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Lanczos Potential and Liénard-Wiechert's field

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Abstract. With the aid of the Newman-Penrose formalism the Lanczos spintensor for some spacetimes and the Weert superpotential for the bound part of the Liénard-Wiechert's electromagnetic field are obtained.

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1. Introduction

In this work we are interesting in constructing potentials for the Weyl tensor and for the Liénard-Wiechert electromagnetic field produced by a puntual charge in arbitrary motion.

The present work is organized as follows. The section 2 has a short exposition of the conventions used in this work. In section 3 we write down, in the Newman-Penrose formalism (NP), the basical equations conecting the Lanczos' potential with the conformal tensor and with an energy tensor in the electromagnetic case. In section 4 we use the technics described in 3 and the Minkowski and Newman-Unti [5] coordinates to obtain the Weert potential [9] for the bound part of the Maxwell tensor associated to the Liénard-Wiechert field. We write down also the superpotential for the corresponding radiative part. Finally, we construct the Lanczos spintensor for some metrics and remark that in all of these examples the NP components of the spintensor are lineal combinations of the spin coefficients in section 5.

2. Conventions in the Newman-Penrose formalism

We shall use the null tetrad formalism [1], so that we consider useful to write the conventions used here.

The null tetrad is written as

$$({z_{(a)}}^r) = (m^r, \bar{m}^r, \ell^r, n^r), \qquad a = 1, \ldots, 4 \tag{1.a}$$

with signature $(+,+,+,-)$, so the orthornormality conditions are

$$({z_{(a)}}^r z_{(b)r}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{1.b}$$

The spin coefficients are given as

$$\begin{array}{lll} \kappa = \gamma_{414}, & \tau = \gamma_{413}, & \lambda = \gamma_{232} \\ \rho = \gamma_{412}, & \nu = \gamma_{233}, & \pi = \gamma_{234} \\ \sigma = \gamma_{411}, & \mu = \gamma_{231}, & \epsilon = \frac{1}{2}(\gamma_{434} + \gamma_{214}) \\ \gamma = \frac{1}{2}(\gamma_{433} + \gamma_{213}), & \alpha = \frac{1}{2}(\gamma_{432} + \gamma_{212}), & \beta = \frac{1}{2}(\gamma_{431} + \gamma_{211}) \end{array} \tag{1.c}$$

which are function of the rotation coefficients

$$\gamma_{abc} = -\gamma_{bac} = -z_{(a)r;t}{z_{(b)}}^r {z_{(c)}}^t \tag{1.d}$$

(semicolon denote covariant derivative).

The Riemann tensor is written as:

$$R^i{}_{jkm} = \Gamma^i{}_{jm,k} - \Gamma^i{}_{jk,m} + \Gamma^i{}_{ck}\Gamma^c{}_{jm} - \Gamma^i{}_{cm}\Gamma^c{}_{jk} \tag{2.a}$$

and the Ricci tensor and the scalar curvature are defined as following

$$R_{jk} = R^i{}_{jki} \quad \text{Ricci}, \quad R = R^a_a \quad \text{scalar curvature}. \tag{2.b}$$

The Weyl tensor is defined in terms of the Riemann tensor

$$C_{ajkm} = R_{ajkm} + \frac{1}{2}(R_{ak}g_{jm} + R_{jm}g_{ak} - R_{jk}g_{am} - R_{am}g_{jk}) + \frac{R}{6}(g_{am}g_{jk} - g_{ak}g_{jm}), \tag{2.c}$$

with the following symmetries

$$C_{ajkm} = -C_{jakm} = -C_{ajmk}, \quad C_{ajkm} + C_{akmj} + C_{amjk} = 0, \quad C_{a\ jm}^{\ j} = 0. \quad (2.d)$$

The Weyl tensor has 10 real-independent components, therefore we can define 5 complex quantities:

$$\begin{aligned} \psi_0 &= C_{abjr} n^a m^b n^j m^r \ , \qquad \psi_1 = C_{abjr} n^a \ell^b n^j m^r, \\ \psi_2 &= -C_{abjr} \ell^a \bar{m}^b n^j m^r \ , \qquad \psi_3 = C_{abjr} \ell^a n^b \ell^j \bar{m}^r, \\ \psi_4 &= C_{abjr} \ell^a \bar{m}^b \ell^j \bar{m}^r, \end{aligned} \quad (3.a)$$

With the quantities $(3.a)$, we can write the conformal tensor

$$\begin{aligned} \tfrac{1}{2}(C_{abcd} + i^* C_{abcd}) = \psi_0 U_{ab} U_{cr} &+ \psi_1 (U_{ab} M_{cr} + U_{cr} M_{ab}) \\ &+ \psi_2 (M_{ab} M_{cr} + V_{ab} U_{cr} + V_{cr} U_{ab}) \\ &+ \psi_3 (V_{ab} M_{cr} + V_{cr} M_{ab}) + \psi_4 V_{ab} V_{cr}, \end{aligned} \quad (3.b)$$

where

$$\begin{aligned} {}^*C_{abcd} &= \tfrac{1}{2} \eta_{abjk} C^{jk}_{\ \ cd}, \qquad \eta_{abjk} = -\sqrt{-g}\,\epsilon_{abjk} \\ V_{ab} &= n_a m_b - n_b m_a, \qquad U_{ab} = -\ell_a \bar{m}_b + \ell_b \bar{m}_a \\ M_{ab} &= m_a \bar{m}_b - m_b \bar{m}_a - n_a \ell_b + n_b \ell_a. \end{aligned} \quad (3.c)$$

And finally we write down the four covariant operators generated from $(1.a)$

$$\delta = m^a \nabla a, \qquad \bar{\delta} = \bar{m}^a \nabla a, \qquad \Delta = \ell^a \nabla a, \qquad D = n^a \nabla a \quad (4)$$

