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Cosmological interaction?

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Abstract. A very simple recurrent formula for estimating the dimensionless coupling constants of four elementary interactions is given. The formula suggests the existence of one unknown interaction. This hypothetical interaction, referred to as cosmological, would be much weaker than the gravitational one. Its coupling constant is defined and estimated. Analogous results are obtained and briefly discussed in the unified electroweak version.

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1. Introduction

Some physicists play a game in which powers of physical fundamental constants are multiplied to obtain dimensionless numbers (for part of the bibliography see [1]; and also [2,3]). They hope that those numbers might be the key to the puzzles of nature, though they are aware of the optional character of such a game. Some of them combine the game with deeper physical considerations [2,3] some others practise only the pure arithmetical exercises. The latter case is commonly called numerology. Some people strongly criticize numerology, even very wittily [4], as a fruitless pursuit [5]. Some others strongly argue in favour of it pointing out the cases that were the origins of important ideas and theories in physics (*e.g.* [6]; the Balmer formula seems to be the most spectacular example). The opponents maintain that numerology does not help us in the understanding of nature. The physical theories, however, do not help us in this either since they are only models describing nature in an approximate way. They enable us to obtain certain formulae, such as in numerology, in the framework of some logical systems (theories), while in numerology we obtain formulae incidentally. This is the only difference between the theories and numerology. The importance of this difference is left to the reader's appreciation. It seems that the statistical verification is a sound approach to numerology [7,8].

The present paper also belongs to such a game. It concerns the strength of the elementary interactions, and the values of the dimensionless coupling constants are used as a kind of measure of this strength. Since these values are not well determined, we shall use the less obligatory term "factor" instead of the obligatory term "constant". These values are discussed in Section 2. In Section 3 a very simple recurrent formula determining these values is given. The formula suggests the existence of one unknown interaction, much weaker than the gravitational one, which we shall refer to as cosmological. The coupling factor of the cosmological interaction is defined and estimated in Section 4. In Section 5 an analogous formula for the unified electro weak interaction case is given and briefly discussed.

2. Dimensionless coupling factors

The elementary interactions have been classified into four kinds, namely gravitational, weak, electromagnetic, and strong. This classification reflects the empiric fact that some intervals of values on the scale (continuous?) of interaction strength are preferred by nature. The tendency to unify all the interactions is not contradictory to this classification. Indeed, if the unification were successful, then this would only mean that all the interactions could be described in one theory including, perhaps, only one coupling constant. However, this would not change the empiric fact hitherto existing and mentioned above, since it could only be changed due to new observations, if they were made.

Those interactions are characterized by their coupling constants. To make comparison possible among the interactions by means of these constants, the latter must have the same dimension. The simplest way of achieving this is to use the dimensionless constants, so this has commonly been done. Therefore in the following we shall speak of dimensionless coupling constants only.

Let α_1, α_2, α_3, and α_4 denote the dimensionless coupling constants of the gravitational, weak, electromagnetic and strong interactions, respectively. α_1, α_2, and α_3 have been defined as products of powers of the appropriate fundamental constants in the framework of the game mentioned in Section 1.

Constant α_1 has been assumed to be comparable with $G\hbar^{-1}c^{-1}m_p^2$ [9], where G is the gravitational constant, $\hbar$ is the Planck constant (reduced), c is the speed of light in vacuum, and m_p is the proton mass. However, the question arises why it is just the proton mass which should be used. For instance, if the electron mass m_e were used instead, then α_1 would be smaller by six orders of magnitude. The situation in the hydrogen atom has commonly been used to compare the gravitational and electric forces, *i.e.* the ratio of the gravitational to electric forces occurring between the proton and electron is taken into account [2,3]. In this case we would have $\alpha_1 \simeq G\hbar^{-1}c^{-1}m_pm_e = 3.215 \times 10^{-42}$ instead of the mentioned $G\hbar^{-1}c^{-1}m_p^2 = 5.904 \times 10^{-39}$. We see that the estimation of α_1 is fairly arbitrary. Following the above course of reasoning we can assume that

$$3 \times 10^{-42} \lesssim \alpha_1 \lesssim 6 \times 10^{-39} \tag{1}$$

The dimensional coupling constant of the weak interactions has been very precisely determined as $G_F\hbar^{-3}c^{-3} = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2} = 4.544\ \mathrm{erg}^{-2}$ [10]. To obtain the dimensionless α_2, the quantity $4.544\ \mathrm{erg}^{-2}$ must be multiplied by the factor m^2c^4. However, the question arises here: m is the mass of what? In accordance with the game, m should be the rest mass of an elementary particle. The electron seems to be the best candidate (and it has been used indeed, *e.g.* [11]) since it is often involved in the weak interaction phenomena and $G_F\hbar^{-3}m_e^2c = 3.046 \times 10^{-12}$ is relatively near to the value of $\sim 10^{-13}$ given by Schweber [9]. Thus we can assume that

