

# Non linear partial differential equations solvable by LAKNS method

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**Abstract.** The LAKNS equation is solved exactly by using a matrix operator method which is different from the method developed by Kingston and Rogers. A series of fundamental non linear equations and their operator representations are obtained, and from their linear combinations other non linear equations can be constructed. It is shown that the non linear equations in the LAKNS equation contain only the first-order time derivative and space differential and integral operators acting on the  $q$  and  $r$  functions.

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## 1. Introduction

Lax [1] and Ablowitz, Kaup, Newell and Segur [2-4] developed the application of the inverse scattering method to solve the initial-value problem for nonlinear evolution equations. The scattering problem is defined by the linear equation

$$\frac{\partial}{\partial x} \psi = \begin{pmatrix} -i\zeta & q(x,t) \\ r(x,t) & i\zeta \end{pmatrix} \psi = R\psi, \quad (1)$$

where  $\zeta$  is the eigenvalue parameter and  $q, r$  are potential functions. The time evolution of the wave function,

$$\frac{\partial}{\partial t} \psi = \begin{pmatrix} A(x,t,\zeta) & B(x,t,\zeta) \\ C(x,t,\zeta) & -A(x,t,\zeta) \end{pmatrix} \psi = S\psi, \quad (2)$$

is chosen in such a way that the eigenvalue parameter remains constant. This condition leads to the LAKNS equation [5],

$$\frac{\partial}{\partial t} R + RS = \frac{\partial}{\partial x} S + SR,$$

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which can be expressed in terms of the elements of  $R$  and  $S$  in the form

$$\begin{aligned}\frac{\partial}{\partial x}A &= qC - rB, \\ \frac{\partial}{\partial x}B &= -2i\zeta B - 2qA + \frac{\partial}{\partial t}q, \\ \frac{\partial}{\partial x}C &= 2i\zeta C + 2rA + \frac{\partial}{\partial t}r.\end{aligned}\quad (3)$$

The authors of Refs. [2-4] used finite expansions of  $A$ ,  $B$  and  $C$  in terms of the eigenvalue parameter  $\zeta$  to determine a broad class of non linear evolution equations for  $q$  and  $r$ , which can be solved by the inverse scattering method. Kingston and Rogers [6] introduced the matrix operator method to delimit an extensive class of such equations.

In this paper, the LAKNS equation is solved exactly by using a matrix operator method, analogous in principle to that of Ref. [6], but different in its implementation and results. A series of fundamental non linear equations and their operator representations are obtained (*Theorem 1*), and from their linear combinations other non linear equations can be constructed (*Theorem 2*). It is shown that the non linear equations in the LAKNS equation can be expressed only in terms of the first-order time derivative and space differential and integral operators acting on the  $q$  and  $r$  functions (*Theorem 3*).

## 2. The LAKNS equation

The main results of this work are contained in *Theorems 1, 2, and 3*. In order to formulate and prove these theorems, *Definitions 1, 2-4, and 5-6* are introduced, and *Lemmas 1-2, 3, and 4-5* are stated and proved, respectively.

*Definition 1:*

$$\begin{aligned}\mathbf{T} &= \begin{pmatrix} A \\ B \\ C \end{pmatrix}, \quad \mathbf{F} = \frac{\partial}{\partial t} \begin{pmatrix} 0 \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ q_t \\ r_t \end{pmatrix}, \quad P = \begin{pmatrix} 0 & -r & q \\ -2q & 0 & 0 \\ 2r & 0 & 0 \end{pmatrix}, \\ \theta &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.\end{aligned}\quad (4)$$

Then the LAKNS equation can be written as

$$\frac{\partial}{\partial x}\mathbf{T} = P\mathbf{T} - 2i\zeta\theta\mathbf{T} + \mathbf{F}, \quad (5)$$

where the determinants of  $P$ ,  $\theta$ , and  $U$  are zeros, and  $U$  and  $\theta^2$  are projection operators

Lemma 1:

$$\begin{aligned} U + \theta^2 &= I, & UPU &= 0, \\ P\theta P &= 0, & \theta P\theta &= 0, \\ UP\theta &= P\theta = \begin{pmatrix} 0 & -r & -q \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{6}$$

where  $I$  is the  $3 \times 3$  unit matrix. The proof is direct using Eqs. (4).