3. Lanczos' spintensor

Lanczos [2] and Bampi-Caviglia [3] found that in all spacetime there exist a tensor K_{ijb} with the algebraic symmetries

$$\begin{aligned} K_{aij} &= -K_{iaj}, \\ K_{a\ r}^{\ r} &= 0 \\ K_{aij} + K_{ija} + K_{jai} &= 0, \end{aligned} \quad (5.a)$$

fulfilling the differential property

$$K_{ab}{}^{c}{}_{;c} = 0 \tag{5.b}$$

being K_{ijb} a superpotential for the Weyl tensor, *i.e.*

$$\begin{aligned} C_{pqjb} = K_{pqj;b} - K_{pqb;j} + K_{jbp;q} - K_{jbq;p} + g_{pb}T_{jq} \\ - g_{pj}T_{qb} + g_{qj}T_{pb} - g_{qb}T_{pj}, \end{aligned} \tag{5.c}$$

where

$$T_{jr} = K_{j}{}^{a}{}_{r;a}. \tag{5.d}$$

Note that all the relations (5) are in agree with (2.d), and that from (5) the relation for T_{jr}

$$T_{jr} = T_{rj}, \qquad T^{r}{}_{r} = 0 \tag{6}$$

hold.

We write now Ecs. (5) in NP formalism. In order to do so, we observe the symmetries (5.a) implying only 16 real components for the Lanczos spintensor, *i.e.*, 8 complex components which are:

$$\begin{aligned} \Omega_0 &= K_{(1)(4)(4)}, & \Omega_4 &= K_{(1)(4)(1)} \\ \Omega_1 &= K_{(1)(4)(2)}, & \Omega_5 &= K_{(1)(4)(3)} \\ \Omega_2 &= K_{(3)(2)(4)}, & \Omega_6 &= K_{(3)(2)(1)} \\ \Omega_3 &= K_{(3)(2)(2)}, & \Omega_7 &= K_{(3)(2)(3)} \end{aligned} \tag{7.a}$$

where we have used the notation $K_{(a)(b)(c)} = K_{pqt}Z^{p}_{(a)}Z^{q}_{(b)}Z^{t}_{(c)}$.

Now we compute the quantities $\frac{1}{2}(K_{abc} + i^{*}K_{abc})$, and arrive at

$$\begin{aligned} \tfrac{1}{2}(K_{abc} + i^{*}K_{abc}) = {} & \Omega_0 U_{ab}\ell_c + \Omega_1(M_{ab}\ell_c - U_{ab}m_c) \\ & + \Omega_2(V_{ab}\ell_c - M_{ab}m_c) - \Omega_3 V_{ab}m_c - \Omega_4 U_{ab}\bar{m}_c \\ & + \Omega_5(U_{ab}n_c - M_{ab}\bar{m}_c) + \Omega_6(M_{ab}n_c - V_{ab}\bar{m}_c) \\ & + \Omega_7 V_{ab}n_c, \end{aligned} \tag{7.b}$$

where

$${}^{*}K_{abc} = \tfrac{1}{2}\eta_{abpq}K^{pq}{}_{c}. \tag{7.c}$$

From the differential equation (5.b) one finds the following relations

$$\Delta\Omega_2 - \delta\Omega_3 - \bar{\delta}\Omega_6 - 2\nu\Omega_1 + (3\mu + \bar{\mu} + \gamma - \bar{\gamma})\Omega_2 + (\bar{\alpha} - 3\beta + \tau - \bar{\pi})\Omega_3 + D\Omega_7 + 2\lambda\Omega_5 + (-\alpha - \bar{\beta} + \bar{\tau} - 3\pi)\Omega_6 + (3\epsilon + \bar{\epsilon} - \rho - \bar{\rho})\Omega_7 = 0$$

$$\Delta\Omega_0 - \delta\Omega_1 - \bar{\delta}\Omega_4 + D\Omega_5 + (\mu + \bar{\mu} - 3\gamma - \bar{\gamma})\Omega_0 + (3\tau - \bar{\pi} + \bar{\alpha} + \beta)\Omega_1 - 2\sigma\Omega_2 + (3\alpha - \bar{\beta} + \bar{\tau} - \pi)\Omega_4 + (\bar{\epsilon} - \epsilon - \bar{\rho} - 3\rho)\Omega_5 + 2\kappa\Omega_6 = 0,$$

$$-\Delta\Omega_1 + \delta\Omega_2 + \bar{\delta}\Omega_5 - D\Omega_6 + \nu\Omega_0 + (\gamma + \bar{\gamma} - 2\mu - \bar{\mu})\Omega_1 + (-\bar{\alpha} + \beta - 2\tau + \bar{\pi})\Omega_2 + \sigma\Omega_3 - \lambda\Omega_4 + (-\alpha + \bar{\beta} - \bar{\tau} + 2\pi)\Omega_5 + (-\epsilon - \bar{\epsilon} + 2\rho + \bar{\rho})\Omega_6 - \kappa\Omega_7 = 0 \quad (8.a)$$

and from (5.c) we obtain

$$\psi_0 = 2[-\delta\Omega_0 + D\Omega_4 + (\bar{\alpha} + 3\beta - \bar{\pi})\Omega_0 - 3\sigma\Omega_1 + (-3\epsilon + \bar{\epsilon} - \bar{\rho})\Omega_4 + 3\kappa\Omega_5],$$

$$2\psi_1 = -\Delta\Omega_0 - 3\delta\Omega_1 + \bar{\delta}\Omega_4 + 3D\Omega_5 + (3\gamma + \bar{\gamma} + 3\mu - \bar{\mu})\Omega_0 + 3(\bar{\alpha} + \beta - \bar{\pi} - \tau)\Omega_1 - 6\sigma\Omega_2 + (-3\alpha + \bar{\beta} - 3\pi - \bar{\tau})\Omega_4 + 3(-\epsilon + \bar{\epsilon} + \rho - \bar{\rho})\Omega_5 + 6\kappa\Omega_6,$$

$$\psi_2 = -\Delta\Omega_1 - \delta\Omega_2 + \bar{\delta}\Omega_5 + D\Omega_6 + \nu\Omega_0 + (2\mu - \bar{\mu} + \gamma + \bar{\gamma})\Omega_1 + (\bar{\alpha} - \beta - \bar{\pi} - 2\tau)\Omega_2 - \sigma\Omega_3 - \lambda\Omega_4 + (-\alpha + \bar{\beta} - 2\pi - \bar{\tau})\Omega_5 + (\epsilon + \bar{\epsilon} - \bar{\rho} + 2\rho)\Omega_6 + \kappa\Omega_7,$$