$$10^{-13} \lesssim \alpha_2 \lesssim 3 \times 10^{-12}. \tag{2}$$

Constant α_3 is an exception among α_k's ($k = 1, 2, 3, 4$) since it has commonly been assumed that it is a precisely determined quantity; namely α_3 has been assumed to be (or calculated as, this depends on the point of view) identical with the fine structure constant $\alpha := e^2\hbar^{-1}c^{-1} = 137.036^{-1}$, where e is the electron charge magnitude. Constant α is considered to be a specially important quantity in physics [1,2], having in the opinion of some physicists an almost mystic meaning. Thus we have

$$\alpha_3 = \alpha = 137.036^{-1} = 7.297 \times 10^{-3}. \tag{3}$$

Constant α_4 has not been expressed, as far as I know, as a product of powers of fundamental constants. Its value has only been directly estimated as ~ 0.3 by Huang [12] and as ~ 1 by Schweber [9]. Thus we can assume that

$$0.3 \lesssim \alpha_4 \lesssim 1. \tag{4}$$

The value of ~ 15 has also been admitted for the strong interaction dimensionless coupling constant (*e.g.*, p. 293 in [9]) but it is commonly treated as secondary with respect to the proper one of the order of magnitude 1. Figuratively, it is treated as, *e.g.*, the van der Waals forces that are secondary with respect to the basic electromagnetic ones.

It is seen that α_k's are not good constants, except for α_3, since they are determined fairly arbitrarily with a high degree of uncertainty, even of few orders of magnitude in the cases of α_1 and α_2. Thus, to be in accordance with semantics, we should not use the obligatory term "constant", but use a less obligatory term, *e.g.* "factor". Therefore we shall call α_k's *dimensionless coupling factors.* We assume that they are running (in appropriate value intervals) quantities serving us as a kind of measure to estimate the elementary interaction strengths with accuracy up to one or even several orders of magnitude.

Factors α_k given by relations (1)–(4), and listed in line 1 of Table I, can be treated as a kind of empiric data since their values had empiric origins.

3. The formula

Let us assume the following recurrent formula:

$$\alpha_{n+1} = \alpha_n^{f(n)}, \qquad f(n) = (n! \times 3)^{-1}, \tag{5}$$

where n's are natural numbers.

	α_0 Cosmological	α_1 gravitational	α_2 weak	α_3 electromagnetic	α_4 strong
Values given by relations (1)–(4)	—	$\sim 3 \times 10^{-42}$ $- \sim 6 \times 10^{-39}$	$\sim 10^{-13}$ $- \sim 3 \times 10^{-12}$	7.297×10^{-3}	$\sim 0.3 - \sim 1$
Values given by equations (3) and (5)	4.08×10^{-116}	3.44×10^{-39}	1.51×10^{-13}	7.297×10^{-3}	0.761
Values given by relations (5) and (8)	$\sim 2 \times 10^{-123}$ $- \sim 3 \times 10^{-120}$	$\sim 10^{-41}$ $- \sim 1.5 \times 10^{-40}$	$(\sim 2 - \sim 5) \times 10^{-14}$	$(\sim 5 - \sim 6) \times 10^{-3}$	~ 0.75

TABLE I .

Since the formula is recurrent, it is necessary to assume one of α_n's as an initial quantity. α_3 given by Eq. (3) is of course the best choice since it is precise and commonly accepted. Then from relations (3) and (5) we obtain the results presented in line 2 of Table I (for α_0 see below). They are in good accordance with the empiric-like (see the last sentence of section 2) line 1 of Table I, *i.e.* with relations (1)–(4).

Let use note two properties of formula (5).

i) The first one is the following. If we take an arbitrary value of α_n ($n = 0, 1, 2, 3, 4$) from those given in Table I as the initial value, then for every $n > 4$ the value of α_n obtained from formula (5) is very close to unity. This means by relation (4) that all the possible further interactions ($n = 5, 6, 7, \ldots$), if they existed, would only be the strong ones. In other words, formula (5) says that the strong interactions are the strongest ones.

ii) The second property is such that formula (5) admits one step in the opposite direction, *i.e.* for $n < 1$, namely it admits $n = 0$ ($0! = 1$). Does this mean that there exists an unknown elementary interaction weaker than the gravitational one? In section 4 we conduct heuristic considerations in favour of such a hypothesis.

Concluding, Eq. (5) admits five and only five kinds of elementary interactions though its domain includes all the natural numbers n.

4. The cosmological interaction

It seems that the physical fundamental constants may be divided into "better" and "worse" ones. The "better" constants are those genuinely universal such as G, h (or $\hbar$), and c, while the "worse" ones are those characterizing the particles such as e or the rest masses of particles. In fact, we have seen in Section 2 what problems are involved in the estimation of the coupling factors if the masses of particles are used. Thus we shall be using only the "better" constants. By using only the constants G, h and c we are unable to obtain dimensionless quantities; this is possible, however, if we additionally use the cosmological constant Λ, which is genuinely a universal constant.