Suppose that in (1)  $\mathbf{T}$  can be expanded as a power series of  $\zeta$

$$\mathbf{T} = \sum_{n=-\infty}^{\infty} \mathbf{T}_n \zeta^n, \tag{7}$$

where

$$\mathbf{T}_n = \begin{pmatrix} A_n \\ B_n \\ C_n \end{pmatrix}$$

and in (2) the power series is finite. Therefore there exist two integers  $M$  and  $N$  ( $M \leq N$ ) such that

$$\begin{aligned} \mathbf{T}_n &\neq 0 & M \leq n \leq N, \\ \mathbf{T}_n &= 0 & n < M \text{ or } n > N. \end{aligned} \tag{8}$$

Substitution of the power series of Eq. (7) in the LAKNS Eq. (5) and comparison of the coefficients of  $\zeta^k$  leads to the equations

$$\frac{\partial}{\partial x} \mathbf{T}_k = P\mathbf{T}_k - 2i\theta\mathbf{T}_{k-1} + \delta_{k0}\mathbf{F}, \tag{9}$$

where  $\delta_{k0}$  is the Kronecker delta.

It is obvious that  $M > 0$  or  $N < -1$  will result in  $\mathbf{F} = 0$ , and therefore  $q$  and  $r$  are both time independent. In such cases the non linear equations will only have the space dependence, becoming ordinary differential equations, which is not the topic that LAKNS studied. The conclusion is that the interval  $[M, N]$  must include at least 0 or  $-1$ . If  $M = N = -1$ , the LAKNS equation is reduced to the sine-Gordon equation; this situation is excluded in the following.

Lemma 2:

$$U \frac{\partial}{\partial x} \mathbf{T}_k = U P \mathbf{T}_k. \quad (10)$$

The proof consists in multiplying Eq. (9) by  $U$  from the left, and recognizing from Eqs. (4) that

$$U\theta = 0, \quad U\mathbf{F} = 0.$$

Theorem 1.

$$\mathbf{T}_{k-1} = \widehat{G} \mathbf{T}_k + \alpha_{k-1} \mathbf{e}_1 - \delta_{k0} \widehat{G}_0 \mathbf{F},$$

where

$$\begin{aligned} \widehat{G}_0 &= \frac{i}{2} \left( \theta + \int^x dx P \theta \right) \\ \widehat{G} &= \widehat{G}_0 \left( \frac{\partial}{\partial x} - P \right) = \frac{i}{2} \left( \theta \frac{\partial}{\partial x} - \theta P + \int^x dx P \theta \right), \\ \mathbf{e}_1 &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \quad (11)$$

and  $\alpha_{k-1}$  is an arbitrary constant, with  $\alpha_k = 0$  for  $k < M$  because  $T_k = 0$  when  $k < M$ .

*Proof:* Multiply Eq. (9) by  $\theta$  from the left to write

$$\theta^2 \mathbf{T}_{k-1} = \frac{i}{2} \left\{ \theta \left( \frac{\partial}{\partial x} - P \right) \mathbf{T}_k - \delta_{k0} \theta \mathbf{F} \right\},$$

and use the first of Eqs. (6) to obtain

$$\mathbf{T}_{k-1} = \frac{i}{2} \left\{ \theta \left( \frac{\partial}{\partial x} - P \right) \mathbf{T}_k - \delta_{k0} \theta \mathbf{F} \right\} + U \mathbf{T}_{k-1}. \quad (12)$$

Then, use lemma 2 to write

$$\frac{\partial}{\partial x} U \mathbf{T}_{k-1} = U P \mathbf{T}_{k-1}. \quad (13)$$

Substitute Eq. (12) in Eq. (13) and use the remaining Eqs. (6) of lemma 1 to obtain

$$\frac{\partial}{\partial x} U \mathbf{T}_{k-1} = \frac{i}{2} \left( P \theta \frac{\partial}{\partial x} \mathbf{T}_k - \delta_{k0} P \theta \mathbf{F} \right),$$

and after integrating

$$U \mathbf{T}_{k-1} = \frac{i}{2} \left( \int^x dx P \theta \frac{\partial}{\partial x} \mathbf{T}_k - \delta_{k0} \int^x dx P \theta \mathbf{F} \right) + \alpha_{k-1} \mathbf{e}_1. \quad (14)$$

Substitution of Eq. (14) in Eq. (12) completes the proof of Eq. (11).