$$2\psi_3 = -3\delta\Omega_2 - \delta\Omega_3 + 3\bar{\delta}\Omega_6 + D\Omega_7 + 3(-\bar{\mu} + \mu + \bar{\gamma} - \gamma)\Omega_2 + 6\nu\Omega_1 + (\bar{\alpha} - 3\beta - 3\tau - \bar{\pi})\Omega_3 - 6\lambda\Omega_5 + 3(\alpha + \bar{\beta} - \bar{\tau} - \pi)\Omega_6 + (3\epsilon + \bar{\epsilon} - \bar{\rho} + 3\rho)\Omega_7,$$

$$\psi_4 = 2[-\Delta\Omega_3 + \bar{\delta}\Omega_7 + 3\nu\Omega_2 + (-\bar{\mu} - 3\gamma + \bar{\gamma})\Omega_3 - 3\lambda\Omega_6 + (3\alpha + \bar{\beta} - \bar{\tau})\Omega_7], \quad (8.b)$$

Ecs. (8) are also written by Zund [4], however this author has some typographical mistakes.

In order to simplify Ecs. (8) we combine Ecs. (8.a) and (8.b). One arrives at the Weyl-Lanczos-equations

$$\psi_0 = 2[-\delta\Omega_0 + D\Omega_4 + (\bar{\alpha} + 3\beta - \bar{\pi})\Omega_0 - 3\sigma\Omega_1 + (-3\epsilon + \bar{\epsilon} - \bar{\rho})\Omega_4 + 3\kappa\Omega_5]$$

$$\psi_1 = 2[-\delta\Omega_1 + D\Omega_5 + \mu\Omega_0 + (\bar{\alpha} + \beta - \bar{\pi})\Omega_1 - 2\sigma\Omega_2 - \pi\Omega_4 + (\bar{\epsilon} - \epsilon - \bar{\rho})\Omega_5 + 2\kappa\Omega_6],$$

$$\psi_2 = 2[-\delta\Omega_2 + D\Omega_6 + 2\mu\Omega_1 + (\bar\alpha - \beta - \bar\pi)\Omega_2 - \sigma\Omega_3 - 2\pi\Omega_5 + (\bar\epsilon + \epsilon - \bar\rho)\Omega_6 + \kappa\Omega_7],$$

$$\psi_3 = 2[-\Delta\Omega_2 + \bar\delta\Omega_6 + 2\nu\Omega_1 + (-\bar\mu + \bar\gamma - \gamma)\Omega_2 - \tau\Omega_3 - 2\lambda\Omega_5 + (\alpha + \bar\beta - \bar\tau)\Omega_6 + \rho\Omega_7],$$

$$\psi_4 = 2[-\Delta\Omega_3 + \bar\delta\Omega_7 + 3\nu\Omega_2 + (-\bar\mu - 3\gamma + \bar\gamma)\Omega_3 - 3\lambda\Omega_6 + (3\alpha + \bar\beta - \bar\tau)\Omega_7],$$

$$\psi_1 = 2\left[-\Delta\Omega_0 + \bar\delta\Omega_4 + (-\bar\mu + 3\gamma + \bar\gamma)\Omega_0 - 3\tau\Omega_1 + (-3\alpha + \bar\beta - \bar\tau)\Omega_4 + 3\rho\Omega_5\right],$$

$$\psi_2 = 2\left[-\Delta\Omega_1 + \bar\delta\Omega_5 + \nu\Omega_0 + (\gamma + \bar\gamma - \bar\mu)\Omega_1 - 2\tau\Omega_2 - \lambda\Omega_4 + (-\alpha + \bar\beta - \bar\tau)\Omega_5 + 2\rho\Omega_6\right],$$

$$\psi_3 = 2[-\delta\Omega_3 + D\Omega_7 + 3\mu\Omega_2 + (\bar\alpha - 3\beta - \bar\pi)\Omega_3 - 3\pi\Omega_6 + (3\epsilon + \bar\epsilon - \bar\rho)\Omega_7] \qquad (9)$$

Ecs. (9) have the same information as the Ecs. (8). The Lanczos' potential is not unique, therefore can be found several solutions of (9). Finally we write the components of the tensor T_{ab} in NP formalism. One finds that

$$T_{(1)(1)} = \delta(\Omega_5 - \bar\Omega_2) - \Delta\Omega_4 + D\bar\Omega_3 + \bar\nu\Omega_0 + \bar\lambda(2\bar\Omega_1 - \Omega_1) + (-\bar\alpha + \beta - 3\bar\pi)\bar\Omega_2 + (-\bar\rho - \epsilon + 3\bar\epsilon)\bar\Omega_3 + (-\mu - \bar\gamma + 3\gamma)\Omega_4 + (\bar\alpha - \beta - 3\tau)\Omega_5 + \sigma(2\Omega_6 - \bar\Omega_6) + \kappa\bar\Omega_7,$$

$$T_{(1)(2)} = -\delta\bar\Omega_5 - \bar\delta\Omega_5 + D(\Omega_6 + \bar\Omega_6) + \bar\mu\Omega_1 + \mu\bar\Omega_1 - \bar\pi\Omega_2 - \pi\bar\Omega_2 + \lambda\Omega_4 + \bar\lambda\bar\Omega_4 + (\alpha - \bar\beta - 2\pi)\Omega_5 + (\bar\alpha - \beta - 2\bar\pi)\bar\Omega_5 + (\epsilon + \bar\epsilon - 2\rho)\Omega_6 + (\epsilon + \bar\epsilon - 2\bar\rho)\bar\Omega_6 + \kappa\Omega_7 + \bar\kappa\bar\Omega_7,$$