Let us define

$$\alpha_0 := Ghc^{-9}\Lambda. \tag{6}$$

In accordance with the game, Eq. (6) can be treated as a definition of the coupling constant (dimensionless) of a hypothetical interaction, which we can call *cosmological interaction.* If it existed, it would be much weaker than the gravitational one. Since both signs are taken into account for Λ, for simplicity we shall discuss the values of Λ in the meaning of $|\Lambda|$.

In distinction from G, h and c that are determined with high precision, the constant Λ is only roughly estimated. Estimations based on the mean density of matter in the universe [13,14] give 1.3×10^{-58} cm^{-2} $\lesssim \Lambda \lesssim 1.8\times10^{-56}$ cm^{-2} if the data on pp. 396 and 397 in [14] are considered. Peach [15] has estimated $\Lambda \lesssim 2\times10^{-55}$ cm^{-2}. Thus we can assume that

$$10^{-58}\mathrm{cm}^{-2} \lesssim \Lambda \lesssim 2\times10^{-55}\mathrm{cm}^{-2}, \tag{7}$$

which by Eq. (6) gives

$$2\times10^{-123} \lesssim \alpha_0 \lesssim 3\times10^{-120}. \tag{8}$$

Such uncertainty of the α_0 value obliges us, in accordance with what was said in Section 2, to refer to α_0 as a coupling factor, not coupling constant, α_0 could be named constant if Λ were determined as a sufficiently precise quantity.

Using the α_0 values from Eq. (8) as the initial ones, we obtain from Eq. (5) the α_n's given in line 3 of Table I, where we also see a fairly good accordance with the empiric-like line 1. There might be another cosmological candidate to define α_0, namely the Hubble constant H (if it is a constant) if the definition $\alpha_0 := Ghc^{-5}H^2$ is assumed. If the best recent estimation of H as $60\pm$ 20 km $s^{-1}Mpc^{-1} = (1.95\pm 0.65)\times10^{-18}s^{-1}$ [16] is used, then we obtain values of α_0 in a value interval included in that given by Eq. (8). Would this be an additional suggestion in favour of the existence of the cosmological interaction (even if $\Lambda = 0$ as it is supposed by some people)?

From estimation (7) we see that a great divergence occurs when establishing the value of Λ. It is not excluded that Λ could be greater (p. 63 in [4]). If we assume after Section 3 the standpoint that α_3 should be the initial quantity, and besides that the cosmological constant and interaction exist in nature, then by Eqs. (3) and (5) we get $\alpha_0 = 4.08\times10^{-116}$ (line 2 of Table I), and hence by Eq. (6) we find that $\Lambda = 2.5\times10^{-51}$ cm^{-2}.

5. Additional comments

We can construct a recurrent formula very similar to formula (5) and repeat our considerations also in the case of unification of the weak and electromagnetic interactions. As it was said in Section 2, the unification concerns the description of nature while the empiric differentiation of the preferred interaction strengths remains. Thus we maintain our standpoint represented in the previous sections, including Eq. (5), and what is said below should be treated as a formal game.

Let β_0, β_1, β_2, and β_3 denote the dimensionless coupling factors of the cosmological, gravitational, electroweak, and strong interactions, respectively. Thus β_0 and α_0, β_1 and α_1, and β_3 and α_4 have the same meaning, respectively. In the unification the electroweak coupling constant has been assumed to be (or calculated as, this depends on the point of view) identical with α, *i.e.,* we have by Eqs. (3) that $\beta_2 = \alpha_3 = \alpha$. Thus the empiric-like line 1 of Table I is the same for β_i's and α_k's are taken into account and α_2 is ignored.

The mentioned formula is

$$\beta_{n+1} = \beta_n^{g(n)}, \qquad g(n) := (3n! \times 3)^{-1}. \tag{9}$$

Let us consider the following equations

$$\beta_0 = \alpha_0, \qquad \beta_1 = \alpha_1, \qquad \beta_2 = \alpha_3. \tag{10}$$

It is easy to see that if any one of them is assumed, then the remaining two result from Eqs. (5) and (9). Thus, if Eq. (9) is used and columns α_2 and α_4 of Table I are ignored, then lines 2 and 3 of Table I are the same in the α's and β's cases. As regards β_3, having the meaning of α_4, Eqs. (9) and (10) give $\beta_3 = 0.998$ in line 2 (initial quantity β_2) and $\beta_3 \simeq 1$ in line 3 (initial quantity β_0).

Eq. (9) has properties analogous to those of Eq. (5) mentioned in Section 3. For the appropriate initial values it gives β_n's very close to unity for every $n > 3$, *i.e.* Eq. (9) admits only four interactions.

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Resumen. Se presenta una fórmula recursiva muy sencilla para evaluar las constantes de acoplamiento sin dimensión. La fórmula sugiere la existencia de una interacción desconocida. Esta interacción hipotética llamada cosmológica, debería ser más débil que la gravitacional. La constante de acoplamiento correspondiente es definida y estimada. Resultados análogos son obtenidos y brevemente discutidos en la versión electro-débil unificada.