Some consequences of *Theorem 1* are the following

i) For  $k = N + 1$  ( $N \neq -1$ ):

$$\mathbf{T}_n = \alpha_N \mathbf{e}_1 \quad (15)$$

ii) For  $k = M$ : Since  $\mathbf{T}_{M-1} = 0$  and  $\alpha_{M-1} = 0$  then Eq. (11) becomes

$$\widehat{G} \mathbf{T}_M - \delta_{M0} \widehat{G}_0 \mathbf{F} = 0,$$

or

$$\widehat{G}_0 \left\{ \left( \frac{\partial}{\partial x} - P \right) \mathbf{T}_M - \delta_{M0} \mathbf{F} \right\} = 0.$$

The result is the non linear equation

$$\left( \frac{\partial}{\partial x} - P \right) \mathbf{T}_M = \delta_{M0} \mathbf{F}. \quad (16)$$

This can also be obtained directly from Eq. (9).

*Definition 2:*

$$\mathbf{f}_k = \left( \frac{\partial}{\partial x} - P \right) \widehat{G}^k \mathbf{e}_1. \quad (17)$$

*Lemma 3:*  $\mathbf{f}_k$  has only  $y, z$  components and can be expressed by the recurrence relation of the two components

$$\mathbf{f}_{k+1} = \frac{i}{2} \sigma_3 \left\{ \frac{\partial}{\partial x} - 2 \left( \frac{q}{r} \right) \int^x dx (r, q) \right\} \mathbf{f}_k, \quad (18)$$

where  $\sigma_3$  is the Pauli  $2 \times 2$  matrix.

*Proof:* From Eqs. (11) and (17)

$$\begin{aligned}\widehat{G} &= \widehat{G}_0 \left( \frac{\partial}{\partial x} - P \right), \\ \mathbf{f}_{k+1} &= \left( \frac{\partial}{\partial x} - P \right) \widehat{G}^{k+1} \mathbf{e}_1 = \left( \frac{\partial}{\partial x} - P \right) \widehat{G}_0 \mathbf{f}_k, \\ \left( \frac{\partial}{\partial x} - P \right) \widehat{G}_0 &= \frac{i}{2} \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \frac{\partial}{\partial x} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2q \int^x dy r & 2q \int^x dy q \\ 0 & -2r \int^x dy r & -2r \int^x dy q \end{pmatrix} \right\}.\end{aligned}$$

It is then obvious that

$$U \mathbf{f}_{k+1} = 0.$$

Thus,  $\mathbf{f}_k$  can have only  $y, z$  components ( $k = 1, 2, \dots$ ). For  $k = 0$ , Eq. (17) becomes

$$\mathbf{f}_0 = \left( \frac{\partial}{\partial x} - P \right) \mathbf{e}_1,$$

and then again

$$U \mathbf{f}_0 = 0.$$

Therefore,  $\mathbf{f}_k$  ( $k = 0, 1, 2, \dots$ ) can be written in the form of a two component vector, Eq. (18).

*Definition 3:*

$$\hat{\sigma} = \frac{i}{2} \sigma_3 \left\{ \frac{\partial}{\partial x} - 2 \begin{pmatrix} q \\ 0 \end{pmatrix} \int^x dy (r \ q) \right\}. \quad (19)$$

Then it follows that

$$\mathbf{f}_k = \hat{\sigma}^k \mathbf{f}_0, \quad \mathbf{f}_0 = 2\sigma_3 \begin{pmatrix} q \\ r \end{pmatrix}. \quad (20)$$

*Definition 4:*

$$\mathbf{F}_M = \hat{\sigma}^M \mathbf{F}. \quad (21)$$

*Theorem 2:* Suppose  $(M = 0, 1, 2, \dots, N = 0, 1, 2, \dots)$

$$\begin{aligned} \mathbf{T}_n &\neq 0 & -M \leq n, \\ \mathbf{T} &= 0 & n < -M \text{ or } n > N, \end{aligned}$$

then the non linear equations determined by the LAKNS equation can be expressed as the linear combinations

$$\mathbf{F}_M = \sum_{k=-M}^N \alpha_k \mathbf{f}_{k+M}, \tag{22}$$

where  $\alpha_{-M}, \dots, \alpha_N$  are arbitrary constants.