$$T_{(1)(3)} = \delta\Omega_6 + \bar\delta\bar\Omega_3 - \Delta(\bar\Omega_2 + \Omega_5) + \bar\nu(\Omega_1 + 2\bar\Omega_1) - \bar\lambda\Omega_2 + (\gamma - \bar\gamma - 3\bar\mu)\bar\Omega_2 + (3\bar\beta - \alpha - \bar\tau)\bar\Omega_3 + \nu\Omega_4 + (\gamma - \bar\gamma - 2\mu)\Omega_5 + (\bar\alpha + \beta - 2\tau)\Omega_6 - \tau\bar\Omega_6 + \sigma\Omega_7 + \rho\bar\Omega_7,$$

$$T_{(1)(4)} = -\delta\bar\Omega_1 - \bar\delta\Omega_4 + D(\bar\Omega_2 + \Omega_5) + \bar\mu\Omega_0 - \bar\pi\Omega_1 + (\bar\alpha + \beta - 2\bar\pi)\Omega_1 + \bar\lambda\bar\Omega_0 + (\bar\epsilon - \epsilon - \bar\rho)\bar\Omega_2 + \bar\kappa\bar\Omega_3 + (3\alpha - \bar\beta - \pi)\Omega_4 + (\bar\epsilon - \epsilon - 3\rho)\Omega_5 - \sigma\bar\Omega_5 + \kappa(2\Omega_6 + \bar\Omega_6),$$

$$T_{(3)(3)} = \delta\Omega_7 + \bar\delta\bar\Omega_7 - \Delta(\Omega_6 + \bar\Omega_6) + \bar\nu\Omega_2 + \nu\bar\Omega_2 - \bar\lambda\Omega_3 - \lambda\bar\Omega_3 + 2(\nu\Omega_5 + \bar\nu\bar\Omega_5) - (\gamma + \bar\gamma + 3\nu)\Omega_6 - (\gamma + \bar\gamma + 3\bar\mu)\bar\Omega_6 + (-\tau + \bar\alpha + 3\beta)\Omega_7$$

$$+(-\bar{\tau}+\alpha+3\bar{\beta})\bar{\Omega}_7,$$

$$T_{(4)(4)} = -\delta\bar{\Omega}_0 - \bar{\delta}\Omega_0 + D(\Omega_1+\bar{\Omega}_1) + (3\alpha+\bar{\beta}-\pi)\Omega_0 + (3\bar{\alpha}+\beta-\bar{\pi})\bar{\Omega}_0$$
$$-(\epsilon+\bar{\epsilon}+3\rho)\Omega_1 - (\bar{\epsilon}+\epsilon+3\bar{\rho})\bar{\Omega}_1 + 2(\kappa\Omega_2+\bar{\kappa}\bar{\Omega}_2) - \bar{\sigma}\Omega_4$$
$$-\sigma\bar{\Omega}_4 + \bar{\kappa}\Omega_5 + \kappa\bar{\Omega}_5. \qquad (10)$$

In the following we make some applications of (9) and (10).

4. Liénard-Wiechert field

If we introduce in the Minkowski metric

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2, \qquad (11)$$

the Newman-Unti [5] coordinates (θ, ϕ, r, u), defined as $(\eta \equiv \theta + i\phi)$:

$$x = q^1(u) + \frac{r}{2\sqrt{2}p}(\bar{\eta}+\eta), \qquad y = q^2(u) + \frac{ir}{2\sqrt{2}p}(\bar{\eta}-\eta)$$
$$z = q^3(u) + \frac{r}{2\sqrt{2}p}(\eta\bar{\eta}-1), \qquad t = q^4(u) + \frac{r}{2\sqrt{2}p}(\eta\bar{\eta}+1)$$
$$p = \frac{1}{2\sqrt{2}}\left[\dot{q}^4 + \dot{q}^3 + (\dot{q}^4-\dot{q}^3)\eta\bar{\eta} - (\dot{q}^1 - i\dot{q}^2)\eta - (\dot{q}^1 + i\dot{q}^2)\bar{\eta}\right] \qquad (12.a)$$

where $q^a(u)$ is an arbitrary time-like curve, u being its corresponding proper-time, one finds

$$ds^2 = \frac{r^2}{2p^2}d\eta d\bar{\eta} - 2drdu - \left(1 - \frac{2\dot{p}}{p}r\right)du^2, \qquad (12.b)$$

(dot denotes $\partial/\partial u$). If we identify $q^a(u)$ with the path of a puntual-charge in arbitrary motion, then this charge q has an electromagnetic field (see [5] and [6])

$$(A_c) = q\left(0, 0, -\frac{1}{r}, \frac{\dot{p}}{p} - \frac{1}{r}\right), \qquad (13.a)$$

corresponding to the Liénard-Wiechert solution in Newman-Unti [5] coordinates.

The Faraday tensor $F_{bc} \equiv A_{c,b} - A_{b,c}$ of $(13.a)$ is

$$F_{14} = q\frac{\partial}{\partial\theta}\left(\frac{\dot{p}}{p}\right), \quad F_{24} = q\frac{\partial}{\partial\phi}\left(\frac{\dot{p}}{p}\right), \quad F_{34} = \frac{q}{r^2}, \qquad (13.b)$$

vanishing on other case.

Teitelboim [7] found that the Maxwell tensor

$$T_{ab} = F_{ac}F_b{}^c - \tfrac{1}{4}(F_{pq}F^{pq})g_{ab} \tag{14.a}$$

for the Liénard-Wiechert case admit a splitting in two tensors

$$T_{ab} = \underset{B}{T}_{ab} + \underset{R}{T}_{ab} \tag{14.b}$$

where $\underset{B}{T}_{ab}$ and $\underset{R}{T}_{ab}$ respectively are the bound and radiative parts of T_{ab}. This tensors fulfill (6) and are dynamically independent outside of the universe line of the charge

$$\underset{B}{T}_a{}^c{}_{;c} \equiv 0, \tag{14.c}$$

$$\underset{R}{T}_a{}^c{}_{;c} \equiv 0. \tag{14.d}$$

A superpotential for the Einstein's canonical pseudotensor was found by Freud [8]. Inspired in this fact we seek a superpotential $\underset{B}{K}_{jrc}$ with the properties (5.*a*), (5.*b*), (5.*d*) and

