are Lagrange multipliers.

One may attempt a covariant quantization by replacing the canonical variables χ^μ and P^ν by operators satisfying the canonical commutations relations:

$$[\widehat{\chi}^{\mu}(\tau,\sigma,\lambda,\rho), \widehat{P}^{\nu}(\tau,\sigma',\lambda',\rho')] = i\eta^{\mu\nu}\delta(\sigma-\sigma')\delta(\lambda-\lambda')\delta(\rho-\rho') \tag{25}$$

$$[\widehat{\chi}(\tau,\sigma,\lambda,\rho), \widehat{\chi}^{\nu}(\tau,\sigma',\lambda',\rho')] = 0, \tag{26}$$

$$[\widehat{P}^{\mu}(\tau,\sigma,\lambda,\rho), \widehat{P}^{\nu}(\tau,\sigma',\lambda',\rho')] = 0 \tag{27}$$

In this context, the constraints (22) are implemented by requiring that the physical state $|\psi\rangle$ satisfy

$$
\begin{aligned}
\widehat{\phi}_1|\psi\rangle &= 0, \\
\widehat{\phi}_2|\psi\rangle &= 0, \\
\widehat{\phi}_3|\psi\rangle &= 0, \\
\widehat{\phi}_4|\psi\rangle &= 0,
\end{aligned}
\tag{28}
$$

where the hat "$\widehat{}$" in the constraints ϕ_1, ϕ_2, ϕ_3, and ϕ_4 denotes operator. Of course, at this stage one may ask whether there are ghost states and whether there is a critical dimension. As in the membrane case, one should expect to encounter many difficulties in answering these questions [11]. Therefore, one may consider an alternative method of quantization.

One possibility is to reduce the independent degrees of freedom by prescribing the gauge, that is, by choosing $\lambda_1, \lambda_2, \lambda_3$, and λ_4. In this case, one may consider the terron to be a closed toroidal surface for which σ, λ, and ρ lie in the range $[0, 2\pi]$. Thus, one may expand quantities in a Fourier series

$$\chi^{\mu} = \sum_{\substack{n,m,l \\ -\infty}}^{\infty} \chi^{\mu}_{nml}(\tau)e^{in\sigma+im\lambda+il\rho}, \tag{29}$$

$$P_{\mu} = \sum_{\substack{n,m,l \\ -\infty}}^{\infty} P^{\mu}_{nml}(\tau)e^{in\sigma+im\lambda+il\lambda}, \tag{30}$$

where χ^{μ}_{nml} and P^{μ}_{nml} are considered complex and

$$
\begin{aligned}
\chi^{\mu}_{nml} &= \chi^{*\mu}_{-n-m-l} \\
P^{\mu}_{nml} &= P^{*\mu}_{-n-m-l}.
\end{aligned}
\tag{31}
$$

However, one may expect to find difficulties of normal ordering as in the membrane [11].

Another interesting possibility that may be used to quantize the terron is to use the path integral formalism, as Polyakov [5] did to quantize the string. The action used by Polyakov to quantize the string is

$$S_s = \int d^2\xi L,$$

with

$$L = -\frac{T}{2}\sqrt{-g}g^{ab}\partial_a\chi^\mu\partial_b\chi_\mu, \tag{32}$$

where T is the tension of the string.

At the classical level, the action is equivalent to the Nambu action for the string

$$S = -T\int d^2\xi\sqrt{-h}, \tag{33}$$

where h_{ab} is defined as in (3) and (4), but the indices a,b run from 0 to 1. An important invariance in the action is the Weyl invariance

$$g^{ab} \rightarrow \lambda g^{ab}. \tag{34}$$

The analog of the action for the membrane and the terron are

$$S_m \equiv -\Omega\int d^3\xi\sqrt{-g}(g^{ab}\partial_a\chi^\mu\partial_b\chi_\mu)^{3/2} \tag{35}$$

$$S_t = -\frac{p}{16}\int d^4\xi\sqrt{-g}(g^{ab}\partial_a\chi^\mu\partial_b\chi_\mu)^2. \tag{36}$$

respectively. These actions are also invariant under the transformation (34), that is to say, they are Weyl invariant. It is known that square root Lagragians such as (35) are troublesome from the point of view of the path integral formalism. This observation suggests to consider the terron theory as a better alternative than the membrane to generalize the string theory.

The generalization of the action (35) and (36) for a d-dimensional extended object is

$$S'_d = \text{const.} \int d^{d+1}\xi \sqrt{-g}(g^{ab}\partial_a\chi^\mu\partial_b\chi_\mu)^{\frac{d+1}{2}}. \tag{37}$$

It is not difficult to show that the action S'_d is Weyl invariant. Variation of S'_d with respect to g^{ab} provides the equation

$$\partial_a\chi^\mu\partial_b\chi_\mu = \frac{g_{ab}}{(d+1)}(g^{cd}\partial_c\chi^\mu\partial_d\chi_\mu). \tag{38}$$

From (38), gives it follows that $g_{ab} = \partial_a\chi^\mu\partial_b\chi_\mu$. This relation immediately gives the equivalence between the action (37) and the Nambu action

$$S_d = \text{const.} \int d^{d+1}\xi \sqrt{-h}. \tag{39}$$

It is important to mention that a number of authors [20,21,22] consider the action

$$S''_d = \text{const.} \int d^{d+1}\xi \sqrt{-g}\left(g^{ab}\partial_a\chi^\mu\partial_b\chi_\mu + (1-d)\right) \tag{40}$$

as a generalization of the action (32) for the string. Observe that the action S''_d, as opposed to the action S'_d, is not Weyl invariant. Variation of the action S''_d with respect to g^{ab} provides the equation

$$\partial_a\chi^\mu\partial_b\chi_\mu = \frac{1}{2}g_{ab}\left(g^{cd}\partial_c\chi^\mu\partial_d\chi_\mu + (1-d)\right). \tag{41}$$

From (41), it follows that $g_{ab} = \partial_a\chi^\mu\partial_b\chi_\mu$. This relation gives the equivalence between (39) and (40).

Finally, I should mention that more progress towards the construction of a membrane theory and a terron theory were made recently by a number of authors [16,17,18,19]. Rayski [16] considered membranes as alternatives of strings and superstrings. His basic idea was to consider a closed membrane to form a compact object: a bubble. He showed that bubbles may perform not only vibrations but also rotations and introduce also infinite towers of particles with higher spins and masses. Kikkawa and Yamasaki [17] demonstrated that for a simple membrane model the masssless particle is unable to be generated in integer critical dimensions. From the point of view of a unification model this results is, of course, a bad new for the membrane theory. Hughes et. al.. [18] considered superterrons (which they called supermembranes) as an example of the spontaneous breaking of $D = 4, N = 2$ global

supersymmetry down to $N = 1$. More recently Bergshoeff *et al.* [19] constructed an action for a superterron (which was also called supermembrane) propagating in $D = 11$ supergravity background.

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Resumen. Se discute la teoría de un objeto extendido relativista libre de tres dimensiones, al cual llamaré un terrón. Se desarrolla una investigación preliminar de la cuantización de un terrón libre usando el método de Dirac de sistemas hamiltonianos con constricciones.