*Proof: Theorem 1,*

$$\mathbf{T}_{k-1} = \widehat{G}\mathbf{T}_k + \alpha_{k-1}\mathbf{e}_1 - \delta_{k0}\widehat{G}_0\mathbf{F},$$

written explicitly for  $k = N + 1, N, \dots, 1, 0, \dots, -M + 1$  gives

$$\begin{aligned} \mathbf{T}_N &= \alpha_N \mathbf{e}_1 \\ \mathbf{T}_{N-1} &= (\alpha_N \widehat{G} + \alpha_{N-1}) \mathbf{e}_1 \\ &\dots \\ \mathbf{T}_0 &= (\alpha_N \widehat{G}^N + \alpha_{N-1} \widehat{G}^{N-1} + \dots + \alpha_1 \widehat{G} + \alpha_0) \mathbf{e}_1 \\ \mathbf{T}_{-1} &= (\alpha_N \mathbf{G}_N^{N+1} + \alpha_{N-1} \widehat{G}^N + \dots + \alpha_0 \widehat{G} + \alpha_{-1}) \mathbf{e}_1 - \widehat{G}_0 \mathbf{F} \\ &\dots \\ \mathbf{T}_{-M} &= (\alpha_N \widehat{G}^{N+M} + \alpha_{N-1} \widehat{G}^{N+M-1} + \dots + \alpha_{-M+1} \widehat{G} + \alpha_{-M}) \mathbf{e}_1 \\ &\quad - \widehat{G}^{M-1} \widehat{G}_0 \mathbf{F}. \end{aligned} \tag{23}$$

Thus the non linear equations are

$$\widehat{G}\mathbf{T}_{-M} = 0. \tag{24}$$

From the relations

$$\widehat{G} = \widehat{G}_0 \left( \frac{\partial}{\partial x} - P \right)$$

and

$$\hat{\sigma} = \left( \frac{\partial}{\partial x} - P \right) G_0,$$

it is immediate to establish that

$$\left( \frac{\partial}{\partial x} - P \right) \hat{G}^k = \hat{\sigma}^k \left( \frac{\partial}{\partial x} - P \right). \quad (25)$$

When Eq. (23) is substituted in Eq. (24) and Eq. (25) is used together with Eqs. (20) and (21), the non linear equations take the form

$$\hat{G}_0 \left\{ \sum_{k=-M}^N \alpha_k \mathbf{f}_{k+M} - \mathbf{F}_M \right\} = 0,$$

leading to Eq. (22). This can also be obtained from the recurrence relation of Eq. (9) by taking  $k = -M$  and using the expression for  $\mathbf{T}_{-M}$ .

*Remark:* The preceding relations also hold when  $N = -1$  and  $M = -2, -3, \dots$

From Eqs. (7), (8) and (23), it is possible to write

$$\begin{aligned} \mathbf{T} &= \sum_{k=-M}^N \left\{ \sum_{s=0}^{N-K} \alpha_{k+s} \hat{G}^s \mathbf{e}_1 \right\} \zeta^k - \sum_{k=i}^M \hat{G}^{k-i} \hat{G}_0 \mathbf{F} \zeta^{-k} \\ &= \sum_{k=-M}^N \alpha_k \zeta^k \mathbf{e}_1 + \hat{G}_0 \left\{ \sum_{k=-M}^N \sum_{s=1}^{N-k} \alpha_{k+s} \mathbf{f}_{s-1} \zeta^k - \sum_{k=i}^M \mathbf{r}_{k-i} \zeta^{-k} \right\}. \end{aligned} \quad (26)$$

When  $|x| \rightarrow \infty$  the functions  $q$ ,  $r$  and their derivatives tend to zero. Thus

$$\mathbf{T}(|x| \rightarrow \infty) \rightarrow \sum_{k=-M}^N \alpha_k \zeta^k \mathbf{e}_1. \quad (27)$$

Then

$$A_{\pm} = \sum_{k=-M}^N \alpha_k \zeta^k, \quad B_{\pm} = 0, C_{\pm} = 0. \quad (28)$$

*Definition 5:* Let  $\tau$  be a function set, such that any function  $w \in \tau$  does not depend explicitly on  $x$ , but is only a function of  $q$ ,  $r$  and their derivatives. If  $w$  is a matrix, then everyone of its elements satisfies the same condition.