$$\underset{B}{T}_{jr} = \underset{B}{K}_j{}^a{}_{r;a}, \tag{15.a}$$

i.e. we construct a superpotential K_{abc} of Lanczos' type for the bound part of the Liénard-Wiechert tensor. In Newman-Unti coordinates one can show that

$$\begin{aligned}
&\underset{B}{T}_{11} = \underset{B}{T}_{22} = \frac{q^2}{4p^2r^2}, \qquad && \underset{B}{T}_{14} = \frac{q^2}{r^2}\frac{\partial}{\partial\theta}\left(\frac{\dot{p}}{p}\right), \\
&\underset{B}{T}_{34} = \frac{q^2}{2r^4}, && \underset{B}{T}_{24} = \frac{q^2}{r^2}\frac{\partial}{\partial\phi}\left(\frac{\dot{p}}{p}\right) \\
&\underset{B}{T}_{44} = \frac{q^2}{2r^4}\left(1 - 2\frac{\dot{p}}{p}r\right).
\end{aligned} \tag{15.b}$$

To solve (15.*a*) is equivalent to solve (10). To do so, we use the NP tetrad

$$(m^a) = \frac{p}{r}(i, -1, 0, 0), \quad (\ell^a) = (0, 0, -\frac{1}{2} + \frac{\dot{p}}{p}r, 1) \quad (n^a) = (0, 0, 1, 0), \tag{15.c}$$

then

$$\tau = \kappa = \sigma = \pi = \epsilon = \lambda = 0, \qquad \rho = 2\mu = -\frac{1}{r}$$

$$\gamma = -\frac{\dot{p}}{2p}, \qquad \nu = 2ip\frac{\partial}{\partial\eta}\left(\frac{\dot{p}}{p}\right), \qquad \alpha = -\bar{\beta} = -\frac{i}{r}\frac{\partial p}{\partial\eta}. \tag{15.d}$$

Additionaly is $\underset{B}{T}_{(a)(b)} = 0$ except for

$$\underset{B}{T}_{(1)(2)} = \frac{q^2}{2r^4}\,, \qquad \underset{B}{T}_{(1)(3)} = -\frac{q^2}{r^3}\bar{\nu}. \tag{15.e}$$

Using (15.c–15.e) in (10) we find the solution

$$\underset{B}{\Omega}_a = 0, \quad a \neq 6,7, \quad \underset{B}{\Omega}_6 = -\frac{q^2}{4r^3}\,,\ \underset{B}{\Omega}_7 = \frac{q^2}{r^2}\nu. \tag{15.f}$$

Substituting (15.c) and (15.f) into (7.b) we compute the corresponding potential $\underset{B}{K}_{jbc}$ of Weert [9]

$$\underset{B}{K}_{jbc} = -\frac{q^2}{4}\omega^{-4}\left[\omega^{-1}(3-4W)(V_j \times K_b)K_c + 4(a_j \times K_b)K_c \right.$$
$$\left. + g_{cj}K_b - g_{cb}K_j\right] \tag{16.a}$$

where we have used the Lowry [10] notation:

$$A_c \times B_j \equiv A_cB_j - A_jB_c, \tag{16.b}$$

and

$$V^c = \frac{1}{2}(n^c + 2\ell^c), \quad \omega = r = -K^c v_c, \quad p^c = r^{-l}K^c - V^c,$$

$$K^c = rn^c, \qquad W = -K^c a_c = r\frac{\dot{p}}{p}, \tag{16.c}$$

being V^c, a^c and w the four-velocity and acceleration and the retarded distance respectively (see Fig. 1). In Fig. 1 q^c is the retarded point associated to x^c.

Using Minkowski and Newman-Unti coordinates in (16.a) one arrives at:

$$\underset{B}{T}_{jc} = q^2\omega^{-4}\left[\tfrac{1}{2}g_{jc} + (K_ja_c + K_ca_j) + B(K_jV_c + K_cV_j)\right.$$
$$\left. - \omega^{-2}(1-2W)K_jK_c\right], \qquad B = \omega^{-1}(1-W); \tag{17.a}$$

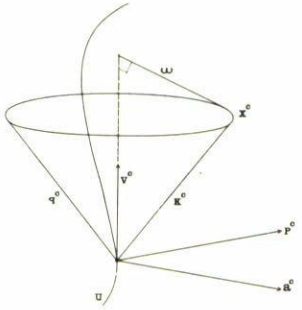

FIGURE 1.

(we have used the notation $,c = \partial/\partial X^c$) also

$$u_{,c} = -\omega^{-1} K_c. \tag{17.b}$$

This potentials are not unique, because if we adhere in (16.a) the superpotential

$$\underset{B}{\tilde{K}}_{jbc} = -q^2\omega^{-3}\left[W(V_j \times K_b)V_c + (a_j \times K_b)(-\omega V_c + K_c) + W(g_{cj}K_b - g_{cb}K_j)\right], \tag{18}$$

(15.a) remains invariant. It is easy to see that $\underset{B}{\tilde{K}}{}_j{}^c{}_{b,c} = 0$.

The investigation of Eq. (14.d) is rather complicated. We want to construct a superpotential $\underset{B}{K}_{jbc}$ for the radiative part of T_{jb}, that means

$$\underset{R}{T}_{jb} = \underset{R}{K}{}_j{}^c{}_{b,c}. \tag{19.a}$$

One finds that $\underset{R}{K}_{jbc}$ can be written as

$$\underset{R}{K}_{jbc} = \frac{q^2}{4}\omega^{-2}\left[(a_j \times K_b)(a_c + 4\omega^{-2}WK_c) + 3\omega^{-2}W^2(g_{cj}K_b - g_{cb}K_j) - \omega^{-1}W(V_j \times K_b)(a_c + \omega^{-2}WK_c) - 2qF_{jb}p_{(\sigma)}p_{(\gamma)}\int_0^u a_{(\sigma)}a_{(\gamma)}V_c du, \quad \sigma,\gamma = 1,2,3 \right. \tag{19.b}$$

where there exist sum in σ and γ. u represents the proper time for the retarded event and $p(\sigma)$ and $a(\sigma)$ are the p^c and a^c components upon a Fermi-tetrad $e_{(\sigma)c}$,

$\sigma = 1,2,3$ in the universe line $(e_{(\sigma)c}V^c = 0)$:

$$p_{(\sigma)} = p^c e_{(\sigma)c}, \quad a_{(\sigma)} = a^c e_{(\sigma)c}, \quad \frac{d}{du} e_{(\sigma)}{}^c = a_{(\sigma)} V^c \, . \tag{19.c}$$

For the Liénard-Wiechert field the relation

$$F_j{}^b p_{(\sigma),b} = 0, \qquad \sigma = 1,2,3 \tag{19.d}$$

holds. Note the nonlocal character of (19.b), it depends on the historically path of the charge because of the radiative effects. $\underset{R}{K_{jbc}}$ does not fulfill relations (5.a) and (5.b), except the antisymmetry relation in the jb subindex.

Substituting (19.b) into (19.a) and using Minkowski coordinates one arrives at

$$\underset{R}{T_{jb}} = q^2\omega^{-4}(a^2 - \omega^{-2}W^2)K_jK_b, \quad a^2 = a^c a_c. \tag{19.e}$$

Ecs. (14.b), (15.a), (19.a) and (19.e) imply

$$T_{jb} = q^2\omega^4 \left[K_jU_b + K_bU_j + (a^2 - B^2)K_jK_b + \tfrac{1}{2}g_{jb}\right] \tag{20.a}$$

$$= \left(\underset{B}{K_j{}^c{}_b} + \underset{R}{K_j{}^c{}_b} \right)_{,c} , \tag{20.b}$$

where

$$U_c = BV_c + a_c. \tag{20.c}$$

Ec. (20.a) was found by Synge [11]. Ec. (20.b) shows that the Maxwell tensor is an exact divergence for the Liénard-Wiechert field. The question what the Weert potential means is still open.

5. Lanczos' potential for some metrics

In this section we obtain the Lanczos superpotential for some well known spacetimes. That is equivalent to solve the equation system (9) for some given metric.

In the following, we only write down the null tetrad for each case, and only when it is necessary we show some spin coefficients. Knowing Ω_j, $j = 0, \ldots, 7$, the corresponding K_{jbc} spintensor can be obtained from (7.b).

1. Gödel metric [12]

$$ds^2 = -(dx^1)^2 - 2(e^{x^4})dx^1dx^2 - \tfrac{1}{2}(e^{2x^4})(dx^2)^2 + (dx^3)^2 + (dx^4)^2$$

$$(m^r) = \left(1, -(e^{-x^4}), 0, \frac{i}{\sqrt{2}}\right), \quad (\ell^r) = \frac{1}{\sqrt{2}}(1,0,-1,0)$$

$$(n^r) = \frac{1}{\sqrt{2}}(1,0,1,0), \qquad \mu = \rho = \frac{i}{2}.$$

then

$$\Omega_r = 0, \quad r \neq 1,6, \qquad \Omega_1 = \Omega_6 = \tfrac{1}{9}\mu \tag{21.a}$$

or

$$\Omega_r = 0, \quad r \neq 1,3,6, \quad \Omega_1 = \Omega_6 = \tfrac{1}{9}\mu, \quad \Omega_3 = c\exp\left(-2x^4 - \frac{i}{\sqrt{2}}x^3\right), \tag{21.b}$$

where c is an arbitrary constant. Substituting $(21.a)$ into $(7.b)$ one finds

$$K_{pqj} = \frac{\sqrt{2}}{12}\eta_{pqab}L^{ab}{}_j$$

$$L_{abj} = \left(e_{(3)a} \times e_{(4)b}\right)e_{(4)j} + \tfrac{1}{3}\left(e_{(3)a}g_{bj} - e_{(3)b}g_{aj}\right)$$

$$(e_{(3)r}) = (0,0,1,0), \qquad (e_{(4)r}) = (-1, -e^{x^4}, 0, 0), \tag{21.c}$$

$(21.c)$ can be compared with the result of Novello-Velloso [13].
2. Taub metric [14]

$$ds^2 = f^{-1}((dx^1)^2 - (dx^4)^2) + f^2((dx^2)^2 + (dx^3)^2),$$

$$f = \sqrt{1 + Kx^1}, \quad K = \text{const.} \neq 0$$

$$(m^r) = \frac{1}{\sqrt{2f}}(0,1,-i,0), \qquad (\ell^r) = \sqrt{\frac{f}{2}}(-1,0,0,1)$$

$$(n^r) = \sqrt{\frac{f}{2}}(1,0,0,1), \quad \rho = \mu = 4\epsilon = 4\gamma = -\frac{K}{2\sqrt{2}}f^{-3/2}$$

then

$$\Omega_r = 0, \qquad r \neq 1,6, \qquad \Omega_1 = \Omega_6 = \tfrac{1}{6}\mu \tag{22.a}$$

3. Schwarzschild metric

$$ds^2 = \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \operatorname{sen}^2\theta d\phi^2) - \left(1 - \frac{2M}{r}\right)dt^2$$

$$(m^a) = \frac{1}{\sqrt{2}r}(0,1,-\frac{i}{\operatorname{sen}\theta},0), \quad (\ell^a) = \frac{1}{\sqrt{2}}\left(-\sqrt{1-\frac{2M}{r}},0,0,\frac{1}{\sqrt{1-\frac{2M}{r}}}\right)$$

$$(n^a) = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{2m}{r}},0,0,\frac{1}{\sqrt{1-\frac{2m}{r}}}\right), \quad \gamma = \epsilon = \frac{M}{2\sqrt{2}}r^2\left(1-\frac{2M}{r}\right)^{-1/2} \qquad (22.b)$$

therefore $\Omega_a = 0$, $a \neq 1,6$, $\Omega_1 = \Omega_6 = \frac{2}{3}\epsilon$