Definition 6: If  $F, w_1, w_2 \in \tau$  and

$$w_1 = w_2 + \frac{\partial}{\partial x} F,$$

we say that  $w_1 \simeq w_2$ .

Let  $f_k$  be a  $2 \times 1$  matrix and  $\tilde{f}_k$  the transposed matrix.

Lemma 4: If  $m \neq n$  and  $f_k (k = 0, 1, \dots, \max(n, m)) \in \tau$ , then  $\tilde{f}_n \sigma_2 f_m \in \tau$  and

$$\begin{aligned} \tilde{f}_n \sigma_2 f_m &\simeq \tilde{f}_{n-i} \sigma_2 f_{n+1} && (\text{when } n > m), \\ \tilde{f}_n \sigma_2 f_m &\simeq \tilde{f}_{n+1} \sigma_2 f_{m-i} && (\text{when } n < m). \end{aligned} \quad (30)$$

Proof: If  $n > m$ ,

$$f_{m+1} = \frac{i}{2} \sigma_3 \left\{ \frac{\partial}{\partial x} - 2 \binom{q}{r} \int^x dy(r \ q) \right\} f_m \in \tau,$$

$$f_n = \frac{i}{2} \sigma_3 \left\{ \frac{\partial}{\partial x} - 2 \binom{q}{r} \int^x dy(r \ q) \right\} f_{n-i} \in \tau,$$

Then

$$\int^x dy(r \ q) f_m \quad \text{and} \quad \int^x dy(r \ q) f_{n-1} \in \tau.$$

Then we obtain

$$\begin{aligned} \tilde{f}_n \sigma_2 f_m &= \frac{i}{2} \left\{ \frac{\partial}{\partial x} \tilde{f}_{n-1} - 2 \left[ \int^x dy \tilde{f}_{n-1} \binom{r}{q} \right] (q \ r) \right\} \sigma_3 \sigma_2 f_m \\ &\simeq \frac{i}{2} \tilde{f}_{n-1} \sigma_2 \sigma_3 \frac{\partial}{\partial x} f_m - \left[ \int^x dy \tilde{f}_{n-1} \binom{r}{q} \right] \frac{\partial}{\partial x} \int^x dz(r \ q) f_m \\ &\simeq \frac{i}{2} \tilde{f}_{n-1} \sigma_2 \sigma_3 \frac{\partial}{\partial x} f_m + \tilde{f}_{n-1} \binom{r}{q} \int^x dz(r \ q) f_m \\ &= \tilde{f}_{n-1} \sigma_2 \left\{ \frac{i}{2} \sigma_3 \left[ \frac{\partial}{\partial x} - 2 \binom{q}{r} \int^x dz(r \ q) \right] f_m \right\} \\ &= \tilde{f}_{n-1} \sigma_2 f_{m+1}. \end{aligned}$$

The same holds for  $n < m$ .

Lemma 5: If  $f_k (k = 1, 2, \dots, \max(n, m)) \in \tau$ , then

$$\tilde{f}_n \sigma_2 f_m \simeq 0. \quad (31)$$

*Proof:* By repeated application of lemma 4 it can be established that

$$\tilde{f}_n \sigma_2 f_m \simeq \begin{cases} \tilde{f}_s \sigma_2 f_s & \text{when } m+n=2s \\ \tilde{f}_{s+1} \sigma_2 f_s & \text{when } m+n=2s+1 \end{cases}$$

It is obvious that  $\tilde{f}_s \sigma_2 f_2 = 0$ . Also

$$\tilde{f}_{s+1} \sigma_2 f_s = \widetilde{\tilde{f}_{s+1} \sigma_2 f_s} = -\tilde{f}_s \sigma_2 f_{s+1} \simeq -\tilde{f}_{s+1} \sigma_2 f_s$$

$$\therefore \tilde{f}_{s+1} \sigma_2 f_s \simeq 0.$$

*Theorem 3:*  $f_m (m=0, 1, 2, \dots) \in \tau$ .