4. Kasner metric [15] (x,y,z,t)

$$ds^2 = t^{2p_1}dx^2 + t^{2p_2}dy^2 + t^{2p_3}dz^2 - dt^2$$

$$p_1 + p_2 + p_3 = 1, \qquad {p^2}_1 + {p^2}_2 + {p^2}_3 = 1$$

$$(m^r) = \frac{1}{\sqrt{2}}\left(t^{-p_1}, -it^{-p_2}, 0, 0\right), \quad (\ell^r) = \frac{1}{\sqrt{2}}\left(0,0,-t^{-p_3},1\right)$$

$$(n^r) = \frac{1}{\sqrt{2}}\left(0,0,t^{-p_3},1\right), \qquad \epsilon = \frac{p_3}{2\sqrt{2}}\frac{1}{t}$$

then

$$\Omega_a = 0, \qquad a \neq 1,6, \qquad \Omega_1 = -\Omega_6 = \tfrac{2}{3}\epsilon \qquad (22.c)$$

5. C metric (x,y,ϕ,τ)

$$ds^2 = (x+y)^{-2}\left(f^{-1}dx^2 + h^{-1}dy^2 + fd\phi^2 - hdt^2\right)$$

$$f = x^3 + ax + b\,, \qquad h = y^3 + ay - b\,,$$

$$(m^r) = \frac{(x+y)}{\sqrt{2}}\left(\sqrt{f},0,\frac{i}{\sqrt{f}},0\right), \qquad (\ell^r) = \frac{(x+y)}{\sqrt{2}}\left(0,-\sqrt{h},0,\frac{1}{\sqrt{h}}\right)$$

$$(n^r) = \frac{(x+y)}{\sqrt{2}}\left(0,\sqrt{h},0,\frac{1}{\sqrt{h}}\right), \qquad \mu = \rho = \sqrt{\frac{h}{2}}$$

$$\tau = -\pi = \sqrt{\frac{f}{2}}, \quad \epsilon = \gamma = -\frac{1}{2}\sqrt{\frac{h}{2}} + \frac{1}{4\sqrt{2h}}(3y^2+a)(x+y)$$

$$\alpha = -\beta = \frac{1}{2}\sqrt{\frac{f}{2}} - \frac{1}{4\sqrt{2f}}(3x^2+a)(x+y)$$

where a and b are constants. Then

$$\Omega_0 = -\Omega_7 = -\frac{\pi}{4}\,, \qquad \Omega_1 = \Omega_6 = \frac{\epsilon}{3} + \frac{\rho}{12}$$

$$\Omega_2 = -\Omega_5 = -\frac{\beta}{3} + \frac{\pi}{12}, \qquad \Omega_3 = \Omega_4 = -\frac{\rho}{4} \tag{22.d}$$

6. Petrov metric [16]

$$ds^2 = f^{4/3}((dx^1)^2 + (dx^2)^2) + f^{-2/3}(dx^3)^2 - (dx^4)^2, \quad f = kx^4 + 1,$$

$$(m^r) = \frac{1}{\sqrt{2}}(f^{-2/3}, -if^{-2/3}, 0, 0), \quad (\ell^r) = \frac{1}{\sqrt{2}}(0, 0, -f^{1/3}, 1)$$

$$(n^r) = \frac{1}{\sqrt{2}}(0, 0, f^{1/3}, 1), \quad \rho = -\mu = 4\epsilon = -\frac{\sqrt{2}k}{3f},$$

being k a constant. One finds that

$$\Omega_a = 0, \qquad a \neq 1, 6, \qquad \Omega_1 = -\Omega_6 = -\tfrac{1}{6}\mu. \tag{22.e}$$

7. Kaigorodov metric [17] (x, y, v, u)

$$ds^2 = 2(kx)^{-2}(dx^2 + dy^2) - 2du\left(dv + 2vx^{-1}dx + xdu\right),$$

$$(m^a) = k\left(\frac{x}{2}, \frac{ix}{2}, -v, 0\right), \qquad (\ell^a) = \left(0, 0, -\sqrt{x}, \frac{1}{\sqrt{x}}\right)$$

$$(n^a) = (0, 0, \sqrt{x}, 0), \qquad \tau = -\pi = \frac{\nu}{3} = \frac{k}{2}$$

then

$$\Omega_a = 0, \qquad a \neq 7, \qquad \Omega_7 = -\pi \tag{22.f}$$

8. Siklos metric [18] (x, y, r, u)

$$ds^2 = r^2x^{-3}(dx^2 + dy^2) - 2dudr + \frac{3}{2}xdu^2$$

$$(m^a) = \frac{x^{3/2}}{\sqrt{2}r}(1, -i, 0, 0)\,, \qquad (\ell^a) = \left(0, 0, -\frac{\sqrt{3x}}{2}, -\frac{2}{\sqrt{3x}}\right)$$

$$(n^a) = \left(0, 0, -\frac{\sqrt{3x}}{2}, 0\right), \qquad \rho = -\mu = \frac{\sqrt{3x}}{2r}, \qquad \beta = -\frac{\alpha}{2} = -\frac{\sqrt{x}}{2\sqrt{2r}}$$

then

$$\Omega_a = 0, \qquad a \neq 3, 7, \qquad \Omega_3 = \frac{1}{4}\mu, \qquad \Omega_7 = \beta$$

or

$$\Omega_0 = \Omega_3 = 0, \qquad \Omega_4 = -3\Omega_1 = -\frac{\rho}{6}, \qquad \Omega_5 = -\Omega_2 = \tfrac{1}{2}\Omega_7 = -\frac{\alpha}{6} \qquad (22.g)$$

9. Gravitational plane waves

$$ds^2 = (dx^1)^2 + (dx^2)^2 - 2dx^3 dx^4 + H(x^1, x^2, x^4)(dx^4)^2, \quad H_{,11} + H_{,22} = 0$$

$$(m^r) = \frac{1}{\sqrt{2}}(1, -i, 0, 0), \qquad (\ell^r) = (0, 0, \sqrt{H}, 0)$$

$$(n^r) = (0, 0, \sqrt{H}, \frac{1}{\sqrt{H}}, \qquad \kappa = \frac{1}{\sqrt{2}H}(H_{,1} - iH_{,2})$$

one finds

$$\Omega_a = 0, \qquad a \neq 0, \qquad \Omega_0 = \frac{\kappa}{2} \qquad (22.h)$$

10. Spherical gravitational waves (θ, ϕ, r, u)

$$ds^2 = \frac{r^2}{4\theta^3}\left(d\theta^2 + d\phi^2\right) - 2dudr + \left(6\theta + \frac{2M}{r}\right)du^2$$

$$(m^a) = \frac{\sqrt{2}}{r}\theta^{3/2}(1, i, 0, 0), \qquad (\ell^a) = \left(0, 0, 3\theta + \frac{M}{r}, 1\right)$$

$$(n^a) = (0, 0, 1, 0), \qquad \rho = -\frac{1}{r}$$

$$\alpha = -\beta = \frac{3\sqrt{2}}{4r}\theta^{1/2}, \qquad \mu = \frac{3\theta}{r} + \frac{M}{r^2}$$

then

$$\Omega_0 = \Omega_7 = 0, \qquad \Omega_4 = -3\Omega_1 = -\tfrac{1}{2}\rho$$

$$\Omega_2 = -\Omega_5 = \tfrac{2}{3}\alpha, \qquad \Omega_3 = -3\Omega_6 = -\tfrac{1}{2}\mu \qquad (22.i)$$

11. McLenaghan-Tariq [19] and Tupper [20] metric

$$ds^2 = \frac{a^2}{x^2}(dx^2 + dy^2) + x^2 d\phi^2 - (dt - 2yd\phi)^2 \qquad a = \text{const.}$$

$$(m^r) = \frac{x}{\sqrt{2}a}(1, -i, 0, 0), \qquad (\ell^r) = \left(0, 0, -\frac{1}{\sqrt{2}x}, -\frac{1}{\sqrt{2}} - \frac{\sqrt{2}y}{x}\right)$$

$$(n^r) = \left(0, 0, \frac{1}{\sqrt{2}x}, -\frac{1}{\sqrt{2}} + \frac{\sqrt{2}y}{x}\right),$$

$$\kappa = -\nu = \frac{1+2i}{2\sqrt{2}a}, \qquad \alpha = -\beta = \frac{1-i}{2\sqrt{2}a}$$

then

$$\Omega_c = 0, \qquad c \neq 0, 7, \qquad \Omega_0 = -\Omega_7 = \tfrac{2}{5}(2\kappa - \bar{\alpha}) \tag{22.j}$$

12. Petrov Type I metric

$$ds^2 = e^f \left(\cos\theta (dx^1)^2 - 2\,\mathrm{sen}\,\theta dx^1 dx^4 - \cos\theta (dx^4)^2\right) + (dx^2)^2 + e^{-2f}(dx^3)^2$$

$$f = Kx^2, \qquad K = \text{const.} \qquad \theta = \sqrt{3}f$$

$$(m^r) = \frac{1}{\sqrt{2}}\left(0, 1, -ie^f, 0\right), \qquad (\ell^r) = \frac{e^{-f/2}}{\sqrt{2}}\left(-\sqrt{\cos\theta}, 0, 0, \frac{1+\mathrm{sen}\,\theta}{\sqrt{\cos\theta}}\right)$$

$$(n^r) = \frac{e^{-f/2}}{\sqrt{2}}\left(\sqrt{\cos\theta}, 0, 0, \frac{1-\mathrm{sen}\,\theta}{\sqrt{\cos\theta}}\right), \qquad \tau = -\pi = -\frac{K}{2\sqrt{2}}$$

$$\kappa = -\frac{K}{2}\sqrt{\frac{3}{2}}\frac{1-\mathrm{sen}\,\theta}{\cos\theta}, \qquad \nu = -\frac{K}{2}\sqrt{\frac{3}{2}}\frac{1+\mathrm{sen}\,\theta}{\cos\theta}$$

$$\alpha = \frac{K}{4\sqrt{2}}\frac{2\cos\theta - \sqrt{3}}{\cos\theta}, \qquad \beta = -\frac{K}{4\sqrt{2}}\frac{2\cos\theta + \sqrt{3}}{\cos\theta},$$

then

$$\Omega_a = 0, \qquad a = 1, 3, 4, 6, \qquad \Omega_0 = \frac{\kappa}{2}, \qquad \Omega_2 = -\frac{\tau}{6}$$

$$\Omega_5 = \tfrac{1}{12}(4\tau + \alpha - \beta), \qquad \Omega_7 = \frac{\nu}{2} \tag{22.k}$$

13. Novotný-Horský metric [21] (x, y, z, t)

$$ds^2 = \mathrm{sen}^{4/3}(az)(dx^2 + dy^2) + dz^2 - \cos^2(az)\,\mathrm{sen}^{-2/3}(az)\,\mathrm{sen}^{-2/3}(az)dt^2,$$

$$(m^r) = \frac{1}{\sqrt{2}}\mathrm{sen}^{-2/3}(az)(1, -i, 0, 0), \qquad (\ell^r) = \frac{1}{\sqrt{2}}\left(0, 0, -1, \mathrm{sen}^{1/3}(az)\sec(az)\right)$$

$$(n^r) = \frac{1}{\sqrt{2}}\left(0, 0, 1, \mathrm{sen}^{1/3}(az)\sec(az)\right), \qquad \mu = \rho = -\frac{a\sqrt{2}}{3}\cot(az)$$

$$\epsilon = \gamma = -\frac{a\sqrt{2}}{4}\left[\tan(az) + \frac{1}{3}\cot(az)\right]$$

where a is a constant. Then

$$\Omega_r = 0, \qquad r \neq 1, 6, \qquad \Omega_1 = \Omega_6 = \tfrac{1}{9}(2\epsilon + \rho) \tag{22.l}$$

Note that Ecs. (21.a), (22.a),...,(22.l) show an internal relation between the Lanczos spintensor and the rotation coefficients. The question wether it yields always so it is possible to find a null-tetrad, so that the quantities Ω_r are lineal combinations of the spin coefficients, has not yet been answered until now. In fact, the 18 NP equations are rather useful to solve (9).

It must be also interesting to compute the Lanczos potential for the Kerr-metric.

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Resumen. Con la ayuda del formalismo de Newman-Penrose obtenemos espintensores de Lanczos para diversos espacio-tiempos, así como el superpotencial de Weert para la parte acotada del campo de Liénard-Wiechert.