*Proof:* Use mathematical induction. Direct calculation indicates that  $f_0, f_1, f_2, f_3 \in \tau$  (see Appendix). Suppose that  $f_0, f_1, f_2, \dots, f_k \in \tau$ , then it has to be proved that  $f_{k+1} \in \tau$ . From Eq. (18)

$$f_{k+1} = \frac{i}{2} \sigma_3 \left\{ \frac{\partial}{\partial x} f_k - 2 \left( \frac{q}{r} \right) \int^x dy (r - q) f_k \right\}.$$

Examine

$$(r - q) f_k = (q - r) \sigma_1 f_k = i(q - r) \sigma_3 \sigma_2 f_k = \frac{i}{2} \tilde{f}_0 \sigma_2 f_k \simeq 0.$$

Therefore,  $f_{k+1} \in \tau$ .

### 3. Conclusion

When  $M = 0$  the LAKNS equation corresponds to non linear differential equations and when  $M > 0$  it corresponds to nonlinear integro-differential equations.

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Appendix

$$f_0 = 2\sigma_3 \begin{pmatrix} q \\ r \end{pmatrix}$$

$$f_1 = i \begin{pmatrix} q_x \\ r_x \end{pmatrix}$$

$$f_2 = -\frac{1}{2}\sigma_3 \begin{pmatrix} q_{xx} - 2rq^2 \\ r_{xx} - 2qr^2 \end{pmatrix}$$

$$f_3 = -\frac{i}{4} \begin{pmatrix} q_{xxx} - 6rqq_x \\ r_{xxx} - 6qrr_x \end{pmatrix}$$

$$f_4 = \frac{1}{8}\sigma_3 \begin{pmatrix} q_{xxxx} - 3r(q^2)_{xx} - 2q(rq)_{xx} + 6r^2q^3 \\ r_{xxxx} - 3q(r^2)_{xx} - 2r(qr)_{xx} + 6q^2r^3 \end{pmatrix}$$

$$f_5 = \frac{i}{16} \begin{pmatrix} q_{xxxxx} - 10[(rqq_{xx})_x + q_x(rq)_{xx} - r_xq_xq_x] + 30q_x(rq)^2 \\ r_{xxxxx} - 10[(qrr_{xx})_x + r_x(rq)_{xx} - q_xr_xr_x] + 30r_x(qr)^2 \end{pmatrix}$$

...

$$F_0 = F = \begin{pmatrix} qt \\ r_t \end{pmatrix}$$

$$F_1 = \frac{i}{2}\sigma_3 \begin{pmatrix} q_{xt} - 2q\frac{\partial}{\partial t} \int^x dyrq \\ r_{xt} - 2r\frac{\partial}{\partial t} \int^x dyrq \end{pmatrix}$$

$$F_2 = -\frac{1}{4} \begin{pmatrix} q_{xxt} - 2q(rq)_t - q(rq_t - qr_t) - 2q_x\frac{\partial}{\partial t} \int^x dyrq + q\frac{\partial}{\partial t} \int^x dy(r_yq - q_yr) \\ r_{xxt} - 2r(qr)_t - r(qr_t - rqt) - 2r_x\frac{\partial}{\partial t} \int^x dyrq + r\frac{\partial}{\partial t} \int^x dy(q_yr - r_yq) \end{pmatrix}$$

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**Resumen.** La ecuación de LAKNS se resuelve exactamente usando un método de operadores matriciales diferente del método desarrollado por Kingston y Rogers. Se obtienen series de ecuaciones fundamentales no lineales y sus representaciones de operadores, y a partir de sus combinaciones lineales se construyen otras ecuaciones no lineales. Se muestra que las ecuaciones no lineales en la ecuación de LAKNS contienen solamente la derivada temporal en primer orden y operadores espaciales diferenciales e integrales que actúan sobre las funciones  $q$  y  $r$ .